Welcome MYP Math 9: Please reflect.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 2/12 Topic: Nothing due... Index Laws Quiz was Friday!	0 1 2	
Tuesday Date: 2/13 Topic: 23B Exponential Functions, 23C Graphs	0 1 2	
Wednesday Date: 2/14 Topic: 23C/D Exponential Growth and Decay	0 1 2	
Thursday Date: 2/15 Topic: Half-life, Exponential Decay	0 1 2	
Friday Date: Topic:	0 1 2	

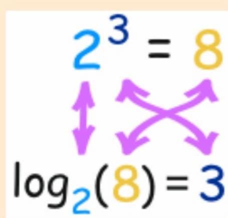
Enjoy your long weekend! :)

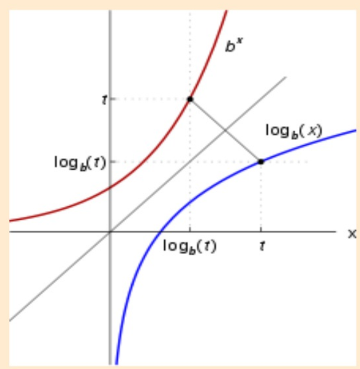
Agenda

1. Warm-up - introduction to solving for exponents

2. Introduction to Logarithms

3. Practice





Warm-up: Solve for x.

$3^x = 9$ $x = 2$	$\boxed{x = 1}$ $\frac{3(4^x)}{3} = \frac{12}{3}$ $4^x = 4^1$ $\frac{1}{2^3} = \frac{2^{-3}}{1}$
$\frac{2(10^x)}{2} = \frac{2000}{2}$ $10^x = 1000$ $\boxed{x = 3}$	$2^{x+1} = \frac{1}{8} = \frac{1}{2^3}$ $2^{x+1} = 2^{-3}$ $x + 1 = -3$ $\boxed{x = -4}$

A EXPONENTIAL EQUATIONS

An **exponential equation** is an equation in which the unknown occurs as part of the index or exponent.

For example: $3^x = 9$ and $5 \times 4^x = 8$ are both exponential equations.

$$\text{If} \quad \boldsymbol{a^x = a^k} \quad \text{then} \quad \boldsymbol{x = k}.$$

$2^x = 8 = 2^3$

For the case $3^x = 9$, we notice that $3^x = 3^2$. Thus $x = 2$ is a solution to the equation, and it is in fact the only solution to the equation.

To solve exponential equations, we try to write both sides of the equation with the **same base**. We can then **equate indices**.

Introduction to Logarithms

Let's look at problem 23D #1(d)...

EXERCISE 23D

You are encouraged to use technology to help answer the following questions.

1 The population of mice in a field after n weeks is given by $P = 500 \times (1.12)^n$.

a How many mice were originally in the field?

b How many mice were in the field after: i 10 weeks ii 20 weeks?

c Sketch the graph of P against n for $n \geqslant 0$.

d How many weeks will it take for the mouse population to reach 2000?

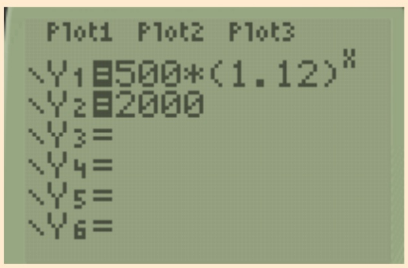
To solve this we need to find *n* such that:

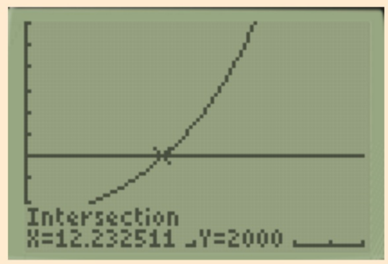
$$\frac{2000}{500} = \frac{500 \times (1.12)^n}{500}$$

or

$$4 = (1.12)^n$$

Way people probably did this yesterday...





Find the intersection of $y = 500 \times (1.12)^x$ and $y = 2000$ on the calculator.

There's gotta be a better way!

Introduction:

1. How many 2s do we need to multiply to get 8? 3

2. How many 5s need to be multiplied together to get 625? 4

$5 \cdot 5 \cdot 5 \cdot 5 = 625$

Let's go back to those warm-up questions:

1. How many 2s do we need to multiply to get 8?

$$2^{?} = 8$$

The <u>*logarithm*</u> tells us what the exponent is!

Definition of the Logarithm

$$\boxed{y = \log_b(x)} \Leftrightarrow \boxed{x = b^y}$$

"log base *b* of *x*"

For $x > 0,\ b > 0,\ \text{and } b \neq 1,$

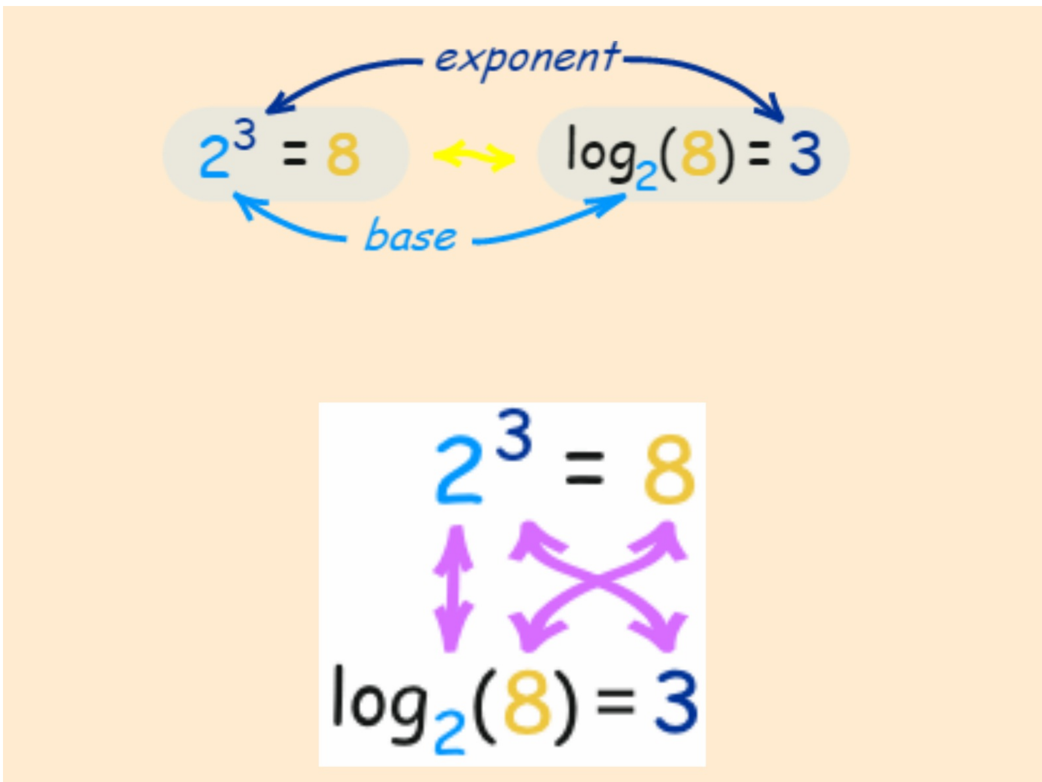
Exponent = log *base* *(Evaluated value)*

More concretely...

$$2^{?} = 8$$

...can be rewritten as...

$$3 \; ? = \log_2(8)$$


exponent
2^3 = 8
log_2(8) = 3
base
2^3 = 8
log_2(8) = 3

Example: Evaluate the logarithms.

$\text{Log}_2(32) = 5 \quad 2^5 = 32$

$\text{Log}_3(81) = 4 \quad 3^4 = 81$

$\text{Log}_4(64) = 3 \quad 4^3 = 64$

$\text{Log}_{10}(1000) = 3 \quad 10^3 = 1000.$

Examples: Rewrite in logarithmic form

$$y = \log_a(x) \Leftrightarrow x = a^y$$

$3^x = 74$ $\log_3(74) = x$	$7^p = 82$	$10^m + 9.5 = 87$ $-9.5 \quad -9.5$ $10^m = 77.5$ $m = \log_{10}(77.5)$

Examples: Rewrite in logarithmic form

$$y = \log_a(x) \Leftrightarrow x = a^y$$

$3^x = 74$	$7^p = 82$	$10^m + 9.5 = 87$
$x = \log_3(74)$	$p = \log_7(82)$	$10^m = 77.5$
		$m = \log_{10}(77.5)$

Examples: Rewrite in logarithmic form

$$y = \log_a(x) \Leftrightarrow x = a^y$$

$4 \cdot 10^k = 4.3$

$6 \cdot 18^v - 6 = 86$

Examples: Rewrite in logarithmic form

$$y = \log_a(x) \Leftrightarrow x = a^y$$

$4 \cdot 10^k = 4.3$

$\frac{4 \cdot 10^k}{4} = \frac{4.3}{4}$

$10^k = 1.075$

$k = \log_{10}(1.075)$

$6 \cdot 18^v - 6 = 86$

$6 \cdot 18^v = 92$

$18^v = \frac{92}{6}$

$v = \log_{18}\left(\frac{92}{6}\right)$

Examples: Solve each equation.

Equate indices? Apply logs?

$$4^{2r+1} = 64 = 4 \cdot 4 \cdot 4$$

$$4^{2r+1} = 4^3$$

$$2r+1 = 3$$

$$2r = 2$$

$$\boxed{r = 1}$$

Verify the r-value!

$$4^{2(1)+1} = 4^{2+1}$$

$$= 4^3$$

$$= 64 \checkmark$$

KEEP CALM
AND
CHOOSE THE RIGHT
Method :)

Examples: Solve each equation.

Equate indices? Apply logs?

$$9 \cdot 8^{6x} = 80$$

$$8^{6x} = \frac{80}{9}$$

$$6x = \log_8\left(\frac{80}{9}\right)$$

$$6x \approx 1.051$$

$$\boxed{x \approx 0.175}$$

Verify the x-value!

$$9 \cdot 8^{(6 \cdot .175)}$$

$$= 9 \cdot 8^{(1.05)}$$

$$\approx 9(8.877)$$

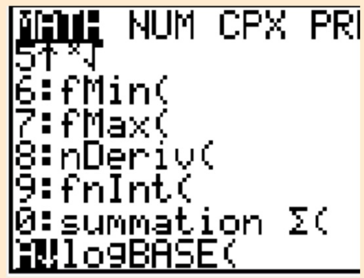
$$\approx 79.9 \approx \boxed{80} \checkmark$$

Examples: How can the calculator help?

1) MATH

2) A *"logBASE("*

3) Enter base & y-value.



$$9 \cdot 8^{6x} = 80$$

$$8^{6x} = \frac{80}{9}$$

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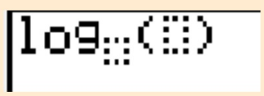
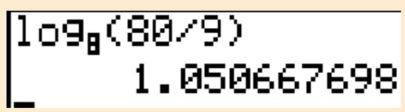
$$\boxed{x \approx 0.175}$$





4) Solve 6*x* = 1.05

Is your base 10?? (Example from earlier)

1) LOG

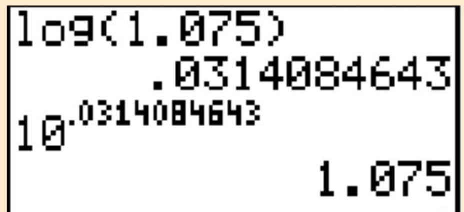
2) Enter values :)

$$\frac{4 \cdot 10^k}{4} = \frac{4.3}{4}$$

$10^k = 1.075$

$k = \log_{10}(1.075)$

$k \approx .0314$



Exponent Verified

Exercises. Rate your confidence:

I'm building confidence: (1 - 8, 17 - 22)

I'm somewhat confident:(1 - 10,17 - 24)

I'm very confident:(5 - 16, 21 - 26)

Quiz Scores posted by 3pm

Exercises

Rewrite each equation in logarithmic form. Solve

1) $17^x = 97$

2) $20^x = 44$

3) $5^x = 77$

4) $3^n = 81$

5) $5^r = 16$

6) $7^x = 26$

7) $10^n = 84.7$

8) $9^x = 0.5$

Exercises Rewrite in logarithmic form

9) $3 \cdot 10^{n} = 69$

10) $10^{r} + 5 = 60$

11) $-5 \cdot 10^{b} = -75$

12) $-4.5 \cdot 10^{x} = -41.8$

13) $-10^{n} + 7 = -34$

14) $1.2 \cdot 10^{n} - 2 = 2$

15) $-9 \cdot 9^{x} + 5 = -81.8$

16) $-20^{m} + 7 = -87$

Solve each equation. Equate indices? Apply logs?

17) $4^{2r} = 4^{-3r}$

18) $3^{-v} = \frac{1}{9}$

19) $8^{-m} = 64$

20) $5^{-3n} = \frac{1}{125}$

21) $6^{3n+3} = 6^{-2n}$

22) $4^{r+2} = \frac{1}{4}$

Solve each equation. Equate indices? Apply logs?

23) $4^{-2p+1} = 16$

24) $3^{2v} = 9$

25) $64^{2x} \cdot 16^{3x-1} = 1$

26) $36^{1-3a} = 6^3$

Solutions

Rewrite each equation in logarithmic form.

1) $17^x = 97$

$x = \log_{17}(97) \approx 1.61$

2) $20^x = 44$

$x = \log_{20}(44) \approx 1.26$

3) $5^x = 77$

$x = \log_5(77) \approx 2.7$

4) $3^n = 81$

$n = \log_3(81) = 4$

5) $5^r = 16$

$r = \log_5(16) \approx 1.73$

6) $7^x = 26$

$x = \log_7(26) \approx 1.67$

7) $10^n = 84.7$

$n = \log_{10}(84.7) \approx 1.93$

8) $9^x = 0.5$

$x = \log_9(.5) \approx -.32$

Solutions

9) $\frac{3 \cdot 10^n}{3} = \frac{69}{3}$ $\quad 10^n = 23$

$n = \log_{10}(23) \approx 1.36$

10) $10^r + 5 = 60$ $\quad 10^r = 55$

$r = \log_{10}(55) \approx 1.74$

$r = LOG(55) \approx 1.74$

11) $-5 \cdot 10^b = -75$

$10^b = 15$

$b = \log_{10}(15) \approx 1.18$

12) $-4.5 \cdot 10^x = -41.8$

$10^x = 9.29$

$x = \log(9.29) \approx .968$

13) $-10^n + 7 = -34$ $\quad -10^n = -44$

$10^n = 44$

$n = \log_{10}(44) \approx 1.62$

14) $1.2 \cdot 10^n - 2 = 2$

$1.2 \cdot 10^n = 4$ $\quad n = LOG\left(\frac{10}{3}\right)$

$10^n = \frac{10}{3}$ $\quad n \approx .523$

15) $-9 \cdot 9^x + 5 = -81.8$

$-9 \cdot 9^x = -86.8$

$9^x = \frac{86.8}{9}$

$x = \log_9\left(\frac{86.8}{9}\right)$

$x \approx 1.032$

16) $-20^m + 7 = -87$ $\quad -20^m = -94$

$20^m = 94$

$m = \log_{20}(94) \approx 1.52$

Solutions

Solve each equation by equating the indices (exponents) OR using logarithms in your graphing calculator.

17) $4^{2r} = 4^{-3r}$

$2r = -3r$

$5r = 0$ $\boxed{r = 0}$

18) $3^{-v} = \frac{1}{9}$ $3^{-v} = \frac{1}{3^v} = \frac{1}{3^2}$

$\boxed{v = 2}$

19) $8^{-m} = 64$ $-m = \log_8(64)$

$\frac{1}{8^m} = 64$ $m = -\log_8(64)$

$\boxed{m = -2}$

20) $5^{-3n} = \frac{1}{125} = \frac{1}{5^{3n}} = \frac{1}{5^3}$

$3n = 3$

$\boxed{n = 1}$

21) $6^{3n+3} = 6^{-2n}$

$3n + 3 = -2n$

$5n + 3 = 0$

$5n = -3$

$\boxed{n = -\frac{3}{5}}$

22) $4^{r+2} = \frac{1}{4} = 4^{-1}$

$r + 2 = -1$

$\boxed{r = -3}$

Solutions

23) $4^{-2p+1} = 16 = 4^{2}$

$-2p+1 = 2$

$-2p = 1$

$\boxed{p = -\frac{1}{2}}$

24) $3^{2v} = 9 = 3^{2}$

$2v = 2$

$\boxed{v = 1}$

25) $64^{2x} \cdot 16^{3x-1} = 1$

$4^{3(2x)} \cdot 4^{2(3x-1)} = 4^{0}$

$6x + 6x - 2 = 0$

$12x - 2 = 0$

$12x = 2$

$\boxed{x = \frac{1}{6}}$

26) $36^{1-3a} = 6^{3}$

$6^{2(1-3a)} = 6^{3}$

$2 - 6a = 3$

$-6a = 1$

$\boxed{a = -\frac{1}{6}}$

Solutions

1) 1.6147	2) 1.2632	3) 2.699	4) 4
5) 1.7227	6) 1.6743	7) 1.9279	8) −0.3155
9) 1.3617	10) 1.7404	11) 1.1761	12) 0.968
13) 1.6128	14) 0.5229	15) 1.0315	16) 1.5166
17) $\{0\}$	18) $\{2\}$	19) $\{-2\}$	20) $\{1\}$
21) $\left\{-\frac{3}{5}\right\}$	22) $\{-3\}$	23) $\left\{-\frac{1}{2}\right\}$	24) $\{1\}$
25) $\left\{\frac{1}{6}\right\}$	26) $\left\{-\frac{1}{6}\right\}$	27) −0.4157	28) 1.1047