Welcome Back MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 2/12 Topic: Nothing due... Index Laws Quiz was Friday!	0 1 2	
Tuesday Date: 2/13 Topic: 23B Exponential Functions, 23C Graphs	0 1 2	
Wednesday Date: 2/14 Topic: 23C/D Exponential Growth and Decay	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

1. Warm-up, HW Questions??
2. Half-life Investigation
3. 23C Growth, 23D Decay
4. Practice

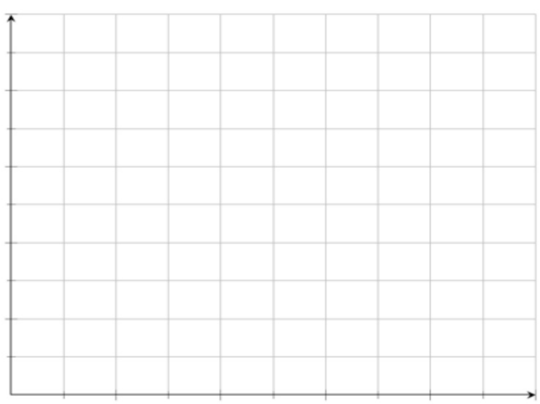
Half-life Investigation

$y_1 = 80(2^{-t}),\ y_2 = 100(2^{-\frac{t}{2}}),\ y_3 = 60(2^{-\frac{t}{5}})$. Mass is measured in grams and time (**t**) in years.

Step 1: Graph all three models on the same set of axes. [Window: $0 \le x \le 10, 0 \le y \le 100$].

Use your graphing calculator and the table function to plot points and draw accurate curves.

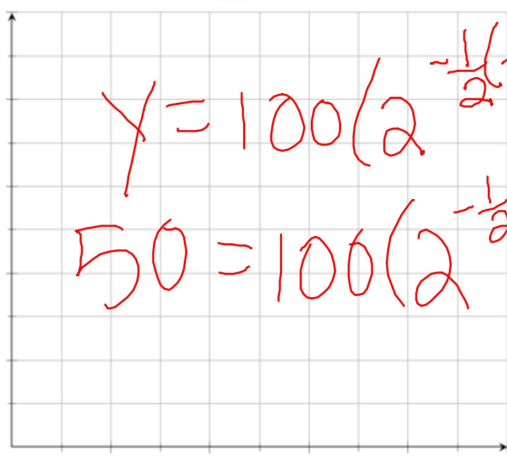
$y_1 = 80(2^{-t})$ $y_2 = 100(2^{\frac{-t}{2}})$ $y_3 = 60(2^{\frac{-t}{5}})$



X	Y_1	Y_2	Y_3
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Half-life Investigation

$y_1 = 80(2^{-t})$ $y_2 = 100(2^{\frac{-t}{2}})$ $y_3 = 60(2^{\frac{-t}{5}})$



$Y = 100(2^{-\frac{1}{2}(t)})$

$50 = 100(2^{-\frac{1}{2}(t)})$

Mass (g)

Time (years)

X	Y_1	Y_2	Y_3
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Half-life Investigation

Step 2

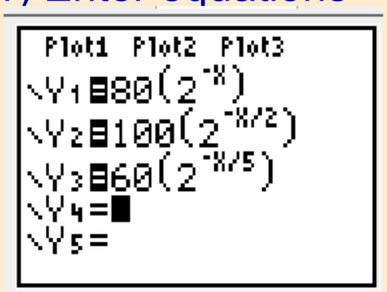
i. Find the original mass of each substance and the half-life of each substance.

iii. Use your graph to determine how long it takes in years for each substance to reach its half-life.

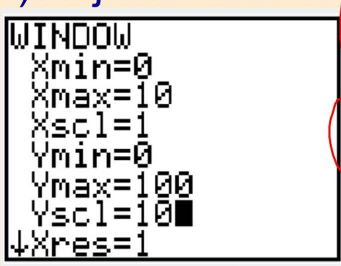
Step 3 Write a formula for the mass *M* of a substance with an initial mass of M_o grams and a half-life of *n* years.

Half-life Investigation: Solution

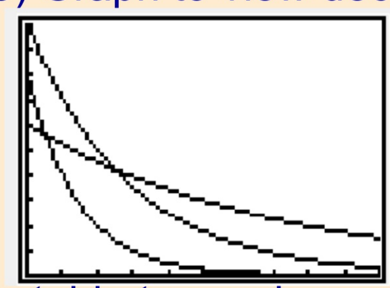
1) Enter equations



2) Adjust window and Graph



3) Graph to view decay



4) Use table to graph curves.

X	Y_1	Y_2	Y_3
0	80	100	60
1	40	70.711	52.233
2	20	50	45.471
3	10	35.355	39.585
4	5	25	34.461
5	2.5	17.678	30
6	1.25	12.5	[illegible]

X=6

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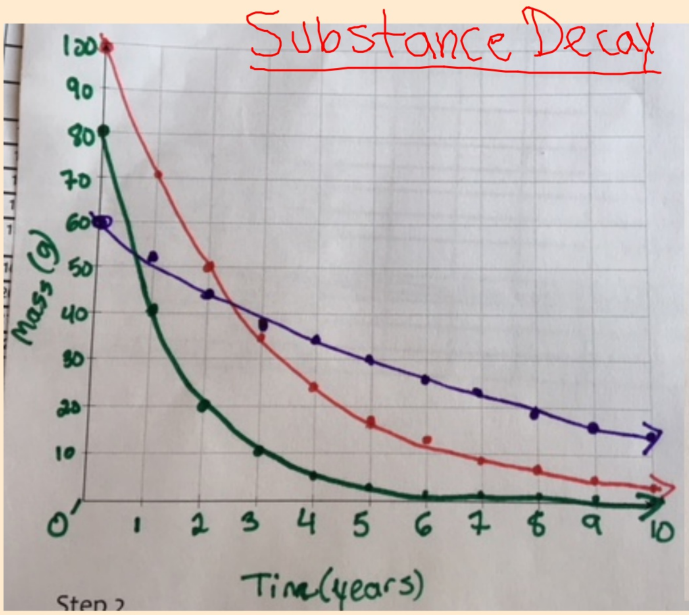
Half-life Investigation: Solution



X	Y_1	Y_2	Y_3
0	80	100	60
1	40	70.711	52.2
2	20	50	45.5
3	10	35.355	39.6
4	5	25	34.5
5	2.5	17.7	30
6	1.25	125	26.1
7	.625	8.84	22.7
8	.313	6.25	19.8
9	.156	4.42	17.2
10	.078	3.125	15

Half-life Investigation: Solution

Step 2

i. Find the original mass of each substance and the half-life of each substance.

Mass 1: 80 g   Mass2: 100g   Mass 3: 60 g

$\frac{1}{2}$ Mass1: 40 g $\frac{1}{2}$ Mass2: 50g $\frac{1}{2}$ Mass3: 30 g

iii. Use your graph to determine how long it takes in years for each substance to reach its half-life.

Mass 1: 1 year   Mass 2: 2 years   Mass 3: 5 years

Step 3 Write a formula for the mass M of a substance with an initial mass of M_0 grams and a half-life of n years.

$M = M_0 (2^{-t})$

$M = M_0 \cdot \left(\frac{1}{2^t}\right)$

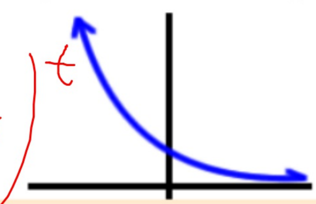
$= \frac{1}{2^{-t}}$

$\frac{2^{-t}}{1} = \frac{1}{2^t}$

This negative means exponential decay!

$M = M_0 \cdot 2^{-t}$

Mass = Initial mass $\left(\frac{1}{2}\right)^t$



Half-life Investigation: Solution

Questions to consider:

1) Will the mass of the substance ever reach zero grams?
2) How should this impact our use of pharmaceuticals?

Drug Metabolism

Medicine has "left" your body when 3% of original amount remains.

The amount of time it takes a drug to leave your body is based on the drug's half-life, or how long it takes for the liver and kidneys to break down and filter half of the amount of the drug in your bloodstream. This means that if a drug's half-life is one hour, after one hour you'd have half as much of the drug in your blood as you did when you first took it. After two hours, you'd have a quarter of the drug left, and after three hours, an eighth. For most medical purposes, a drug is considered to have cleared your system after five half-lives, when only about three percent of the drug is left. Each drug has its own half-life, ranging anywhere from seconds to days.

http://theoakstreatment.com/drug-addiction/long-drugs-stay-system/

ACTIVITY — CARBON DATING

You have probably heard of **carbon dating** as a technique for estimating the age of artefacts or bones. To understand how carbon dating works, we first need to understand the concept of **half-life**.

HALF-LIFE

The **half-life** of a radioactive substance is the time it takes for the substance to decay to half of its original amount.

If we know the half-life of a substance, we can use an exponential function to model the mass of the substance over time.

What to do:

Suppose the masses of substances A, B, and C are modelled by $M_A = 80 \times 2^{-t}$ grams, $M_B = 100 \times 2^{-\frac{t}{2}}$ grams, and $M_C = 60 \times 2^{-\frac{t}{5}}$ grams respectively, where t is in years.

1. Draw the graphs of M_A, M_B, and M_C on the same set of axes. Find the original mass of each substance.
2. From the graphs, determine how long it takes for each substance to decay to half of its original mass.
3. Hence, conjecture a formula for the mass M of a substance with an initial mass of M_0 grams and a half-life of n years.

Exercises...
23E Decay

Wednesday After school: Garages :)
No After School Thursday 2-15

(Thursday 2-15)
Parent - Teacher Conferences
Thursday 4 - 8 pm
Friday 8am - 12 pm

Exercises...

EXERCISE 23E

1 When a container of liquid is left to cool, its temperature in °C is given by $T = 100 \times (0.933)^t$, where t is the time in minutes.

a Find the initial temperature of the liquid.

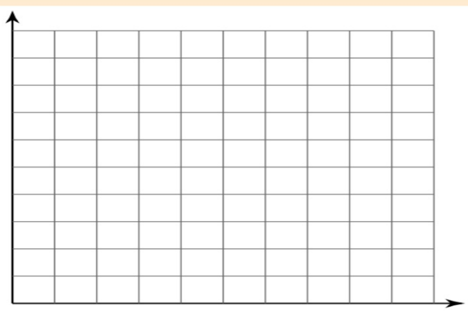
b Find the temperature after:

i 10 minutes **ii** 20 minutes **iii** 30 minutes.

c Draw the graph of T against t for $t \geqslant 0$, using your results from **a** and **b**.

d How long will it take for the liquid's temperature to fall to:

i 40°C **ii** 10°C?



Exercises...

2 The weight of a radioactive substance t years after being discovered is given by $W = 150 \times (0.997)^t$ grams.

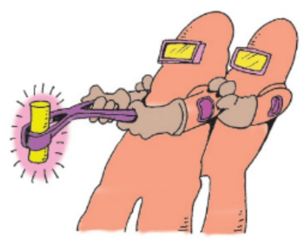
a How much radioactive substance was discovered?

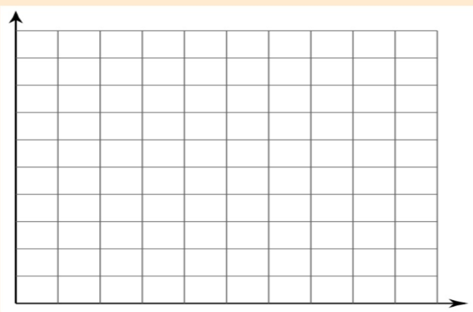
b Determine the weight of the substance after:

i 100 years **ii** 200 years **iii** 400 years.

c Sketch the graph of W against t for $t \geqslant 0$, using your results from **a** and **b**.

d How long will it take for the substance to decay to 25 grams?





Exercises...

3 The marsupial *Eraticus* is endangered. There is only one colony remaining, and research into its population has shown its decline over the last 25 years:

Year	1985	1990	1995	2000	2005	2010
Population	255	204	164	131	105	84

Let n be the time since 1985 and P be the population size.

a Graph P against n with P on the vertical axis.

b It is believed that P and n are connected by the function $P = a \times b^n$ where a and b are constants. Find:

- **i** a, using the 1985 population
- **ii** b, using the 2010 population.

c Using the values of a and b found in **b** check that your exponential function fits the data from 1990, 1995, 2000, and 2005.

d In what year do you expect the population size to be reduced to 50?

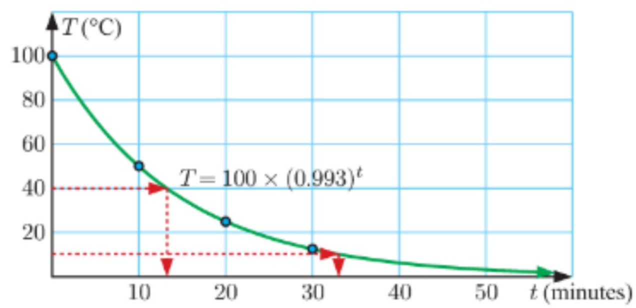
Solutions
(23E Decay)

EXERCISE 23E

1 **a** 100°C

b **i** 50.0°C **ii** 25.0°C **iii** 12.5°C

c



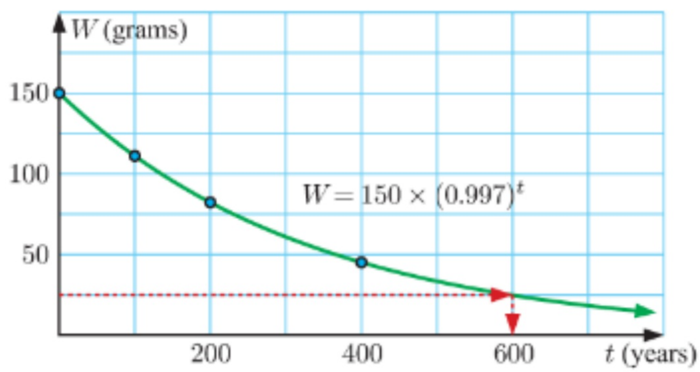
d **i** $\approx$ 13 mins **ii** $\approx$ 33 mins

Solutions
(23E Decay)

2 **a** 150 g

b **i** ≈ 111 g **ii** ≈ 82.2 g **iii** ≈ 45.1 g

c

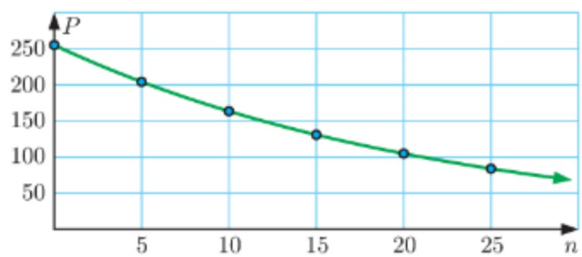


d ≈ 596 years

Solutions (23E Decay)

3 **a**

n	0	5	10	15	20	25
P	225	204	163	131	104	84



b **i** $a = 255$ **ii** $b \approx 0.957$

c So, $P = 255 \times (0.9566)^n$

1990, $P \approx 204$; 1995, $P \approx 164$; 2000, $P \approx 131$; 2005, $P \approx 105$

d 2021