Welcome Back MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 2/12 Topic: Nothing due... Index Laws Quiz was Friday!	0 1 2	
Tuesday Date: 2/13 Topic: 23B Exponential Functions, 23C Graphs	0 1 2	
Wednesday Date: ______ Topic: ______________	0 1 2	
Thursday Date: ______ Topic: ______________	0 1 2	
Friday Date: ______ Topic: ______________	0 1 2	

Warm-up: For Tomorrow...

Solve a Valentine message,
or create a "Solution Key" to a Valentine Card

Solve for "i"

$$9x-7i > 3(3x-7u)$$

$$9x-7i > 9x-21u$$
$$-9x \quad -9x$$

$$\frac{-7i}{-7} > \frac{-21u}{-7}$$

$$i < 3u$$

i <3 u

GraphJam.com

Class Plan:

1. Warm-up
2. 23C Growth, 23D Decay Investigations
3. Examples

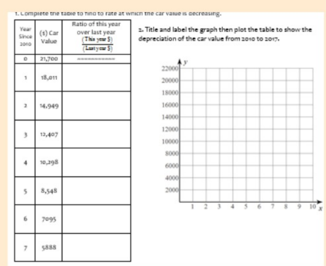
D GROWTH

E DECAY

4. Practice

Investigate: Growth *& D*ecay.

1) Choose Car value or Ant population.



1. Complete the table to find to rate at which the car value is decreasing.

Year Since 2010	($) Car Value	Ratio of this year over last year (This year $) / (Last year $)
0	21,700	------------
1	18,011	
2	14,949	
3	12,407	
4	10,298	
5	8,548	
6	7095	
7	5888	

2. Title and label the graph then plot the table to show the depreciation of the car value from 2010 to 2017.

2) Complete table & graph.

3) Examine patterns

4) Make predictions

Car value in 2024?

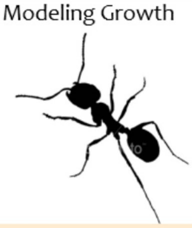
Number of ants after spring break?

5) Defend realism -

or ***unrealism.***

Investigate: Growth

Modeling Growth



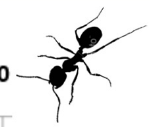
Ants have moved into the corner of our classroom! On Friday I noticed 16 ants. On Saturday Engineers found there were 24 ants. When I came to school Sunday to make copies I noticed there were 36 ants. Ms. Berg reported 54 ants in the corner on Monday!!!

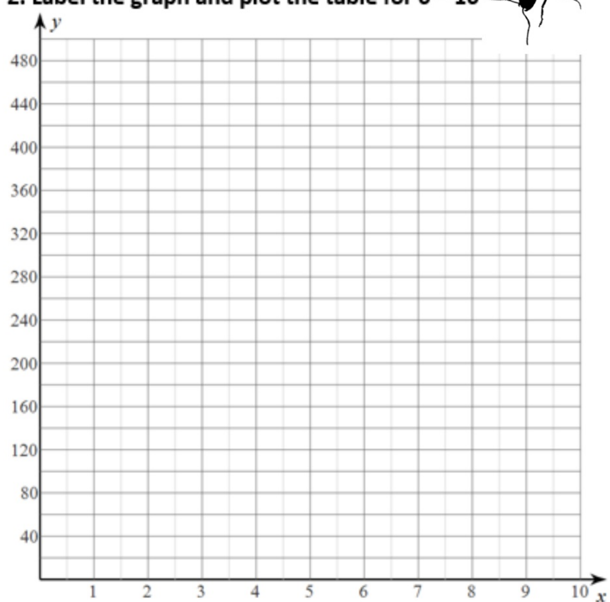
1) Verify your equation works using day 5. Show that your equation will produce 121.5 ants.
2) Use the pattern or equation to predict the population of ants after we return from parent teacher conferences. (Eek! ***11 days*** of population growth!!) Show your work.
3) Use the pattern or equation to predict the population of ants after we return from spring break. (Eek! ***58 days*** of population growth!!) Show your work.
4) When would we have 100,000,000,000 ants??? How did you determine this many days?
5) Defend whether your predictions are realistic, ***or not realistic.***

1) Create a table showing the number of ants for 0 – 10 days.

Day	Ants	Ratio of todays total to yesterday's total
0	16	~~~~~~~~~~~~~~~~~~
1	24	$\frac{24}{16}$ =
2	36	
3	54	
4		
5		
6		
7		
8		
9		
10		

2. Label the graph and plot the table for 0 – 10





Modeling Investigation, Key

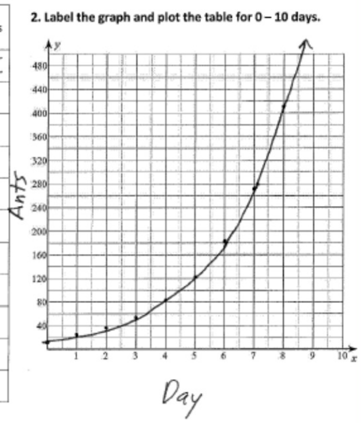
Ants have moved into the corner of our classroom! On Friday I noticed 16 ants. On Saturday Engineers found there were 24 ants. When I came to school Sunday to make copies I noticed there were 36 ants. Aidan reported 54 ants in the corner on Monday!!!

The ant population is increasing! Find the pattern of increase and predict how many ants will be in our classroom when we return from parent teacher conferences **AND** from spring break.

1) Create a table showing the number of ants for 0 – 10 days.

Day	Ants	Ratio of todays total to yesterday's total
0	16	————
1	24	$\frac{24}{16}=\frac{3}{2}$
2	36	$\frac{36}{24}=\frac{3}{2}$
3	54	$\frac{54}{36}=\frac{6}{4}=\frac{3}{2}$
4	81	$\frac{3}{2}$
5	121.5	$\frac{3}{2}$
6	182.25	$\frac{3}{2}$
7	273.375	$\frac{3}{2}$
8	410.0625	$\frac{3}{2}$
9	615.09375	$\frac{3}{2}$
10	922.640625	$\frac{3}{2}$

2. Label the graph and plot the table for 0 – 10 days.



Questions:

1) What do you notice about the graph? Describe the shape and any patterns you see.

It increases at a greater rate each day. The ratio between each day is $\frac{3}{2}$. (Exponential)

2) Writing a rule: Complete each sentence.

a) The starting population of the ants is 16

b) Each day the ant population is multiplying by a constant rate of $\frac{3}{2}$

3) Use parts **a)** and **b)** of part 2) to predict the population of ants after we return from parent teacher conferences. (Eek! ***8 days*** of population growth!!) Show your work.

$$16\cdot\left(\frac{3}{2}\right)^8 = 410.0625$$

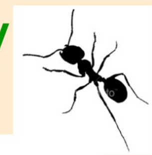
4) Use parts **a)** and **b)** of part 2) to predict the population of ants after we return from spring break. (Eek! ***59 days*** of population growth!!) Show your work.

$$16\cdot\left(\frac{3}{2}\right)^{59} \approx 3.92197\times10^{11}$$

5) Defend whether your predictions are realistic, ***or not realistic.***

This is not realistic. The biggest ant colonies ever found have had ants in the hundreds of millions, and this prediction is way bigger than that. Things like food and space will eventually limit the colony's growth.

Modeling Investigation, Key



1. verify day 5 ant pop.

$y = 16 \cdot (1.5)^5$

$y = 121.5$ ants (same as table)

122...? ½ an ant! ö

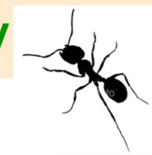
2. predict day 11 ant pop.

$y = 16 \cdot (1.5)^{11}$

$y \approx 1{,}383.96$ ants

(let's say 1,384 ☺)

Modeling Investigation, Key



3. PREDICT day 58 ANT POP.

$y = 16 \cdot (1.5)^{58}$

$y = 2.615 \times 10^{11}$

261,464,666,400 ANTS!

4. PREDICT DAY # of 100,000,000,000 ANTS (100 billion)

$1.0 \times 10^{11} = 16 \cdot (1.5)^x$

$6,250,000,000 = (1.5)^x$

$(\log_{1.5} 6250000000)$

$55.6 \approx x$ DAYS

55-56 days, as 58 days was 261 billion

Investigate: Decay

A brand new Mini Cooper cost $21,700 when it was purchased in 2010. As soon as the car is driven off the lot, the value of the car begins to depreciate- decrease in value. The table below shows the value of the car since 2010.

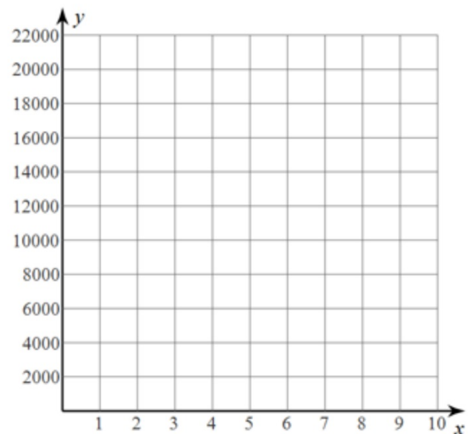
<u>Questions:</u> (Record responses in your notebook)

1) What do you notice about the graph? Describe the shape and any patterns you see.
2) Verify your equation works using 2014. Show that your equation will produce a value of $10,298.
3) Predict the value of the car in 2018. (*The value of the car after it is owned for 8 years*). Show work.
4) Predict the value of the car in 2024. Show your work.
5) When would the car be worth ***less than*** $200? How did you determine this many years?
6) Defend whether your predictions are realistic, ***or not realistic.***

1. Complete the table to find to rate at which the car value is decreasing.

Year Since 2010	($) Car Value	Ratio of this year over last year $\frac{(\text{This year \$})}{(\text{Last year \$})}$
0	21,700	~~~~~~~~~~~~
1	18,011	$\frac{18,011}{21,700} \approx$
2	14,949	
3	12,407	
4	10,298	
5	8,548	
6	7095	
7	5888	

2. Title and label the graph then plot the table to sho depreciation of the car value from 2010 to 2017.



<u>Writing a rule:</u>

a) The starting value of the car is ___________________

b) Each year the car value is multiplying by a fractional - constant rate of ___________________.

Equation: ______________________

Modeling Investigation, Key

IB MYP 9 Math Name: Key

Modeling Growth and Decay

A brand new Mini Cooper cost $21,700 when it was purchased in 2010. As soon as the car is driven off the lot, the value of the car begins to depreciate- decrease in value. The table below shows the value of the car since 2010.

1. Complete the table to find to rate at which the car value is decreasing.

Year Since 2010	($) Car Value	Ratio of this year over last year (This year $)/(Last year $)
0	21,700	----------
1	18,011	18,011/21,700 = .83
2	14,949	14,949/18,011 ≈ .83
3	12,407	12,407/14,949 ≈ .83
4	10,298	10,298/12,407 ≈ .83
5	8,548	8,548/10,298 ≈ .83
6	7095	7095/8,548 ≈ .83
7	5888	5888/7095 ≈ .83

2. Title and label the graph then plot the table to show the depreciation of the car value from 2010 to 2017.

Questions:

1) What do you notice about the graph? Describe the shape and any patterns you see.

- curve
- car value ($) is decreasing as time increases

2) Writing a rule: Complete each sentence.

a) The starting value of the car is $21,700

b) Each year the car value is multiplying by a fractional - constant rate of ≈ .83

3) Use parts a) and b) of part 2) to predict the value of the car in 2018. (*The value of the car after it is owned for 8 years*). Show your work.

$5888(.83) \approx \$4887.04$

4) Use parts a) and b) of part 2) to predict the value of the car in 2024. (*The value of the car when you may graduate from college!*) Show your work.

$y = 21{,}700(.83)^{14}$

$y \approx \$1597.91$

6) 5) Defend whether your predictions are realistic, ***or not realistic.***

From my experience with cars, they lose their value quickly!... But $20,102.09 less than the original value ö ah! (in 14 yrs)

Modeling Investigation, Key

2) verify 2014 $y = 21{,}700(.83)^4$

My equation almost exactly generated 10,298

$y \approx \$10{,}298.46$

5) worth less than $200?

$$200 = 21{,}700(.83)^x$$

($\log_{.83} .0092166$)

$$.0092166 = (.83)^x$$

$.83^{20} \approx .02$

$.83^{30} \approx .003$ (too low)

25 ≈ x yrs

Conclusion: 23D Growth

How did you know the data was *growing* based on the equation?

$$y = a(1+r)^x = a \cdot b^x$$

a: starting value (y-int)

r: rate of growth

Example: Ant Population

$$y = 16(1+.5)^x = 16(1.5)^x$$

50% growth in population each year

Conclusion: 23E Decay

How did you know the data was *decaying* based on the equation?

$$y = a(1-r)^x$$

a: starting value

r: rate of decay

Example: Car Value

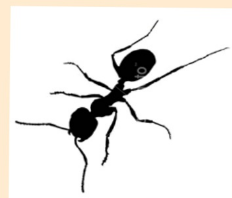
$$y = 21{,}700(1-.17)^x$$

$y = 21{,}700(.83)^x$

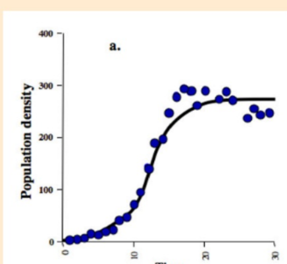
17% depreciation each year

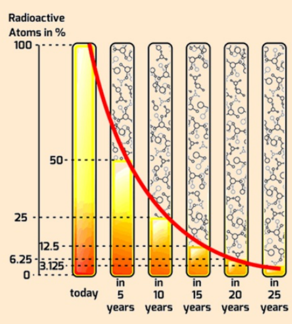
Compare growth vs. decay.

-Which scenario is growing, decreasing the fastest? How do you know?



-What other scenarios are similar to these situations?

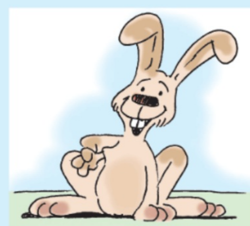




D GROWTH

Example 6 Self Tutor

The population of rabbits on a farm is given by the function $R = 50 \times (1.07)^n$ where n is the number of weeks after the rabbit farm was established.



a What was the original rabbit population?

b How many rabbits were present after 15 weeks?

c How many rabbits were present after 30 weeks?

d Sketch the graph of R against n for $n \geqslant 0$.

e How long will it take for the population to reach 500?

a When $n = 0$, $R = 50 \times (1.07)^0$

$= 50 \times 1$

$= 50$ $\therefore$ there were 50 rabbits originally.

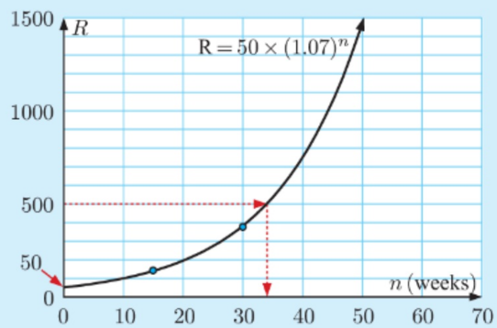
b When $n = 15$, $R = 50 \times (1.07)^{15}$

≈ 137.95 $\therefore$ there were 138 rabbits after 15 weeks.

c When $n = 30$, $R = 50 \times (1.07)^{30}$

≈ 380.61 $\therefore$ there were 381 rabbits after 30 weeks.

d

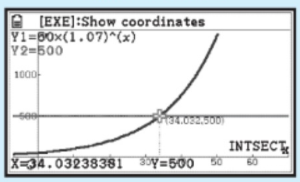


e From the graph, the number of weeks to reach 500 rabbits is approximately 34.

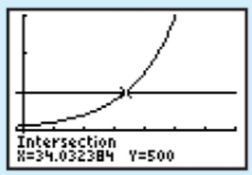
e From the graph, the number of weeks to reach 500 rabbits is approximately 34.

Alternatively, we could use technology to find where the graph cuts the line $R = 500$.

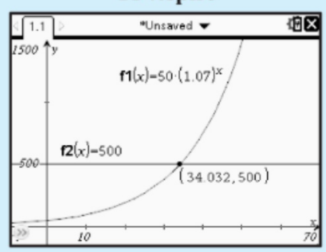
Casio fx-CG20



TI-84 Plus



TI-$n$spire



$\therefore$ it will take about 34 weeks.

E DECAY

When the value of a variable decreases exponentially over time, we call it **exponential decay**.

Example 7 — Self Tutor

When a diesel-electric generator is switched off, the current decays according to the formula $I = 24 \times (0.25)^t$ amps, where t is the time in seconds.

- **a** Find the value of I when $t = 1$, 2, and 3.
- **b** What current flowed in the generator at the instant when it was switched off?
- **c** Plot the graph of I against t for $t \geqslant 0$ using your results from **a** and **b**.
- **d** How long will it take for the current to fall to 4 amps?

a When $t = 1$,
$I = 24 \times (0.25)^1$
$= 6$ amps

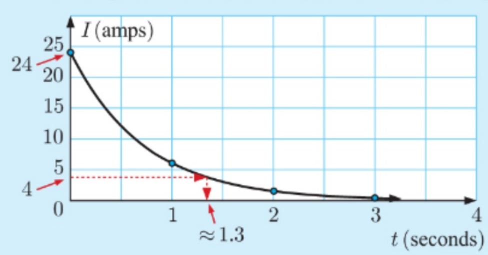
When $t = 2$,
$I = 24 \times (0.25)^2$
$= 1.5$ amps

When $t = 3$,
$I = 24 \times (0.25)^3$
$= 0.375$ amps

b When $t = 0$, $I = 24 \times (0.25)^0 = 24$

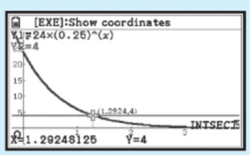
$\therefore$ 24 amps of current flowed at the instant the generator was switched off.

c

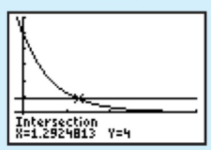


d

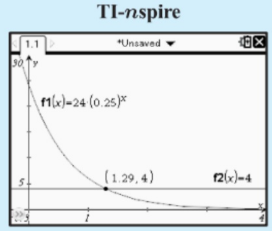
Casio fx-CG20



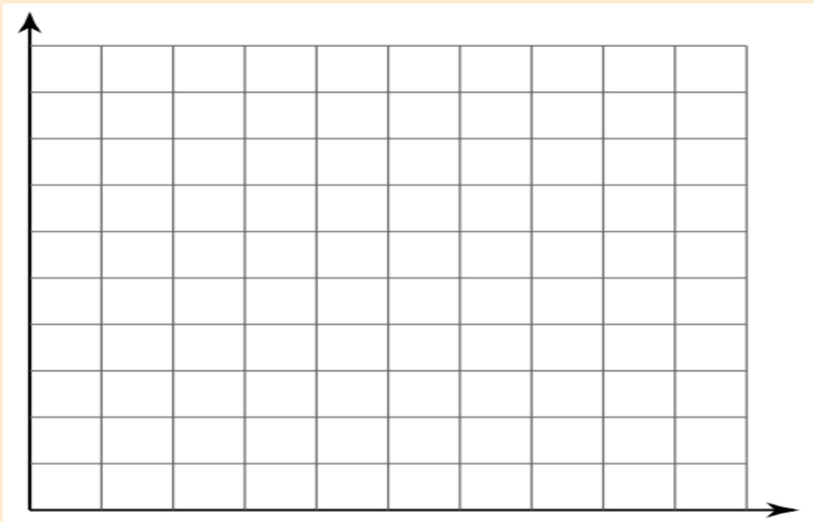
TI-84 Plus



TI-$n$spire



It will take ≈ 1.29 seconds for the current to fall to 4 amps.



Exercises...
23D Growth

Tomorrow: 23E Decay

EXERCISE 23D

You are encouraged to use technology to help answer the following questions.

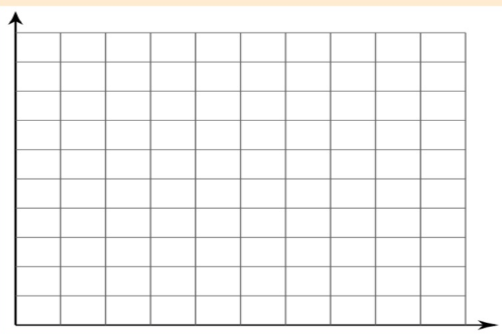
1 The population of mice in a field after n weeks is given by $P = 500 \times (1.12)^n$.

  a How many mice were originally in the field?

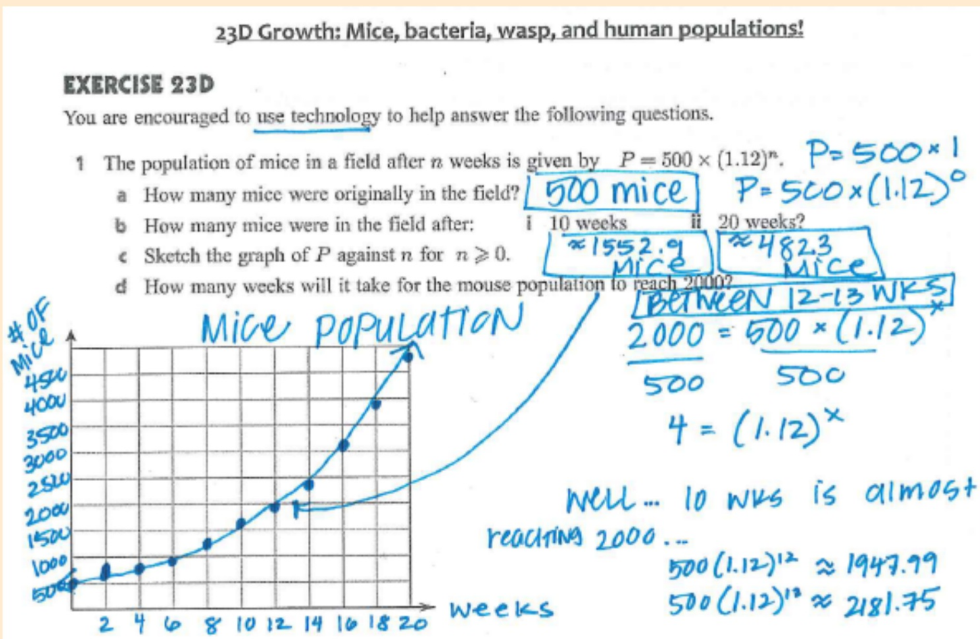
  b How many mice were in the field after: **i** 10 weeks **ii** 20 weeks?

  c Sketch the graph of P against n for $n \geqslant 0$.

  d How many weeks will it take for the mouse population to reach 2000?



Exercises...



23D Growth: Mice, bacteria, wasp, and human populations!

EXERCISE 23D

You are encouraged to use technology to help answer the following questions.

1 The population of mice in a field after n weeks is given by $P = 500 \times (1.12)^n$.

a How many mice were originally in the field?

b How many mice were in the field after: i 10 weeks ii 20 weeks?

c Sketch the graph of P against n for $n \geqslant 0$.

d How many weeks will it take for the mouse population to reach 2000?

Exercises...

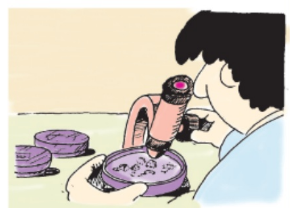
2 The weight of bacteria in a culture t hours after it has been established is given by the formula $W = 20 \times (1.007)^t$ grams.

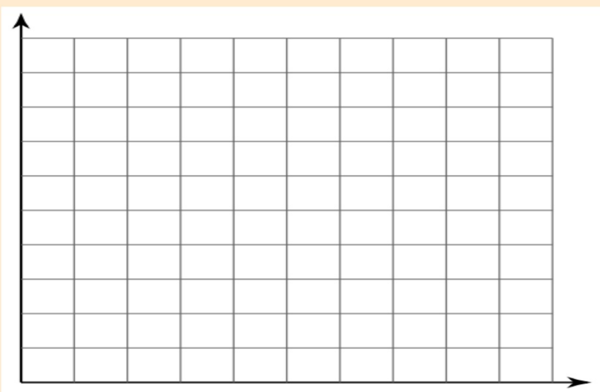
a Find the original weight of bacteria in the culture.

b Find the weight of the bacteria after 24 hours.

c Sketch the graph of W against t for $t \geqslant 0$.

d How long will it take for the weight to reach 50 grams?





Exercises...

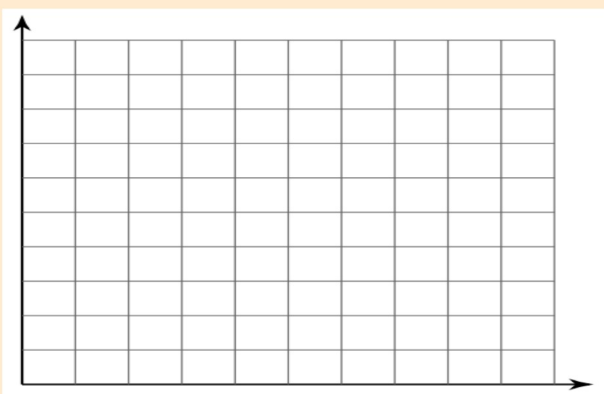
3 The population of wasps in a nest n days after it was discovered is given by $P = 250 \times (1.06)^n$.

a How many wasps were in the nest originally?

b Find the number of wasps after: **i** 25 days **ii** 8 weeks.

c Sketch the graph of P against n for $n \geqslant 0$.

d How long will it take for the population to double?



Exercises...

4 The population of a city was determined by census at 10 year intervals:

Year	1970	1980	1990	2000	2010
Population (thousands)	23.0	27.6	33.1	39.7	47.7

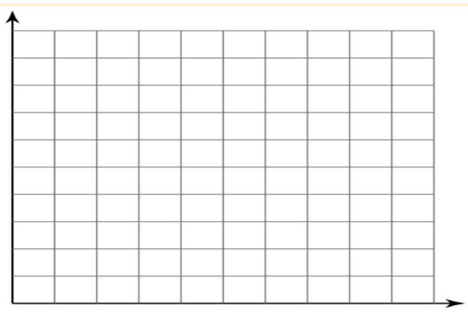
Suppose x is the number of years since 1970, and P is the population (in thousands).

a Draw the graph of P against x with P on the vertical axis.

b The variables P and x are connected by the exponential function $P = a \times b^x$ where a and b are constants.
Find the value of:
  i a, using the 1970 population data
  ii b, using the 2010 population data.

c Using the values of a and b found in **b**, check that your exponential formula fits the data from 1980, 1990, and 2000.

d Use your formula to predict the city's population in the year: **i** 2020 **ii** 2060.

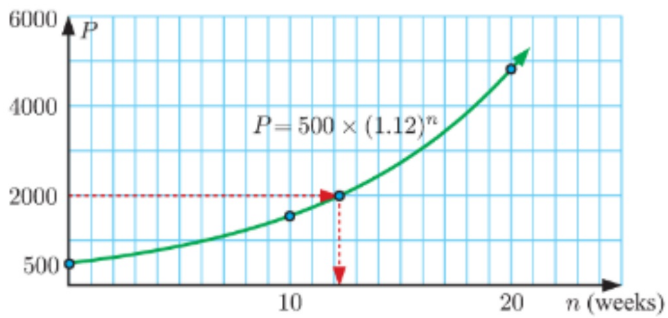


Solutions
(23D Growth)

EXERCISE 23D

1 **a** 500 mice **b** **i** 1550 mice **ii** 4820 mice

c

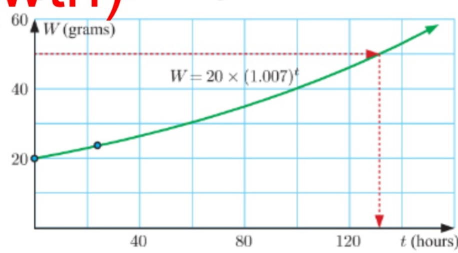


d $\approx$ 12 weeks

Solutions
(23D Growth)

2 **a** 20 g **b** ≈ 23.6 g

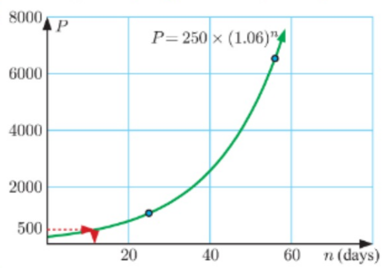
c



d ≈ 131 hours

3 **a** 250 wasps **b** **i** 1070 wasps **ii** 6530 wasps

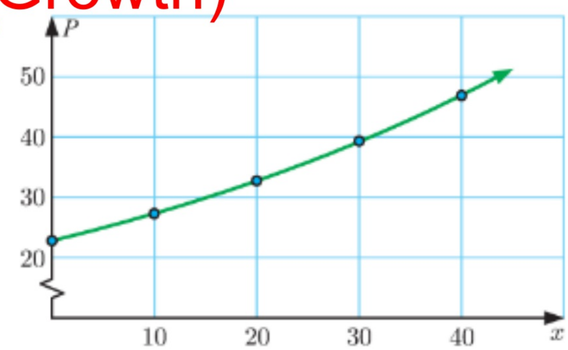
c



d ≈ 12 days

Solutions
(23D Growth)

4 **a**



b **i** $a = 23$ **ii** $b \approx 1.018$

c 1980, $P \approx 27.6$; 1990, $P \approx 33.1$; 2000, $P \approx 39.7$

d **i** $\approx 57\,300$ **ii** $\approx 119\,000$

Exercises...

EXERCISE 23E

1 When a container of liquid is left to cool, its temperature in °C is given by $T = 100 \times (0.933)^t$, where t is the time in minutes.

a Find the initial temperature of the liquid.

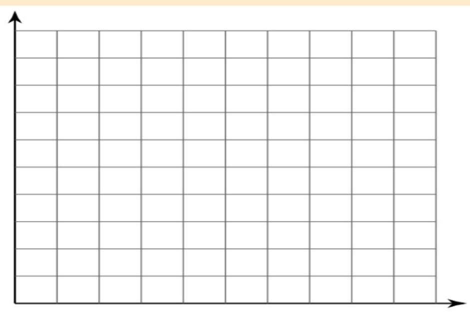
b Find the temperature after:

i 10 minutes **ii** 20 minutes **iii** 30 minutes.

c Draw the graph of T against t for $t \geqslant 0$, using your results from **a** and **b**.

d How long will it take for the liquid's temperature to fall to:

i 40°C **ii** 10°C?



Exercises...

2 The weight of a radioactive substance t years after being discovered is given by $W = 150 \times (0.997)^t$ grams.

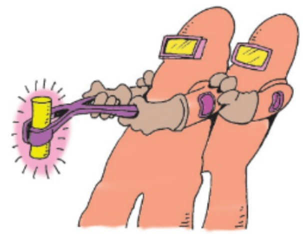
a How much radioactive substance was discovered?

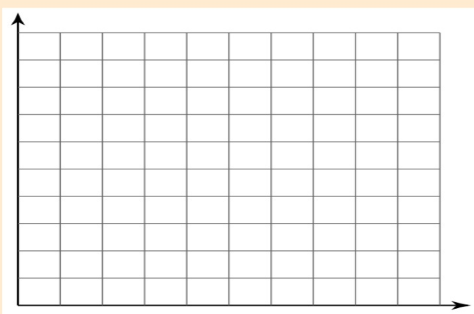
b Determine the weight of the substance after:

i 100 years **ii** 200 years **iii** 400 years.

c Sketch the graph of W against t for $t \geqslant 0$, using your results from **a** and **b**.

d How long will it take for the substance to decay to 25 grams?





Exercises...

3 The marsupial *Eraticus* is endangered. There is only one colony remaining, and research into its population has shown its decline over the last 25 years:

Year	1985	1990	1995	2000	2005	2010
Population	255	204	164	131	105	84

Let n be the time since 1985 and P be the population size.

a Graph P against n with P on the vertical axis.

b It is believed that P and n are connected by the function $P = a \times b^n$ where a and b are constants. Find:

- **i** a, using the 1985 population
- **ii** b, using the 2010 population.

c Using the values of a and b found in **b** check that your exponential function fits the data from 1990, 1995, 2000, and 2005.

d In what year do you expect the population size to be reduced to 50?

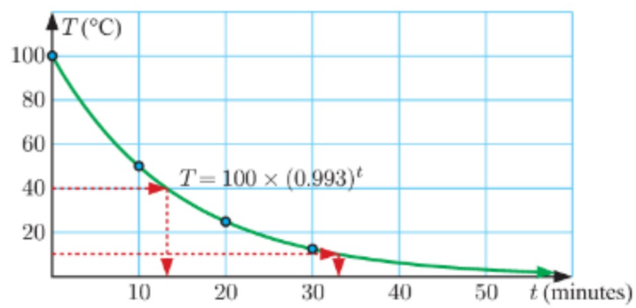
Solutions
(23E Decay)

EXERCISE 23E

1 **a** 100°C

b **i** 50.0°C **ii** 25.0°C **iii** 12.5°C

c



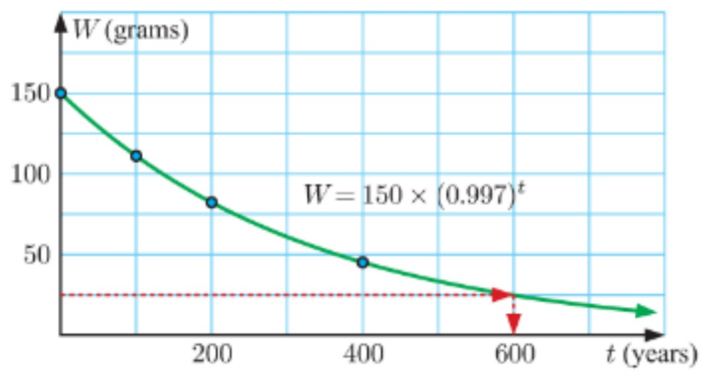
d **i** ≈ 13 mins **ii** ≈ 33 mins

Solutions
(23E Decay)

2 **a** 150 g

b **i** ≈ 111 g **ii** ≈ 82.2 g **iii** ≈ 45.1 g

c

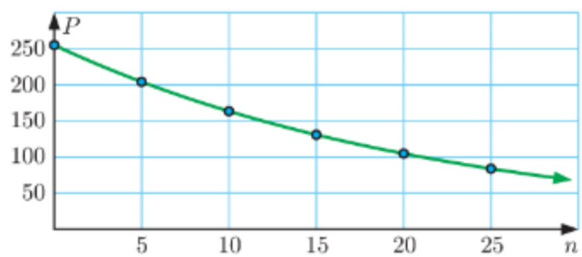


d ≈ 596 years

Solutions
(23E Decay)

3 **a**

n	0	5	10	15	20	25
P	225	204	163	131	104	84



b **i** $a = 255$ **ii** $b \approx 0.957$

c So, $P = 255 \times (0.9566)^n$
1990, $P \approx 204$; 1995, $P \approx 164$; 2000, $P \approx 131$;
2005, $P \approx 105$

d 2021