

Welcome Back!

1. Staple rubric on top of project.
2. Turn into basket

| | Assignment Effort Grade (Circle One) | Comments (What was interesting or challenging?) |
|--|--|---|
| Monday Date: <u>2/26</u> Topic: <u>Exponential Project Due!</u> | 0 1 2 | |
| Tuesday Date: _____ Topic: _____ | 0 1 2 | |
| Wednesday Date: _____ Topic: _____ | 0 1 2 | |
| Thursday Date: _____ Topic: _____ | 0 1 2 | |
| Friday | | |

Plan for Tuesday 2-27

Knox: (Room N135) Last names: *materials*

A - ELLIS

Paulson: (Room E122) Last names:

ERDRICH - KYLLONEN *video*

Mohamud: (Room 207) Last names:

LACOURSIERE - ROMERO-R *materials*

Kohnert: (Room E102) Last names:

RUSSELL - ZWACK-LAFAURIE *video*

Warm-up:

$$x = \log_3(3^2)$$

$$x = \log_3(9)$$

$$3^x = 9$$

$$x = 2$$

$$x = 2 \log_3(3)$$

$$x = 2(\log_3(3))$$

$$3^a = 3^1$$

$$\Rightarrow x = 2(1) = 2$$

Warm-up:

$$x = \log_5(5^3)$$

$$x = 3 \log_5(5)$$

Class Plan:

1. Warm-up

2. Properties of Logarithms
Part 2

3. Practice & Finish Project

Introduction to Investigation:

Find the value of x .

$$x = \log_4(8^2)$$

Power Property Investigation of Logs

1) Fill in the blanks to discover the property.

a)

$$4^{\frac{3}{2}} = \frac{2}{\sqrt{4}} \boxed{3} = \underline{8}$$

b)

$$\log_4(8) = \underline{1.5}, \log_4(2^3) = \underline{\frac{3}{2}}$$

$$3 \log_4(2) = 3(\underline{.5}) = \underline{1.5}$$

Try conclude

What can we conclude?

$$\text{ii. } \log_4(2^3) = \underline{3} \log_4(\underline{2})$$

"How many factors of 4 produce 8?"

$$4^1 \cdot 4^{\frac{1}{2}} = 4 \cdot 2 = 8$$

"How many factors of 4 produce 2?"

$$4^{\frac{1}{2}} = 2 = \sqrt{4}$$

Investigation: Properties of Logarithms

5) "Power property of exponents"

a)

$$4^{\frac{3}{2}} = \frac{2\sqrt{4^3}}{1} = \underline{8}$$

b)

i. $\log_4(8) = \underline{\frac{3}{2}}, \log_4(2^3) = \underline{\frac{3}{2}}$

ii. $3\log_4(2) = 3(\underline{\frac{1}{2}}) = \underline{\frac{3}{2}}$

Question: What can we conclude?

iii. $\log_4(2^3) = \underline{3} \log_4(\underline{2})$

Solutions

Power Property Investigation of Logs

c)

$$2^{\frac{10}{2}} = \sqrt{2^{\boxed{}}} = \underline{\hspace{2cm}}$$

d)

$$\log_2(32) = \underline{\hspace{1cm}}, \log_2(2^5) = \underline{\hspace{1cm}}$$

$$5\log_2(2) = 5(\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$$

What can we conclude?

$$\text{iii. } \log_2(2^5) = \underline{\hspace{1cm}} \log_2(\underline{\hspace{1cm}})$$

Examine parts a), b), c) and d) when finding the log of a value that is raised by an exponent.

$$\log_a(x^y) = \underline{\hspace{1cm}} \log_a(\underline{\hspace{1cm}})$$

Investigation: Properties of Logarithms

c)

$$2^{\frac{10}{2}} = \sqrt[2]{2^{10}} = 32$$

d)

i. $\log_2(32) = 5, \log_2(2^5) = 5$

ii. $5 \log_2(2) = 5(\underline{1}) = 5$

Question: What can we conclude?

iii. $\log_2(2^5) = 5 \log_2(\underline{2})$

6) Examine parts a), b), c) and d) when finding the log of a value that is raised by an exponent.

$$\log_a(x^y) = y \log_a(x)$$

Solutions

$$(x^m)^n$$

Change of Base Property

$$\log_3(70) = n$$

$$(3^n = 70) \log_x(\quad)$$

$$\log_x(3^n) = \log_x(70)$$

$$\frac{n \log_x(3)}{\log_x(3)} = \frac{\log_x(70)}{\log_x(3)} =$$

$$\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$$

a) Why does this work?

1. Rewrite as an exponential expression
2. Take the log base x of both sides
3. Apply the Power Property
4. Rearrange your equation.

$$\log_3(70) = n$$

$$3^n = 70$$

$$\log_x(3^n) = \log_x(70)$$

$$n \cdot \log_x(3) = \log_x(70)$$

$$n = \frac{\log_x(3)}{\log_x(70)} \text{ hence,}$$

$$\log_3(70) = \frac{\log_x(3)}{\log_x(70)}$$

Change of Base Property

a) Why does this work?

1. Rewrite as an exponential expression
2. Take the log base x of both sides
3. Apply the Power Property
4. Rearrange your equation.

Division, isolate n

$$\log_b(a) = \frac{\log_x(\underline{a})}{\log_x(\underline{b})}$$

Notes

Properties of Exponents and Logarithms

Definition of Logarithm

If $x = a^m$, then $\log_a x = m$.

Product Property

$$a^m \cdot a^n = a^{m+n} \quad \text{or} \quad \log_a xy = \log_a x + \log_a y$$

Quotient Property

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{or} \quad \log_a \frac{x}{y} = \log_a x - \log_a y$$

Power Property

$$\log_a x^n = n \log_a x \quad (\text{Today's property!})$$

Power of a Power Property

$$(a^m)^n = a^{mn}$$

Power of a Product Property

$$(ab)^m = a^m b^m$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Change-of-Base Property

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Definition of Rational Exponents

$$a^{m/n} = (\sqrt[n]{a})^m \quad \text{or} \quad \sqrt[n]{a^m}$$

Definition of Negative Exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Product Property

$$a^m \cdot a^n = a^{m+n} \quad \text{or} \quad \log_a xy = \log_a x + \log_a y$$

Quotient Property

$$\frac{a^m}{a^n} = a^{m-n} \quad \text{or} \quad \log_a \frac{x}{y} = \log_a x - \log_a y$$

Power Property

$$\log_a x^n = n \log_a x$$

Today

Power of a Power Property

$$(a^m)^n = a^{mn}$$

Power of a Product Property

$$(ab)^m = a^m b^m$$

Power of a Quotient Property

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Change-of-Base Property

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Exercises:

REVIEW: Evaluate each expression. (Show conversion to an exponential expression)

1) $\log_3 81$

2) $\log_6 36$

3) $\log_5 125$

4) $\log_2 32$

$2^x = 32$
 $x = 5$

5) $\log_4 \frac{1}{16}$

6) $\log_2 \frac{1}{4}$

7) $\log_6 216$

8) $\log_4 \frac{1}{64}$

$4^{-3} = \frac{1}{4^3} = \frac{1}{64} = 4^x$

$x = -3$

Exercises:

Expand each logarithm.

9) $\log 5^2$

10) $\log (7 \cdot 3)$

11) $\log \sqrt{2} = \frac{1}{2} \log 2$

12) $\log 5^5$

13) $\log 8^4$

14) $\log \sqrt[3]{12}$

$$\frac{\log 12}{3} = \log 12^{\frac{1}{3}} = \boxed{\frac{1}{3} \log 12}$$

Exercises: Expand each logarithm

$$15) \log \frac{3}{7} = \log 3 - \log 7$$

$$16) \log (7 \cdot 8)$$

$$17) \log (7 \cdot 6 \cdot 11^6)$$

$$18) \log_2 \sqrt[3]{5 \cdot 3 \cdot 8}$$

$$\log 7 + \log 6 + \log(11^6)$$

$$\log 7 + \log 6 + 6 \log 11$$

Exercises: Expand each logarithm

15) $\log \frac{3}{7}$

16) $\log (7 \cdot 8)$

17) $\log (7 \cdot 6 \cdot 11^6)$

18) $\log_2 \sqrt[3]{5 \cdot 3 \cdot 8}$

$$\frac{1}{3} \log_2 5 + \frac{1}{3} \log_2 3 + \frac{1}{3} \log_2 8$$

$$\frac{1}{3} \log_2 5 + \frac{1}{3} \log_2 3 + 1$$

Exercises: Expand each logarithm

$$19) \log_6 (11\sqrt[3]{3 \cdot 2})$$

$$20) \log_2 (6^2 \cdot 5^4)$$

$$21) \log_5 \left(\frac{11^4}{10} \right)^2$$

$$22) \log_3 \sqrt[3]{12 \cdot 7 \cdot 5}$$

Exercises: Expand each logarithm

$$21) \log_5 \left(\frac{11^4}{10} \right)^2$$

$$22) \log_3 \sqrt[3]{12 \cdot 7 \cdot 5}$$

$$\begin{aligned} & \log_3 (12 \cdot 7 \cdot 5)^{\frac{1}{3}} \\ & \log_3 12^{\frac{1}{3}} + \log_3 7^{\frac{1}{3}} + \log_3 5^{\frac{1}{3}} \\ & \frac{1}{3} (\log_3 12 + \log_3 7 + \log_3 5) \end{aligned}$$

Exercises: Expand each logarithm

$$21) \log_5 \left(\frac{11^4}{10} \right)^2$$

$$22) \log_3 \sqrt[3]{12 \cdot 7 \cdot 5}$$

$$2 \log_5 11^4 - 2 \log_5 10$$

$$8 \log_5 11 - 2 \log_5 10$$

$$x^2 = \frac{x^5}{x^3}$$

Exercises: Expand each logarithm

23) $\log_5 (11\sqrt[3]{7 \cdot 10})$

24) $\log_9 (3^6 \cdot 10)^3$

$\log_9 3 = \frac{1}{2}$
 $9^{\frac{1}{2}} = \sqrt{9} = 3$

$\log_9 (3^6)^3 + \log_9 10^3$
 $18 \log_9 3 + 3 \log_9 10$
 $18 \left(\frac{1}{2}\right) + 3 \log_9 10$

$9 + 3 \log_9 10$

Exercises: Expand each logarithm

25) $\log_2 \sqrt[3]{7 \cdot 5 \cdot 8}$

26) $\log_2 (8^3 \sqrt[3]{3})$

$$\begin{aligned} & \log_2 (8^3) + \log_2 3^{1/3} \\ & 3 \log_2 (8) + \frac{1}{3} \log_2 (3) \\ & 3(3) + \frac{\log_2 (3)}{3} \\ & \boxed{9 + \frac{\log_2 (3)}{3}} \end{aligned}$$

Exercises:

Use a calculator and the change of base formula to approximate each to the nearest thousandth.

27) $\log_3 4.7$

28) $\log_7 13$

29) $\log_6 6.9$

30) $\log_5 20$

Exercises:

Use a calculator and the change of base formula to approximate each to the nearest thousandth.

31) $\log_4 6$

32) $\log_2 1.2$

33) $\log_6 8$

34) $\log_2 2$

Exercises:

Solutions!

- 1) 4 2) 2 3) 3 4) 5
5) -2 6) -2 7) 3 8) -3
9) $2\log 5$ 10) $\log 7 + \log 3$ 11) $\frac{\log 2}{2}$ 12) $5\log 5$
13) $4\log 8$ 14) $\frac{\log 12}{3}$ 15) $\log 3 - \log 7$ 16) $\log 7 + \log 8$
17) $\log 7 + \log 6 + 6\log 11$ 18) $\frac{\log_2 5}{3} + \frac{\log_2 3}{3} + 1$ 19) $\log_6 11 + \frac{\log_6 3}{3} + \frac{\log_6 2}{3}$
20) $2\log_2 6 + 4\log_2 5$ 21) $8\log_5 11 - 2\log_5 10$ 22) $\frac{\log_3 12}{3} + \frac{\log_3 7}{3} + \frac{\log_3 5}{3}$
23) $\log_5 11 + \frac{\log_5 7}{3} + \frac{\log_5 10}{3}$ 24) $18\log_9 3 + 3\log_9 10$
25) $\frac{\log_2 7}{3} + \frac{\log_2 5}{3} + 1$ 26) $9 + \frac{\log_2 3}{3}$ 27) 1.409
28) 1.318 29) 1.078 30) 1.861 31) 1.292
32) 0.263 33) 1.161 34) 1