

## Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
<b>Monday</b> Date: <u>2/19</u> Topic: _____	0 1 2	No School
<b>Tuesday</b> Date: <u>2/20</u> Topic: <u>Logarithms</u>	0 1 2	
<b>Wednesday</b> Date: _____ Topic: _____	0 1 2	
<b>Thursday</b> Date: _____ Topic: _____	0 1 2	
<b>Friday</b> Date: _____ Topic: _____	0 1 2	

Warm-up:

$$b^x = y \quad x = \log_b(y)$$

Solve for y. (Try without a calculator!)

$$y = \log_2(128)$$

$$2^y = 128$$

$$y = 7$$

$$y = \log_3(81)$$

$$3^y = 81$$

$$y = 4$$

## Class Plan:

1. Warm-up

2. Investigation:

Properties of Logarithms

3. Practice

## Investigation: Properties of Logarithms

Product Property

$$\log_a(x) + \log_a(y) = \log_a(\underline{\quad})$$

Quotient Property

$$\log_a(x) - \log_a(y) = \log_a(\underline{\quad})$$

Done? Write Properties in your notebook  
& raise your hand

# 1. Product/Multiplication Property

$$2^3 \cdot 2^4 = 2^{\boxed{7}}, 2^3 = \underline{8}, 2^4 = \underline{16}, 2^7 = \underline{128}$$
$$\underline{8} \cdot \underline{16} = \underline{128}$$

- i.  $\log_2(8) = \underline{3}, \log_2(16) = \underline{4}$   
ii.  $\log_2(8) + \log_2(16) = \underline{7}$   
iii.  $\log_2(8) + \log_2(16) = \log_2(\underline{128})$

Question: How are parts **i, ii, iii** related?



$\log_2(\quad)$

# 1. Product/Multiplication Property

$$4^2 \cdot 4^3 = 4^{\boxed{\phantom{00}}}, 4^2 = \underline{\phantom{00}}, 4^3 = \underline{\phantom{00}}, 4^5 = \underline{\phantom{0000}} \\ \underline{\phantom{00}} \cdot \underline{\phantom{00}} = \underline{\phantom{000}}.$$

- i.  $\log_4(16) = \underline{\phantom{00}}, \log_4(64) = \underline{\phantom{00}},$
- ii.  $\log_4(16) + \log_4(64) = \underline{\phantom{00}}$
- iii.  $\log_4(16) + \log_4(64) = \log_4(\underline{\phantom{0000}})$

Question: How are parts **i, ii, iii** related?

2) Examine parts **a), b), and c)**. Complete the rule when adding logarithms with a common **base**.

$$\log_a(x) + \log_a(y) = \log_a(\underline{\phantom{0000}})$$

## Investigation: Properties of Logarithms

1) "Product property of exponents"

a)

$$2^3 \cdot 2^4 = 2^{\boxed{7}}, 2^3 = \underline{8}, 2^4 = \underline{16}, 2^7 = \underline{128}$$
$$\underline{8} \cdot \underline{16} = \underline{128}.$$

b)

i.  $\log_2(8) = \underline{3}, \log_2(16) = \underline{4},$

ii.  $\log_2(8) + \log_2(16) = \underline{7}$

iii.  $\log_2(8) + \log_2(16) = \log_2(\underline{128})$

Question: How are parts i, ii, iii related?

The sum of the logs is equal to the log of  $(8 \cdot 16)$

## Solutions

## Investigation: Properties of Logarithms

c)  $4^2 \cdot 4^3 = 4^{\boxed{5}}$ ,  $4^2 = \underline{16}$ ,  $4^3 = \underline{64}$ ,  $4^5 = \underline{1024}$

i.  $\log_4(16) = \underline{2}$ ,  $\log_4(64) = \underline{3}$ ,

ii.  $\log_4(16) + \log_4(64) = \underline{5}$

iii.  $\log_4(16) + \log_4(64) = \log_4(\underline{16 \cdot 64})$

Question: How are parts i, ii, iii related?

The sum of the logs is equal to the log of  $(16 \cdot 64)$

2) Examine parts a), b), and c). Complete the rule when adding logarithms with a common **base**.

$$\log_a(x) + \log_a(y) = \log_a(\underline{xy})$$

## Solutions



## 2. Quotient/Division Property

$$\frac{4^5}{4^3} = 4^{\square}, 4^5 = \underline{\hspace{2cm}}, 4^3 = \underline{\hspace{2cm}}, 4^2 = \underline{\hspace{2cm}}$$

- i.  $\log_4(1024) = \underline{\hspace{1cm}}, \log_4(64) = \underline{\hspace{1cm}},$
- ii.  $\log_4(1024) - \log_4(64) = \underline{\hspace{1cm}}$
- iii.  $\log_4(1024) - \log_4(64) = \log_4(\underline{\hspace{2cm}})$

Question: How are parts **i, ii, iii** related?

## 2. Quotient/Division Property

$$\frac{3^6}{3^2} = 3^{\boxed{4}}, 3^6 = \underline{729}, 3^2 = \underline{9}, 3^4 = \underline{81}$$

- i.  $\log_3(729) = \underline{6}, \log_3(9) = \underline{2}$   
ii.  $\log_3(729) - \log_3(9) = \underline{4}$   
iii.  $\log_3(729) - \log_3(9) = \log_3(\underline{81})$

Question: How are parts **i, ii, iii** related?

Division of values  
= difference of logs

4) Examine parts **a), b), c) and d)**. Complete the rule when subtracting logs with a common **base**.

$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$

## Investigation: Properties of Logarithms

3) "Division property of exponents"

a)

$$\frac{4^5}{4^3} = 4^{\boxed{2}}, 4^5 = \underline{1024}, 4^3 = \underline{64}, 4^2 = \underline{16}$$

b)

i.  $\log_4(1024) = \underline{5}, \log_4(64) = \underline{3},$

ii.  $\log_4(1024) - \log_4(64) = \underline{2}$

iii.  $\log_4(1024) - \log_4(64) = \log_4\left(\frac{1024}{64}\right)$

Question: How are parts i, ii, iii related?

The difference of the logs is equal to the log of  $\left(\frac{1024}{64}\right)$

## Solutions

## Investigation: Properties of Logarithms

c)

$$\frac{3^6}{3^2} = 3^{\boxed{4}}, 3^6 = \underline{729}, 3^2 = \underline{9}, 3^4 = \underline{81}$$

d)

i.  $\log_3(729) = \underline{6}, \log_3(9) = \underline{2},$

ii.  $\log_3(729) - \log_3(9) = \underline{4}$

iii.  $\log_3(729) - \log_3(9) = \log_3\left(\frac{729}{9}\right)$

Question: How are parts i, ii, iii related?

The difference of logs is equal to the log of  $\left(\frac{729}{9}\right)$

4) Examine parts a), b), c) and d). Complete the rule when subtracting logs with a common **base**.

$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$

## Solutions

## NOTES: Properties of Logarithms

### Product Property

$$\log_a(x) + \log_a(y) = \log_a(\text{_____})$$

### Quotient Property

$$\log_a(x) - \log_a(y) = \log_a(\text{_____})$$

## Properties of Exponents and Logarithms

### Definition of Logarithm

If  $x = a^m$ , then  $\log_a x = m$ .

### Product Property

$$a^m \cdot a^n = a^{m+n} \quad \text{or} \quad \log_a xy = \log_a x + \log_a y$$

### Quotient Property

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{or} \quad \log_a \frac{x}{y} = \log_a x - \log_a y$$

### Power Property

$$\log_a x^n = n \log_a x$$

### Power of a Power Property

$$(a^m)^n = a^{mn}$$

### Power of a Product Property

$$(ab)^m = a^m b^m$$

### Change-of-Base Property

$$\log_a x = \frac{\log_b x}{\log_b a}$$

### Definition of Rational Exponents

## Notes

$$\log_{\square} (\square)$$

LOG

$$\log_{(12)} (47) = \frac{\log 47}{\log 12}$$

### Power of a Quotient Property

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

### Example: Evaluate

Evaluate each expression.

$$4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

$$10) \log_4 \frac{1}{64} = \log_4(1) - \log_4(64)$$

$$= 0 - 3$$

$$\boxed{-3}$$

**Example: Expand using properties**

Expand each logarithm.

$$17) \log_6 (2 \cdot 5) = \log_6 (2) + \log_6 (5)$$



**Example:** Expand using properties

Expand each logarithm.

$$19) \log_6 \frac{7}{3} = \log_6(7) - \log_6(3)$$

Solve each equation.

**Example:** Expand using properties

$$23) \log_5(8) + \log_5(x - 4) = 2 \quad \text{Exponent}$$

$$\log_5(8x - 32) = 2$$

$$8x - 32 = 25$$

$$\frac{8x}{8} = \frac{57}{8}$$

$$x = \frac{57}{8}$$

Solve each equation.

**Example:** Expand using properties

$$23) \log_5 8 + \log_5 (x - 4) = 2$$

$$\log_5 (8(x-4)) = 2 \quad 8x - 32 = 25$$

$$\log_5 (8x - 32) = 2 \quad 8x = 57$$

$$8x - 32 = 5^2$$

$$x = \frac{57}{8}$$

## Exercises

- 1) Practice applying log properties
- 2) Examine your quiz with EXEMPLARS (posted 3pm.)

**Simplify.**

1)  $2^2 \cdot 2^2$

2)  $(2^3)^3 \cdot 2^2$

3)  $(2 \cdot 2^{-2})^3$

4)  $2^{-2} \cdot (2^4)^{-2}$

5)  $\frac{3^3 \cdot 3^2}{3^4}$

6)  $\frac{4^{-1} \cdot 4^{-2}}{4}$

## Exercises...

Evaluate each expression.

7)  $\log_4 16$

8)  $\log_3 \frac{1}{81}$

9)  $\log_6 216$

10)  $\log_4 \frac{1}{64}$

11)  $\log_4 64$

12)  $\log_7 343$

## Exercises...

11)  $\log_4 64$

12)  $\log_7 343$

13)  $\log_5 \frac{1}{125}$

14)  $\log_2 \frac{1}{16}$

15)  $\log_5 \frac{1}{25}$

16)  $\log_3 \frac{1}{243}$

## Exercises...

Expand each logarithm.

17)  $\log_6 (2 \cdot 5)$

18)  $\log_5 (3 \cdot 10)$

19)  $\log_6 \frac{7}{3}$

20)  $\log_2 \frac{12}{11}$

21)  $\log_5 6^5$

22)  $\log_4 (8 \cdot 3)$

## Exercises...

Solve each equation.

23)  $\log_5 8 + \log_5 (x - 4) = 2$

24)  $\log_2 3 + \log_2 (x - 5) = \log_2 27$

25)  $\log_3 6 - \log_3 -2x = 2$

26)  $\log_5 2 + \log_5 -5x = 1$

27)  $\log_2 10 - \log_2 5x = \log_2 69$

28)  $\log_6 -x + \log_6 9 = 4$



# Solutions

## Answers to Exponent & Logarithm Properties (ID: 1)

- |                                   |                                   |                                   |                             |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------|
| 1) $2^4$                          | 2) $2^{11}$                       | 3) $\frac{1}{2^3}$                | 4) $\frac{1}{2^{10}}$       |
| 5) 3                              | 6) $\frac{1}{4^4}$                | 7) 2                              | 8) -4                       |
| 9) 3                              | 10) -3                            | 11) 3                             | 12) 3                       |
| 13) -3                            | 14) -4                            | 15) -2                            | 16) -5                      |
| 17) $\log_6 2 + \log_6 5$         | 18) $\log_5 3 + \log_5 10$        | 19) $\log_6 7 - \log_6 3$         | 20) $\log_2 12 - \log_2 11$ |
| 21) $5\log_5 6$                   | 22) $\log_4 8 + \log_4 3$         | 23) $\left\{\frac{57}{8}\right\}$ | 24) $\{14\}$                |
| 25) $\left\{-\frac{1}{3}\right\}$ | 26) $\left\{-\frac{1}{2}\right\}$ | 27) $\left\{\frac{2}{69}\right\}$ | 28) $\{-144\}$              |