Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 2/19 Topic: No School	0 1 2	
Tuesday Date: 2/20 Topic: Exponents and Logarithms	0 1 2	
Wednesday Date: 2/21 Topic: Logarithm Properties	0 1 2	
Thursday Date: _____ Topic: _____	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Warm-up: Logarithm Property

Evaluating $\log_2(50)$

Change-of-Base Property

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\frac{\log_{10}(50)}{\log_{10}(2)} = \frac{\log(50)}{\log(2)} \approx 5.64$$

$$2^{5.64} \approx 49.86\ldots$$

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/change-of-base-formula-for-logarithms/a/logarithm-change-of-base-rule-intro

Solution:

$$\log_2(50) = \frac{\log_{10}(50)}{\log_{10}(2)} \qquad \text{Change of base rule}$$

$$= \frac{\log(50)}{\log(2)} \qquad \text{Since } \log_{10}(x) = \log(x)$$

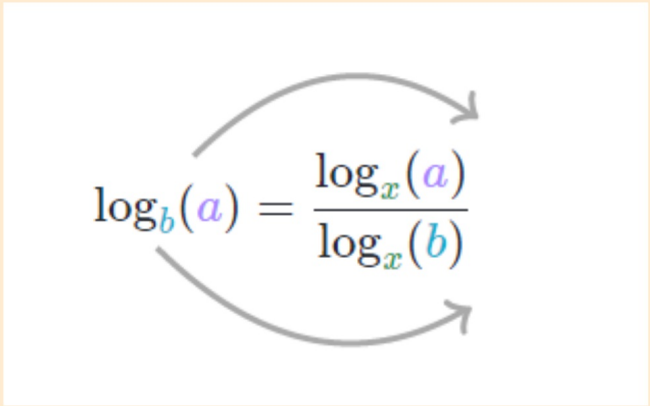
We can now find the value using the calculator.

$$\approx 5.644$$

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/change-of-base-formula-for-logarithms/a/logarithm-change-of-base-rule-intro

The change of base rule

We can change the base of any logarithm by using the following rule:



Why it works...

To examine this, let's return to the original expression $\log_2(50)$. If we let $\log_2(50) = n$, then it follows that $2^n = 50$.

Because the two values are equal, we can take the log *in any base* of both sides. Now we have:

$$2^n = 50$$

$$\log_x(2^n) = \log_x(50) \qquad \text{If } Y = Z, \text{ then } \log_x(Y) = \log_x(Z)$$

$$n\log_x(2) = \log_x(50) \qquad \text{Power Rule}$$

$$n = \frac{\log_x(50)}{\log_x(2)} \qquad \text{Divide both sides by } \log_x(2)$$

Since $n = \log_2(50)$, we have that $\log_2(50) = \dfrac{\log_x(50)}{\log_x(2)}$ as desired!

https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/change-of-base-formula-for-logarithms/a/logarithm-change-of-base-rule-intro

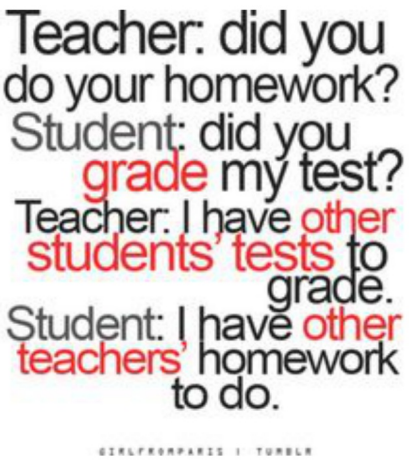
Class Plan:

1. Warm-up: Logarithm Property.

2. Practice project. 

3. Consider topics for project.

Joke of the day.....Teacher roast!



Thank you for your patience! (Study guide scores will be in the portal by Thursday evening!

Exponential Practice Project:

1) Read directions
2) Calculate ***b***, Model data

$$y = a \cdot b^x$$

3) Verify equation
4) Make predictions

Directions for the practice project:
1) Using the Hamline data, scale and create a graph.
2) Build your equation. Use your notes and checklist to assist you.
3) Interpret the real-life meanings of your equation.
4) Verify your equation.
5) Make predictions using your graph and/or equation.
6) Write down and answer questions that can be answered from your models.
7) Organize your work as neatly as possible.

Year since 1999	$ In-state tuition and fees	Ratio of current tuition $ ÷ previous tuition $
0	20461	
1	20351	
2	20966	
3	22894	
4	24331	
5	25063	
6	25544	
7	26790	
8	27203	

Average of ratios _____

Exponential Equation:

$$y = a \cdot b^x$$

a: Starting value

b: Constant multiplier (multiplier is always positive.)

r: Rate of growth/decay, interpreted as a %

Exponential Growth

b>1

$$y = a(1+r)^x$$

Ant Population Example:

$$y = 16(1.5)^x$$

r = 50% growth

Exponential Decay

0<b<1

$$y = a(1-r)^x$$

Car Value Example:

$$y = 21{,}700(0.83)^x$$

r = 17% depreciation

Table

Year since 1999	$ In-state tuition and fees	Ratio of current tuition $ ÷ previous tuition $
0	20461	
1	20351	20351/20461 ≈ .995
2	20966	
3	22894	
4	24331	
5	25063	
6	25544	
7	26790	
8	27203	

Average of ratios _____

Table

The data below is the Hamline University tuition from 1999 to 2010.

Year since 1999	$ In-state tuition and fees	Ratio of current tuition $ ÷ previous tuition $
0 1999	20461	
1 2000	20351	$\frac{20351}{20461} \approx .995$
2 2001	20966	$\frac{20966}{20351} \approx 1.03$
3 2002	22894	$\frac{22894}{20966} \approx 1.09$
4 2003	24331	$\frac{24331}{22894} \approx 1.06$
5 2004	25063	$\frac{25063}{24331} \approx 1.03$
6 2005	25544	$\frac{25544}{25063} \approx 1.02$
7 2006	26790	$\frac{26790}{25544} \approx 1.05$
8 2007	27203	$\frac{27203}{26790} \approx 1.02$

Average of ratios $(.995 + 1.03 + 1.09 + 1.06 + 1.03 + 1.02 + 1.05 + 1.02)/8 = \frac{8.295}{8}$

Directions for the practice project:

1) Using the Hamline data, scale and create a graph.

Graph

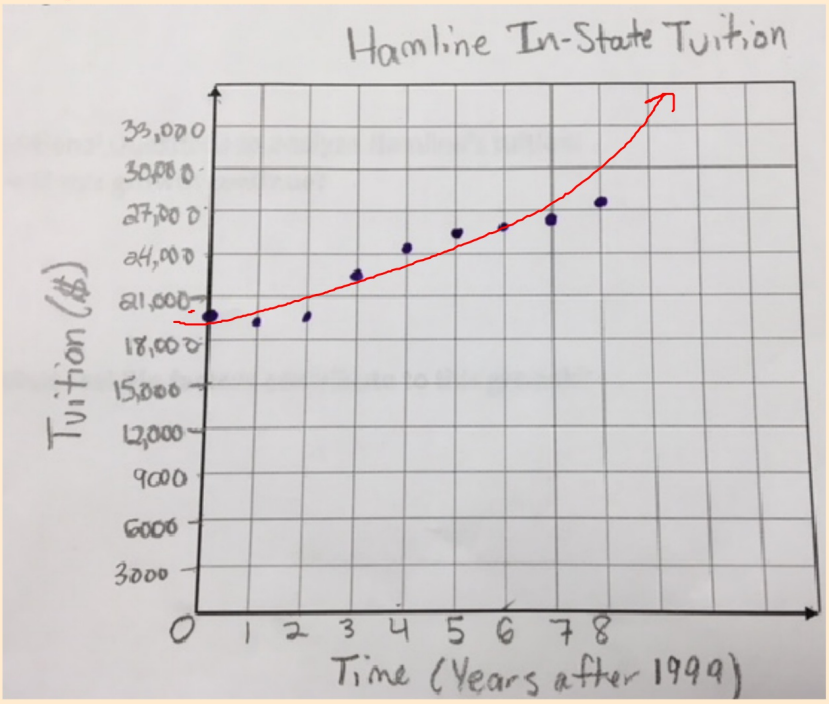
Year since 1999	$ In-state tuition and fees
0	20461
1	20351
2	20966
3	22894
4	24331
5	25063
6	25544
7	26790
8	27203

TUITION

27,000
26,000
25,000
24,000
23,000
22,000
21,000
20,000
0

"break"

Graph


Hamline In-State Tuition
Tuition ($)
33,000
30,000
27,000
24,000
21,000
18,000
15,000
12,000
9000
6000
3000
0 1 2 3 4 5 6 7 8
Time (Years after 1999)

Equation

2) Build your equation. Use your notes to assist you. $y = a \cdot b^x = a(1+r)^x$

a) The starting tuition of Hamline University is 20,461

b) Each year the tuition price is multiplying by a constant rate of 1.04

c) Write an equation modeling tuition growth. $y = 20{,}461(1.04)^x$

3) Interpret the real-life meanings of your equation.

Y: The total (current) cost of Hamline tuition.

X: Time in years after 1999.

a: Hamline tuition during 1999

b: Amount multiplied to each year of tuition

r: 4% Growth rate of tuition costs

Verify:

Does the equation model the data well?

4) Verify your equation. Use year 6 (2005) in your equation to show tuition is $25,063.

$$y = 20461(1.04)^x$$

1.036

equation $345.7th too high

$y = \$25,889.70$

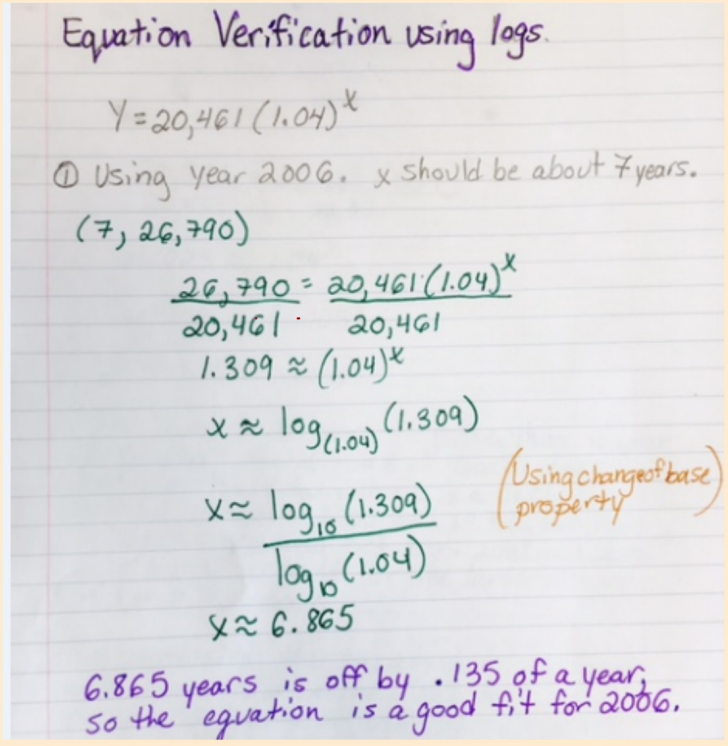
Pretty close.

(Based on our equation)

ACTUAL: $25,544

Good model

Verify:



Equation Verification using logs.

$Y = 20{,}461(1.04)^x$

① Using year 2006. x should be about 7 years.

(7, 26,790)

$$\frac{26{,}790}{20{,}461} = \frac{20{,}461(1.04)^x}{20{,}461}$$

$$1.309 \approx (1.04)^x$$

$$x \approx \log_{(1.04)}(1.309)$$

(Using change of base property)

$$x \approx \frac{\log_{10}(1.309)}{\log_{10}(1.04)}$$

$$x \approx 6.865$$

6.865 years is off by .135 of a year, so the equation is a good fit for 2006.

Verify:

② Using year 2001.

(2, 20,966)

$$\frac{20,966}{20,461} = \frac{20,461\,(1.04)^x}{20,461}$$

$$1.025 \approx 1.04^x$$

$$x = \log_{(1.04)}(1.025)$$

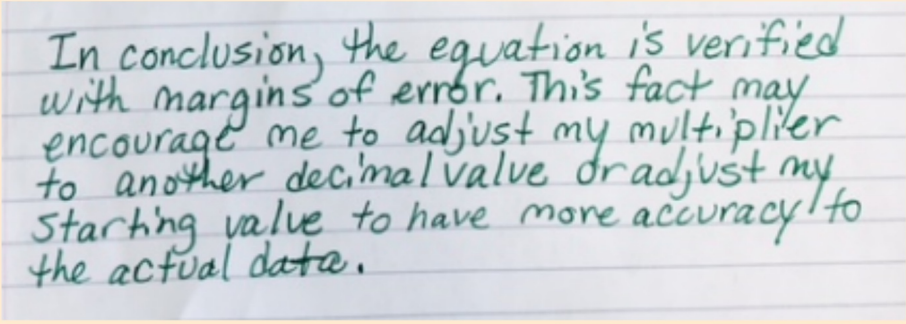
$$x \approx 0.630 \text{ years.}$$

This number of years is more than a year away from the actual 2 years. Looking back at the data, there is a decline in tuition between 2000 and 1999, but then it starts growing between 2001 and 2000. This could account for the larger margin of error for 2001.

Verify:

Is the equation an accurate model for the data?

How could it be more accurate?

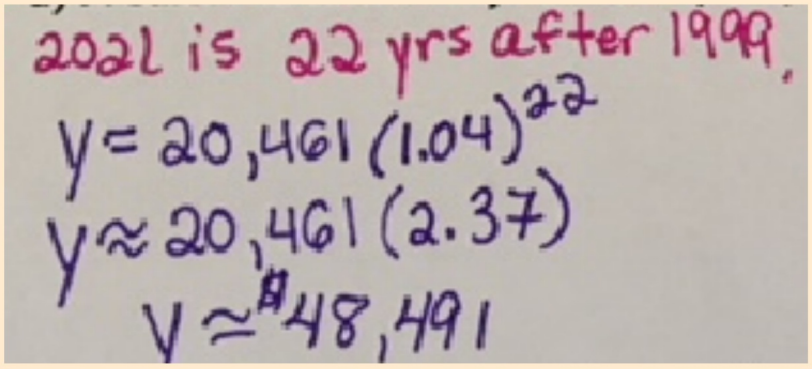


Predictions:

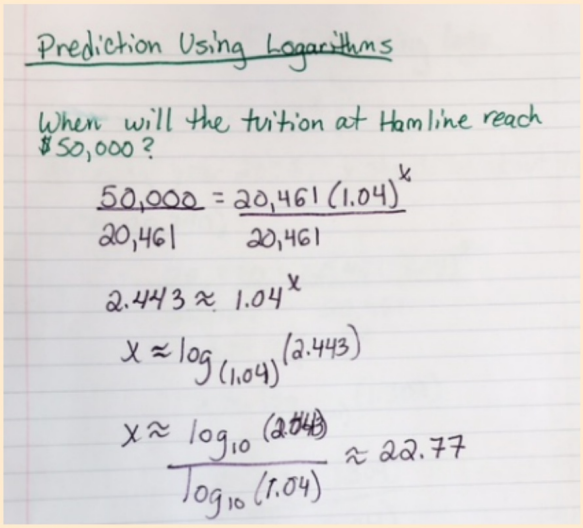
5) Make predictions using your graph and/or equation.

a) Predict the tuition in year 2021. (The year you graduate high school!)

$$y = 20461(1.04)^{x}$$



Predictions:



Prediction Using Logarithms

When will the tuition at Hamline reach $50,000?

$$\frac{50,000}{20,461} = \frac{20,461(1.04)^x}{20,461}$$

$$2.443 \approx 1.04^x$$

$$x \approx \log_{(1.04)}(2.443)$$

$$x \approx \frac{\log_{10}(2.443)}{\log_{10}(1.04)} \approx 22.77$$

According to this model, Hamline tuition will be $50,000 by 2021 or 2022. This seems unrealistic and over projected since Hamline tuition is $40,284 for the 2019 school year. It is hard to believe tuition would increase by $10,000 over 2 school years.

Conclusion Topics:

Additional Questions to analyze Hamline's tuition:

6) Will this growth continue?

It is likely that tuition costs will continue to grow, but the growth may slow down and not be as high as 4% each year. 2018 tuition costs ($40,284) are $8000 ***less*** than what the model predicts for 2021. It is unlikely that tuition will grow by **$8000** in 3 school years. This evidence shows the growth will likely slow down.

Conclusion Topics:

7) What real-life factors contribute to this growth?

- Factors that contribute to this growth include growing costs to maintain the schools buildings, technology, and personel.
- There are federal subsidies to higher education that have been reduced, which require the schools to charge more.
- Student Services have also increased as more students are suffering from mental health related issues. This requires additional counselors, therapists, and psychologits.

Conclusion Topics:

HAMLINE UNIVERSITY

Undergraduate Tuition Rates

View Tuition Due Dates »

Deposits

Title	Amount
Non-Refundable Deposit (new students only)	$400

Cost Estimator

The Undergraduate Cost Estimator may help undergraduate students, who have an award package, estimate their cost of attendance at Hamline

2018-2019 Tuition Rates

FULL TIME (12-18 CREDITS PER SEMESTER)

Installment	Amount
Per year	$40,284

Consider topics for project!

Project Introduction - EL checklist

Step 1: Data Intro – Table – Graph

___Project **Title**

___Organize data in a table

___Display data using a scatterplot

Step 2: Equation $y = a(b)^x$ $b = (1+r)^x$ OR $b = (1-r)^x$

___Calculate the constant multipliers between each data value

___Identify the starting value ____Justify your starting value

___Identify the constant multiplier ____Justify your constant multiplier

___Write Equation

___Use the collected data values and verify the equation using ***logs.***

$$x = a^y \Leftrightarrow y = \log_a(x)$$

____ Write a statement commenting on the validity of the equation

Step 3: Analysis (Using Table-Graph-Equation)

Interpret the real-life meanings of your equation:

____Starting value _____Constant multiplier

____ **r** (*rate of growth / decay*) *What % is the data growing/decaying ?*

____Dependent variable (*y-value*) ____Independent variable (*x-value*)

___Use ***logarithms*** to make a prediction ***outside*** the collected data set.

___Discuss the accuracy of the prediction

___Use ***logarithms*** to make a prediction ***inside*** the collected data set.

___ Discuss the accuracy of the prediction

____ Write a conclusion of the project.

Title, table Due monday
Start graph 2.26

Rate

EL checklist: Possible questions to explore in your project.

Question Bank:

1) How fast or slow is the data growing or decaying?

2) Will this pattern continue forever? Why/Why not?

3) What real-life factors could change this pattern?

4) How will this pattern affect my life?

5) What impacts could this growth or decay have on society?

6) What other scenarios behave similar to this growth or decay pattern?