Welcome Back!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 2/26 Topic: Exponential Project Due!	0 1 2	
Tuesday Date: 2/27 Topic: **ACT For Juniors**	0 1 2	
Wednesday Date: 2/28 Topic: Power Property of Logarithms	0 1 2	
Thursday Date: 3/1 Topic: Review of Logarithms	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

1. Homework Questions?

2. Examples

3. Unit Test Review Time

-Use old handouts to practice

-Examine notes (online)

-Redo quiz problems

Properties of Exponents

Example:

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

$$\left(yx^{-1} \cdot 2y^{2}\right)^{2}$$

$b^m \cdot b^n = b^{m+n}$	$(b^m)^n = b^{m \cdot n}$	$(ab)^n = a^n \cdot b^n$	$b^0 = 1$
$\frac{b^m}{b^n} = b^{m-n}$	$b^{-n} = \frac{1}{b^n}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

Properties of Exponents

Example:

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

$$\left(yx^{-1} \cdot 2y^{2}\right)^{2} = \frac{4y^6}{x^2} = 4y^6 \cdot \frac{1}{x^2}$$

$$(y\,x^{-1}\,2\,y^2)(y\,x^{-1}\,2\,y^2)$$

$$2 \cdot 2 \cdot x^{-1} \cdot x^{-1} \cdot y^1 \cdot y^1 \cdot y^2 \cdot y^2$$

$$4x^{-2}y^6$$

Properties of Exponents

$$\frac{1}{a^{-m}} = \frac{a^m}{1}$$

Example:

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

$$\frac{x^0 y^0}{y^2 \cdot \left(x^{\frac{1}{4}} y^{-\frac{5}{3}}\right)^{\frac{5}{3}}}$$

$$x^{\frac{1}{4}\left(\frac{5}{3}\right)} = x^{\frac{5}{12}}$$

$$y^{-\frac{5}{3}\left(\frac{5}{3}\right)} = y^{-\frac{25}{9}}$$

$$= \frac{1}{y^2 x^{5/12} \cdot y^{-25/9}} = \frac{1}{y^{18/9} \cdot y^{-25/9} x^{5/12}}$$

$$= \frac{1}{y^{-7/9} \cdot x^{5/12}} = \frac{y^{7/9}}{x^{5/12}} \left(\frac{x^{7/12}}{x^{7/12}}\right)$$

$$= \frac{y^{7/9} \cancel{x}^{7/12}}{x}$$

Example:

Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.

$$\frac{x^0 y^0}{y^2 \cdot \left(x^{\frac{1}{4}} y^{-\frac{5}{3}}\right)^{\frac{5}{3}}} = \frac{1}{y^2 \cdot x^{\frac{5}{12}} y^{-\frac{25}{9}}}$$

$$\frac{1}{4} \cdot \frac{5}{3} = \frac{5}{12}$$

$$\frac{-5}{3} \cdot \frac{5}{3} = \frac{-25}{9}$$

$$\frac{y^{\frac{25}{9}}}{y^{2} \cdot x^{\frac{5}{12}}} = \frac{y^{\frac{7}{9}}}{x^{\frac{5}{12}}} \cdot \frac{x^{\frac{7}{12}}}{x^{\frac{7}{12}}}$$

$$2 = \frac{18}{9}$$

$$\frac{25}{9} - \frac{18}{9} = \frac{7}{9}$$

$$= \frac{y^{\frac{7}{9}} x^{\frac{7}{12}}}{x}$$

Properties of Exponents

Example: Simplify. (No calculator)

$$\left(4x^{4}\right)^{\frac{3}{2}} = \sqrt[2]{(4x^4)^3} = \sqrt{64x^{12}}$$

$$= \boxed{8x^6}$$

Properties of Exponents

Example: Simplify. (No calculator)

$$(4x^4)^{\frac{3}{2}} = 4^{\frac{3}{2}} x^{4 \cdot \frac{3}{2} = \frac{12}{2}}$$

$$= 4^{3 \cdot \frac{1}{2}} x^6$$

$$= \sqrt{4^3}\, x^6$$

$$= \sqrt{64}\, x^6 = 8x^6$$

Properties of Logarithms

Example: (4, + 16) base 2

Show/verify this property works using base 2.

$$\log_b(a) - \log_b(c) = \log_b\left(\frac{a}{c}\right)$$

$$\log_2(4) - \log_2(16) = \log_2\left(\frac{4}{16}\right)$$

$$2 - 4 = -2$$

$$-2 = -2 \checkmark$$

Properties of Logarithms

Example:

Show/verify this property works using base 2.

$$\log_b(a) - \log_b(c) = \log_b\left(\frac{a}{c}\right)$$

$$\log_2 32 - \log_2 4 = \log_2\left(\frac{32}{4}\right)$$

$$5 - 2 = \log_2(8)$$

$$3 = 3 \checkmark$$

Properties of Logarithms

<u>Example:</u>

$$\log_b(a) - \log_b(c) = \log_b\left(\frac{a}{c}\right)$$

$$\log_2(32) - \log_2(8) = \log_2\left(\frac{32}{8}\right)$$

$$5 - 3 = \log_2(4)$$

$$5 - 3 = 2$$

$$\boxed{2 = 2} \quad \checkmark$$

Exponential Growth/Decay

Example: $y = 500(1.002)^x$

Miles deposited \$500 into a bank account with .2% annual interest.

a) How much will Miles have after 10 years?

$y = 500(1.002)^{10} \approx 510.09$

b) When will Miles have over \$600?
Show all work.

$\frac{600}{500} < \frac{500(1.002)^x}{500}$

$1.2 < 1.002^x$

$x = \log_{1.002}(1.2)$

91.25 yrs

Exponential Growth/Decay

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$y = 500(1.002)^{10}$

$y \approx \$510.09$

b) When will Miles have oer $600?
Show all work.

$\frac{600}{500} = \frac{\cancel{500}(1.002)^x}{\cancel{500}}$

$1.2 = 1.002^x$

$\log_{1.002} 1.2 = x$

$91 \text{ yrs} \approx x$

Recall the Log Properties

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p \log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$
Equality Rule	If $\log_a x = \log_a y$ then $x = y$

Properties of Exponents and Logarithms

Notes

Definition of Logarithm

If $x = a^m$, then $\log_a x = m$.

Product Property

$a^m \cdot a^n = a^{m+n}$ or $\log_a xy = \log_a x + \log_a y$

Quotient Property

$\frac{a^m}{a^n} = a^{m-n}$ or $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Property

$\log_a x^n = n \log_a x$ (Monday's property!)

Power of a Power Property

$(a^m)^n = a^{mn}$

Power of a Product Property

$(ab)^m = a^m b^m$

Power of a Quotient Property

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Change-of-Base Property

$\log_a x = \frac{\log_b x}{\log_b a}$

Definition of Rational Exponents

$a^{m/n} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$

Definition of Negative Exponents

$a^{-n} = \frac{1}{a^n}$ or $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Product Property

$a^m \cdot a^n = a^{m+n}$ or $\log_a xy = \log_a x + \log_a y$

Quotient Property

Notes...zoome

$\dfrac{a^m}{a^n} = a^{m-n}$ or $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

Power Property

$\log_a x^n = n \log_a x$

Power of a Power Property

$(a^m)^n = a^{mn}$

Power of a Product Property

$(ab)^m = a^m b^m$

Power of a Quotient Propert

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Change-of-Base Property

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Exercises:

Solutions!

Rewrite each equation in logarithmic form.

1) $3^5 = 243$

$5 = \log_3(243)$

2) $17^{-2} = \frac{1}{289}$

$-2 = \log_{17}\left(\frac{1}{289}\right)$

3) $v^{-5} = u$

$-5 = \log_v(u)$

4) $y^{10} = x$

$10 = \log_y(x)$

Expand each logarithm.

5) $\log\left(\frac{3}{5^6}\right)^5$

$5\log 3 - 5\log 5^6$

$= 5\log 3 - 30\log 5$

6) $\log_8(5 \cdot 12 \cdot 7^4)$

$\log_8(5) + \log_8(12) + 4\log_8(7)$

Exercises:

Solutions!

7) $\log_8\left(c\sqrt{a\cdot b}\right) = \log_8 c\cdot a^{\frac{1}{2}}\cdot b^{\frac{1}{2}}$

$= \log_8 c + \frac{1}{2}\log_8 a + \frac{1}{2}\log_8 b$

8) $\log_4\left(z^5\sqrt{x}\right) = \log_4\left(z^5 x^{\frac{1}{2}}\right)$

$= 5\log_4(z) + \frac{1}{2}\log_4(x)$

Condense each expression to a single logarithm.

9) $3\log_6 x - 6\log_6 y = \dfrac{3\log_6(x)}{6\log_6(y)}$

$= \dfrac{\log_6 x^3}{\log_6 y^6} = \log_6\left(\dfrac{x^3}{y^6}\right)$

10) $6\log_5 c + \dfrac{\log_5 a}{2} = 6\log_5 c + \frac{1}{2}\log_5 a$

$= \log_5 c^6 + \log_5 a^{\frac{1}{2}}$ $\left\{a^{\frac{1}{2}} = \sqrt{a}\right\}$

$= \log_5\left(c^6\cdot\sqrt{a}\right)$

11) $36\log_8 10 + 6\log_8 3$

$\log_8\left(10^{36}\cdot 3^6\right)$

12) $\log_7 11 + \log_7 10 + 5\log_7 3$

$\log_7\left(11\cdot 10\cdot 3^5\right) = \log_7\left(110\cdot 3^5\right)$

Exercises:

Solutions!

Rewrite using an exponential equation, then evaluate each expression.

13) $\log_7 49$

$7^x = 49$

$\boxed{x = 2}$

14) $\log_{216} \frac{1}{6}$

$216^x = \frac{1}{6}$ $\quad 216^{\frac{1}{3}} = 6$

$\boxed{x = -\frac{1}{3}}$ $\quad 216^{-\frac{1}{3}} = \frac{1}{6}$

15) $\log_9 3$

$9^x = 3$

$\boxed{x = \frac{1}{2}}$

16) $\log_2 64$

$2^x = 64$ $\boxed{x = 6}$

Rewrite using an exponential equation. Then use a calculator to approximate each to the nearest thousandth.

17) $\log_3 14$

$3^x = 14$

$x \approx 2.402$

18) $\log_3 17$

$3^x = 17$ $\quad x \approx 2.579$

19) $\log_2 2.7$

$2^x = 2.7$

$x \approx 1.433$

20) $\log_7 62$

$7^x = 62$ $\quad x \approx 2.121$

Exercises:

Solutions!

Solve each equation. (Note, 24 requires solving a quadratic equation.)**

21) $\log_9 (3n+2) = \log_9 (n+7)$

$3n+2 = n+7$

$2n = 5$

$\boxed{n = 5/2}$

22) $\log_{19} (-3k+8) = \log_{19} -4k$

$-3k+8 = -4k$

$8 = -k$

$-8 = k$

Exercises: **Solutions!**

23) $\log_7(4a-2)=\log_7 5a$

$4a-2=5a$

$\boxed{-2=a}$

But $\log_7(5(-2))$

$=\log_7(-10)$

$\underbrace{7^x=-10}$

No value will ever make 7 multiply to a negative value.

24) $\log_{12}(4x^2+4x)=\log_{12}(5+3x^2)$

$4x^2+4x=5+3x^2$

$-3x^2 \qquad -3x^2$

$x^2+4x=5$

$-5 \quad -5$

$x^2+4x-5=0$

$(x+5)(x-1)=0$ {Factoring method}

$\boxed{x=-5,1}$

$x=\dfrac{-4\pm\sqrt{16-4(1)(-5)}}{2(1)}$ {QUAD Formula method}

$x=\dfrac{-4\pm\sqrt{36}}{2}$

$x=\dfrac{-4+6}{2} \qquad x=\dfrac{-4-6}{2}$

$\boxed{x=1}$ and $\boxed{x=-5}$

Exercises...

Study!

Unit 5 Exponetials Test

Tomorrow!