Happy Friday! Please reflect on the week :)

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 1/29 Topic:	0 1 2	I rested after FINALS :)
Tuesday Date: 1/30 Topic:	0 1 2	New Semester!
Wednesday Date: 1/31 Topic: 1A & 2A: Exponent Notation	0 1 2	
Thursday Date: 2/1 Topic: 2B: Index Laws (Mult./Power/Division)	0 1 2	
Friday Date: 2/2 Topic: 2B: Index Laws (Mult./Power/Division)	0 1 2	

Unit 5: Exponentials and Logarithms

Warm-up: Simplify the expression.

$(2n^3)^2 \cdot (n^4)^3 = 4n^6 \cdot n^{12} = \boxed{4n^{18}}$

$(2n^3)(2n^3) \cdot n^4 \cdot n^4 \cdot n^4 = \boxed{4n^{18}}$

Class Plan:

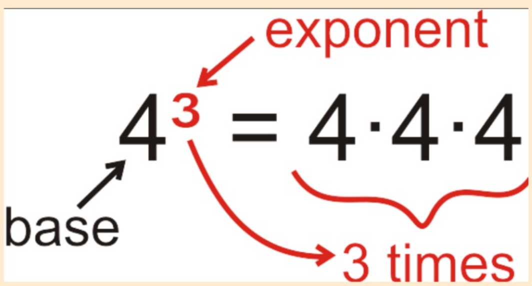
1. Warm-up
2. Laws of Indices Investigation (Zero and Negative Exponents)
3. Video Break
4. Practice

Today's joke!

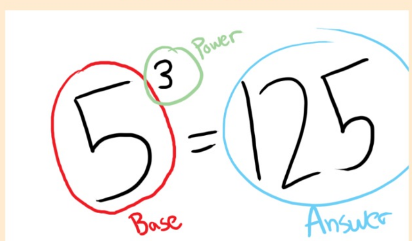
I'd like to buy a new boomerang please.
Also, can you tell me how to throw the old one away?

Unit 5: Exponentials and Logarithms

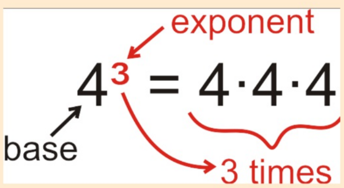
- What is an exponent?
- What is the base?



{Expanded form.}



Investigation:Index Laws



(Calculator may help as well)

<--- Use expanded form to discover the laws/properties.

(Properties of Exponents)

$$b^0 = \square$$

$$b^{-m} = \frac{\square}{\square}$$

$$\frac{1}{b^m} = \square$$

When done: Record in notebook & show teacher

Property Investigation

Original = expanded form = Simplified Exonential form

$$\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = x^{\boxed{0}} = ?$$

1. Rewrite each expression in expanded form (***shown above***). Then rewrite it in ***simplified*** exponential form with a single base.

a) $\frac{4^3}{4^3} = \frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = 4^{\boxed{0}} = 1$ **b)** $\frac{x^2}{x^2} = \frac{x \cdot x}{x \cdot x} = x^{\boxed{0}} = 1$

2. Examine the simplified form and the exponents in parts **a** and **b** of #1.

i. Write a rule for the exponent of ***zero***.

$$b^0 = \boxed{1}$$

Answers to Investigation

Original = expanded form = Simplified Exonential form

$$\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = x^{\boxed{0}} = ?$$

1. Rewrite each expression in expanded form (***shown above***). Then rewrite it in ***simplified*** exponential form with a single base.

a) $\frac{4^3}{4^3} = \frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = 4^{\boxed{0}} = 1$ b) $\frac{x^2}{x^2} = \frac{x \cdot x}{x \cdot x} = x^{\boxed{0}} = 1$

2. Examine the simplified form and the exponents in parts **a** and **b** from # 1.

i. Write a rule for the exponent of **zero.**

$$b^0 = \boxed{1}$$

Property Investigation

3. Rewrite in expanded form. Then rewrite it in ***simplified*** exponential form with a single base.

a) $\frac{3^2}{3^5} = \frac{3 \cdot 3}{3\,3\,3\,3\,3} = \frac{1}{3^{\boxed{3}}} = 3^{\boxed{-3}}$

b) $\frac{r}{r^3} = \frac{r}{r\,r\,r} = \frac{1}{r^{\boxed{2}}} = r^{\boxed{-2}}$

$b^m = \frac{1}{b^{-m}}$

4. Examine the simplified form and the exponents in parts a and **b** from #3.

i. Write a rule for simplifying **negative exponents**.

$\frac{b^{-m}}{1} = \boxed{\frac{1}{b^m}}$ **AND** $\frac{1}{b^m} = \boxed{b^{-m}}$

Answers to Investigation

3. Rewrite in expanded form. Then rewrite it in ***simplified*** exponential form with a single base.

a) $\frac{3^2}{3^5} = \frac{3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{3^{\boxed{3}}} = 3^{\boxed{-3}}$

b) $\frac{r}{r^3} = \frac{r}{r \cdot r \cdot r} = \frac{1}{r^{\boxed{2}}} = r^{\boxed{-2}}$

4. Examine the simplified form and the exponents in parts **a** and **b** from 3.

i. Write a rule for the **negative property of exponents.**

$b^{-m} = \boxed{\frac{1}{b^m}}$ **AND** $\frac{1}{b^m} = \boxed{b^{-m}}$

B Chapter 2 INDEX LAWS

If the bases a and b are both positive, and the indices m and n are integers, then:

$a^m \times a^n = a^{m+n}$	To **multiply** numbers with the **same base**, keep the base and **add** the indices.
$\dfrac{a^m}{a^n} = a^{m-n}$	To **divide** numbers with the same base, keep the base and **subtract** the indices.
$(a^m)^n = a^{mn}$	When **raising** a **power** to a **power**, keep the base and **multiply** the indices.
$(ab)^n = a^n b^n$	The power of a product is the product of the powers.
$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	The power of a quotient is the quotient of the powers.
$a^0 = 1, \quad a \neq 0$	Any non-zero number raised to the power of zero is 1.

$a^{-n} = \dfrac{1}{a^n}$ and in particular $a^{-1} = \dfrac{1}{a}$.

B Chapter 2 INDEX LAWS

Product Property of Exponents

$$a^m \cdot a^n = a^{m+n}$$

Quotient Property of Exponents

$$\frac{a^m}{a^n} = a^{m-n}$$

Definition of Negative Exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Zero Exponents

$$a^0 = 1$$

Power of a Power Property

$$(a^m)^n = a^{mn}$$

Power of a Product Property

$$(ab)^m = a^m b^m$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

***<u>Important:</u> Recognize how connected the properties are.

B Video Break **INDEX LAWS**

1) How do positive exponents relate to the negative exponents??

2) Simplify:

Positive exponents!

$x^{-8} \cdot x^{5} = ?$

https://www.youtube.com/watch?v=wQmtsgRMGmU

B EXAMPLES — INDEX LAWS

1) $x^0 = 1$

2) $2 + 5^0 = 2 + 1 = 3$

3) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

4) $\left(\frac{2}{5}\right)^{-2} = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

B EXAMPLES **INDEX LAWS**

5) $\dfrac{(5x)^{-1}}{1} = \dfrac{1}{5x}$

6) $\dfrac{5x^{-1}}{1} = \dfrac{5}{x}$

7) $5^0 - 5^{-1}$

$1 - \dfrac{1}{5} = \dfrac{4}{5}$

8) $\dfrac{(3b^2)^{-2}}{1} = \dfrac{1}{(3b^2)^2} = \dfrac{1}{9b^4}$

B EXAMPLES — INDEX LAWS

Example 8

Se

Simplify:

a 7^0 **b** x^0 **c** $\frac{y^4}{y^4}$ **d** $2 + 5^0$

a $7^0 = 1$

b $x^0 = 1$ {provided $x \neq 0$}

c $\frac{y^4}{y^4} = y^{4-4}$
$= y^0$
$= 1$ {provided $y \neq 0$}

d $2 + 5^0$
$= 2 + 1$
$= 3$

Example 9

Se

Simplify using the index laws:

a $3x^2 \times 5x^5$

b $\frac{20a^9}{4a^6}$

c $\frac{b^3 \times b^7}{(b^2)^4}$

a
$$3x^2 \times 5x^5$$
$$= 3 \times 5 \times x^2 \times x^5$$
$$= 15 \times x^{2+5}$$
$$= 15x^7$$

b
$$\frac{20a^9}{4a^6} = \frac{20}{4} \times a^{9-6}$$
$$= 5a^3$$

c
$$\frac{b^3 \times b^7}{(b^2)^4} = \frac{b^{3+7}}{b^{2\times 4}}$$
$$= \frac{b^{10}}{b^8}$$
$$= b^{10-8}$$
$$= b^2$$

B EXAMPLES

INDEX LAWS

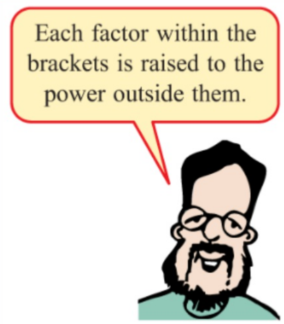
Example 11 Self Tutor

Remove the brackets of:

a $(3a)^2$ **b** $\left(\frac{2x}{y}\right)^3$

a $(3a)^2 = 3^2 \times a^2$
$= 9a^2$

b $\left(\frac{2x}{y}\right)^3 = \frac{2^3 \times x^3}{y^3}$
$= \frac{8x^3}{y^3}$



Example 12

Express in simplest form, without brackets: **a** $(3a^3b)^4$ **b** $\left(\dfrac{x^2}{2y}\right)^3$

a
$$\begin{aligned} &(3a^3b)^4 \\ &= 3^4 \times (a^3)^4 \times b^4 \\ &= 81 \times a^{3\times 4} \times b^4 \\ &= 81a^{12}b^4 \end{aligned}$$

b
$$\begin{aligned} \left(\frac{x^2}{2y}\right)^3 &= \frac{(x^2)^3}{2^3 \times y^3} \\ &= \frac{x^{2\times 3}}{8 \times y^3} \\ &= \frac{x^6}{8y^3} \end{aligned}$$

B EXAMPLES — INDEX LAWS

Example 13 — Self Tutor

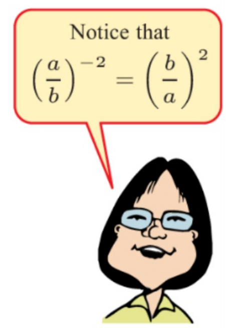
Write without brackets or negative indices:

a 2^{-2} **b** $5^0 - 5^{-1}$ **c** $\left(\frac{2}{5}\right)^{-2}$

a 2^{-2}
$= \frac{1}{2^2}$
$= \frac{1}{4}$

b $5^0 - 5^{-1}$
$= 1 - \frac{1}{5}$
$= \frac{4}{5}$

c $\left(\frac{2}{5}\right)^{-2}$
$= \left(\frac{5}{2}\right)^2$
$= \frac{25}{4}$
$= 6\frac{1}{4}$



B EXAMPLES — INDEX LAWS

Example 14 — Self Tutor

Write without brackets or negative indices:

a $(5x)^{-1}$ **b** $5x^{-1}$ **c** $(3b^2)^{-2}$

a $(5x)^{-1} = \frac{1}{5x}$

b $5x^{-1} = \frac{5}{x}$

c $(3b^2)^{-2} = \frac{1}{(3b^2)^2} = \frac{1}{3^2b^4} = \frac{1}{9b^4}$

In $5x^{-1}$ the index -1 refers to the x only.

Today's joke!

I'd like to buy a new boomerang please.

Practice (***2B, Page 28 - 32***) Rate your confidence of manipulating indices:

I'm building confidence: (**a - f**)
(4,7, 9)

I'm somewhat confident: (**a - f**)
(4, 5, 7, 9)

I'm very confident: (**a - f**)
(4, 5, 8, 10)

4 Simplify:

a 4^0 b $\left(\frac{2}{3}\right)^0$ c 12^0 d $2^0 + 2^2$

e a^0 f $(13^3)^0$ g $3y^0$ h $(3y)^0$

i $d^2 \times d^0$ j $\frac{c^7}{c^7}$ k $2 + 3^0$ l $8 - 9^0$

5 Simplify using one or more of the index laws:

a $\frac{b^4}{b}$ b $3c^2 \times 5c^3$ c $\frac{a^3b^2}{ab^2}$ d $\frac{18d^8}{3d^3}$

e $\frac{14p^3q^4}{2pq}$ f $12st^3 \times 3s^3$ g $\frac{y^{12}}{(y^2)^5}$ h $\frac{q^3 \times q^6}{(q^2)^3}$

i $\frac{6x^3y^2}{3x^3y}$ j $\frac{w^2z^4}{(wz^2)^2}$ k $\frac{a^4 \times a^2}{b(a^2)^3}$ l $\frac{16p^7q^6}{(2pq^2)^3}$

7 Remove the brackets of:

a $(pq)^3$ b $(ab)^5$ c $(xy)^4$ d $(pqr)^5$

e $(3a)^3$ f $(5b)^2$ g $(2n)^5$ h $(4ab)^3$

i $(9ab)^2$ j $\left(\frac{x}{y}\right)^4$ k $\left(\frac{a}{b}\right)^7$ l $\left(\frac{c}{2d}\right)^6$

m $\left(\frac{3x}{y^2}\right)^2$ n $\left(\frac{w^2}{2v}\right)^2$ o $\left(\frac{2r}{3s^2}\right)^3$ p $\left(\frac{3a^2}{ab}\right)^4$

8 Express in simplest form, without brackets:

a $(3a^2)^2$ **b** $\left(\frac{2}{p^2q}\right)^4$ **c** $(3cd^4)^3$ **d** $\left(\frac{a^4}{3n^7}\right)^2$

e $\left(\frac{2x^2}{y^6}\right)^3$ **f** $(2s^3t^4)^5$ **g** $\left(\frac{4g^5}{h^2}\right)^2$ **h** $\left(\frac{3x^2y^3}{4z}\right)^4$

9 Write without brackets or negative indices:

a 2^{-1} **b** 3^{-1} **c** 6^{-1} **d** 8^{-1}

e 3^2 **f** 3^{-2} **g** 2^3 **h** 2^{-3}

i 5^2 **j** 5^{-2} **k** 10^2 **l** 10^{-2}

10 Write without brackets or negative indices:

a $\frac{3^2}{3^4}$ **b** $\frac{2^{10}}{2^{15}}$ **c** $\left(4^{-1}\right)^2$ **d** $\left(7^2\right)^{-1}$

e $\left(\frac{1}{3}\right)^{-1}$ **f** $\left(\frac{2}{5}\right)^{-1}$ **g** $\left(\frac{4}{3}\right)^{-1}$ **h** $\left(\frac{1}{12}\right)^{-1}$

i $\left(3\frac{1}{2}\right)^{-1}$ **j** $3^0 - 3^{-1}$ **k** $7^{-1} + 7^0$ **l** $2^0 + 2^1 + 2^{-1}$

m $\left(\frac{3}{4}\right)^{-2}$ **n** $5 + 5^{-2}$ **o** $\left(\frac{5}{2}\right)^{-2}$ **p** $\left(2\frac{1}{3}\right)^{-2}$

Solutions to 2B

EXERCISE 2B

1	**a** 5^6	**b** 6^{11}	**c** a^6	**d** a^{10}
	e b^{14}	**f** a^{2+n}	**g** b^{m+9}	**h** p^7
2	**a** 3^7	**b** 7^4	**c** 5^3	**d** a^5
	e b^6	**f** p^{n-2}	**g** y^{5-b}	**h** b^{2n-1}
3	**a** 2^{18}	**b** 3^{21}	**c** 5^{28}	**d** a^{12}
	e q^9	**f** d^{6n}	**g** x^{8y}	**h** g^{10a}

4 **a** 1 **b** 1 **c** 1 **d** 5 **e** 1 provided $a \neq 0$
f 1 **g** 3 provided $y \neq 0$ **h** 1 provided $y \neq 0$
i d^2 provided $d \neq 0$ **j** 1 provided $c \neq 0$ **k** 3 **l** 7

5	**a** b^3	**b** $15c^5$	**c** a^2	**d** $6d^5$
	e $7p^2q^3$	**f** $36s^4t^3$	**g** y^2	**h** q^3
	i $2y$	**j** 1	**k** $\frac{1}{b}$	**l** $2p^4$

Solutions to 2B

6	a	2^4	b	3^3	c	5^3	d	2^6
	e	3^6	f	2^{a+2}	g	7^{t-1}	h	2^{4k}
	i	2^{5-x}	j	5^3	k	$7^{4(a-1)}$	l	3^2
	m	3^{y-2x}	n	2^{2y-3x}	o	11^{2x}	p	5^3
7	a	p^3q^3	b	a^5b^5	c	x^4y^4	d	$p^5q^5r^5$
	e	$27a^3$	f	$25b^2$	g	$32n^5$	h	$64a^3b^3$
	i	$81a^2b^2$	j	$\frac{x^4}{y^4}$	k	$\frac{a^7}{b^7}$	l	$\frac{c^6}{64d^6}$
	m	$\frac{9x^2}{y^4}$	n	$\frac{w^4}{4v^2}$	o	$\frac{8r^3}{27s^6}$	p	$\frac{81a^4}{b^4}$

Solutions to 2B

8 **a** $9a^4$ **b** $\frac{16}{p^8q^4}$ **c** $27c^3d^{12}$ **d** $\frac{a^8}{9n^{14}}$

e $\frac{8x^6}{y^{18}}$ **f** $32s^{15}t^{20}$ **g** $\frac{16g^{10}}{h^4}$ **h** $\frac{81x^8y^{12}}{256z^4}$

9 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **d** $\frac{1}{8}$ **e** 9 **f** $\frac{1}{9}$

g 8 **h** $\frac{1}{8}$ **i** 25 **j** $\frac{1}{25}$ **k** 100 **l** $\frac{1}{100}$

10 **a** $\frac{1}{9}$ **b** $\frac{1}{32}$ **c** $\frac{1}{16}$ **d** $\frac{1}{49}$ **e** 3 **f** $2\frac{1}{2}$

g $\frac{3}{4}$ **h** 12 **i** $\frac{2}{7}$ **j** $\frac{2}{3}$ **k** $1\frac{1}{7}$ **l** $3\frac{1}{2}$

m $1\frac{7}{9}$ **n** $5\frac{1}{25}$ **o** $\frac{4}{25}$ **p** $\frac{9}{49}$

11 **a** $\frac{1}{2x}$ **b** $\frac{2}{x}$ **c** $\frac{3}{b}$ **d** $\frac{1}{3b}$ **e** $\frac{1}{4b^2}$ **f** $\frac{b^2}{4}$

g n^2 **h** $\frac{n^2}{3}$ **i** $\frac{1}{ab}$ **j** $\frac{a}{b}$ **k** $\frac{a}{b^2}$ **l** $\frac{1}{a^2b^2}$

m $\frac{1}{2ab}$ **n** $\frac{2}{ab}$ **o** $\frac{2a}{b}$ **p** a^2b^3