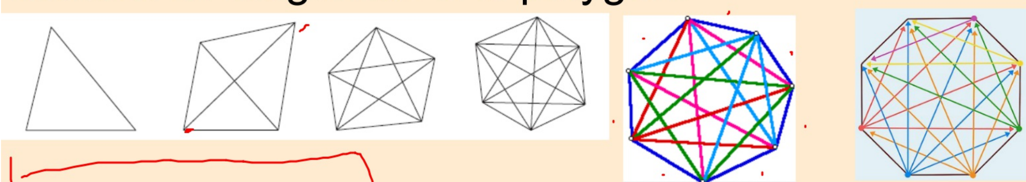
Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 3/5 Topic:____________	0 1 2	No Homework (Test Friday)
Tuesday Date: 3/6 Topic:____________	0 1 2	No Homework
Wednesday Date: ______ Topic:____________	0 1 2	
Thursday Date: ______ Topic:____________	0 1 2	
Friday Date: ______ Topic:____________	0 1 2	

Warm-up: Complete the table that shows the relationship between the number of sides and the number of diagonals of a polygon.

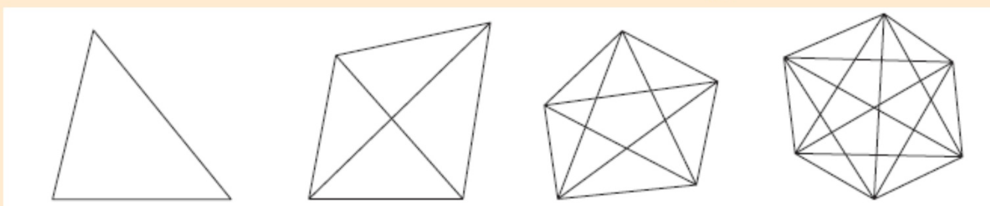


Number of sides *x*	Number of diagonals *y*
3	0
4	2
5	5
6	9
7	14
8	20

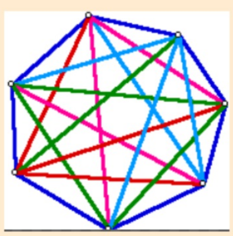
Number of sides x	Number of diagonals y
3	0
4	2
5	5
6	9
7	14
8	20

n: number of sides

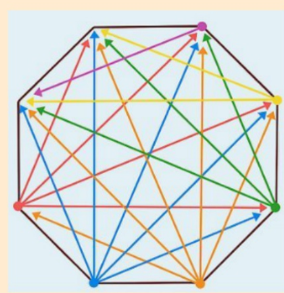
d: total number of diagonals



n=3, d=0 | n=4, d=2 | n=5, d=5 | n=6, d=9



n=7, d=14

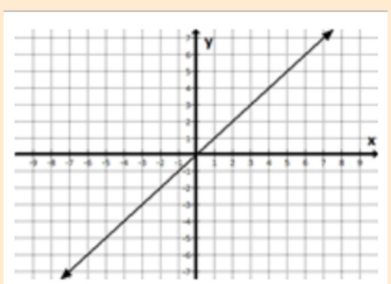


n=8, d=20

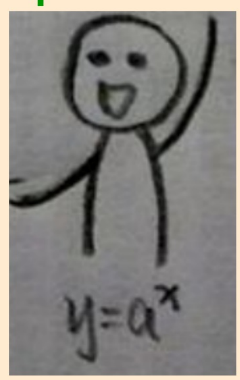
Class Plan:

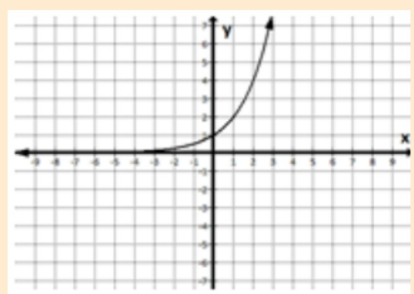
1. Warm-up
2. Introduction: Unit 6 Polynomials
3. Polynomial Degree
4. Practice

Linear

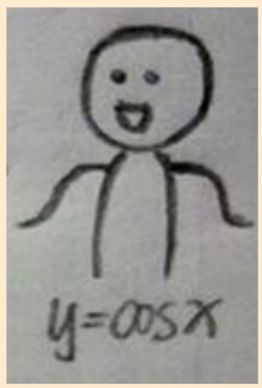


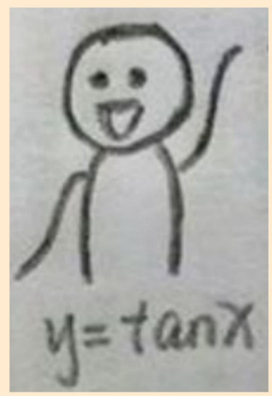
Exponential

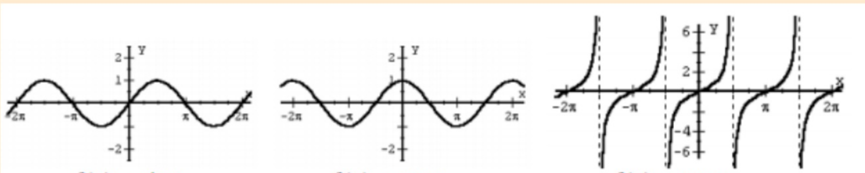




Trigonometric Functions





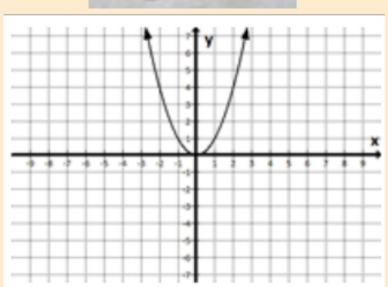


f(x) = sin x   f(x) = cos x   f(x) = tan x

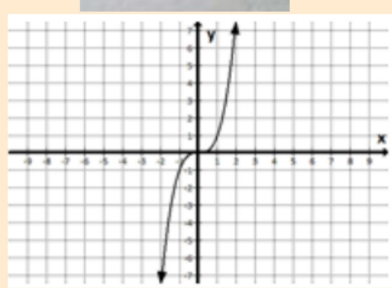
Trigonometric Functions

Polynomials

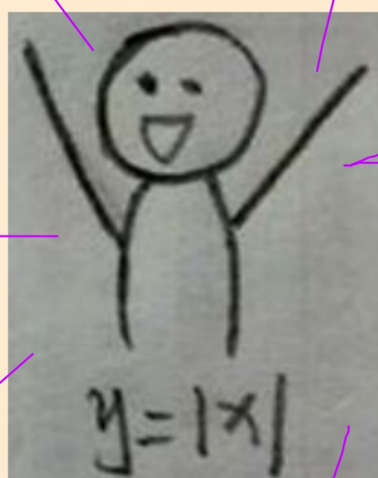
Quadratic

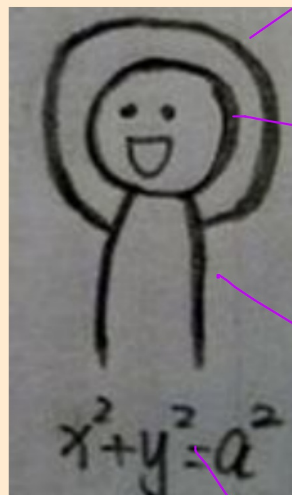


Cubic



...Other sweet dance moves!





What is a **Polynomial**?

Polynomial comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms"

http://www.mathsisfun.com/algebra/polynomials.html

A **polynomial** expression is a sum of **terms** containing the same variable raised to different powers. Each term is a product of numbers and variables. When a polynomial is set equal to a second variable, such as *y*, you have a **polynomial function.**

Definition of a Polynomial

A **polynomial** in one variable is any expression that can be written in the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$$

where x is a variable, the exponents are nonnegative integers, and the coefficients are real numbers.

What is a **Polynomial**?

$y = 2^{-4} + x^3$

Definition of a Polynomial

A **polynomial** in one variable is any expression that can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$$

where x is a variable, the exponents are nonnegative integers, and the coefficients are real numbers.

...An expression with:

1) **Variables:** no fractional or negative exponents
2) Coefficients are real numbers.

Degree - Power of the term that has the greatest exponent

General Form - degrees of terms decrease left to right $y = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0$

degree 3

Coefficients (if a term has no coefficient, the coefficient is 1)

Polynomial Function: $y = 1x^3 + 9x^2 + 26x^1 + 24\,x^0$

Highest-degree term

Constant term

Polynomial

What **degree** is a **linear** function? $y = mx^1 + b$

(one)

A polynomial that has only one term is called a **monomial.** A polynomial with two terms is a **binomial,** and a polynomial with three terms is a **trinomial.** Polynomials with more than three terms are usually just called "polynomials."

Monomial, Binomial, Trinomial

There are special names for polynomials with 1, 2 or 3 terms:

$3xy^2$	$5x - 1$,	$3x + 5y^2 - 3$
Monomial (1 term)	Binomial (2 terms)	Trinomial (3 terms)

How do you remember the names? Think cycles!

http://www.mathsisfun.com/algebra/polynomials.html

What is the degree of a function?

Finite Differences Method:

***Used to find the degree of a polynomial.

You can use this method to find the degree of the polynomial function that models a certain set of data. Analyzing differences to find a polynomial's degree is called the **finite differences method**.

Subtract y-values if x-values are increasing ***or*** decreasing at a constant rate.

Example 1: $y = 2x^2 - 5x - 7$

What degree is the function?

x	y
3.7	1.88
3.8	2.88
3.9	3.92
4.0	5.00
4.1	6.12
4.2	7.28

D_1 D_2

1
1.04
1.08
1.12
1.16

.04
.04
.04
.04

Degree 2

Example 2: $y = 0.1x^3 - x^2 + 3x - 5$

What degree is the function? 3

x	y
−5	−57.5
0	−5
5	−2.5
10	25
15	152.5
20	455

D_1	D_2	D_3
52.5	−50	75
2.5	25	75
27.5	100	75
127.5	175	
302.5		

Looking over the examples...

Note that in each case the x-values are spaced equally. You find the first set of differences, D_1, by subtracting each y-value from the one after it. You find the second set of differences, D_2, by finding the differences of consecutive D_1 values in the same way. Notice that for the 2nd-degree polynomial function, the D_2 values are constant, and that for the 3rd-degree polynomial function, the D_3 values are constant. What do you think will happen with a 4th- or 5th-degree polynomial function?

A Chapter 21 Notes QUADRATIC FUNCTIONS

A **quadratic function** is a relationship between two variables which can be written in the form $\boldsymbol{y = ax^2 + bx + c}$, where x and y are the variables, and a, b, and c are constants, $a \neq 0$.

general $y = ax^2 + bx + c$

The constants a, b, and c are called the **coefficients** of the quadratic function.

For example:

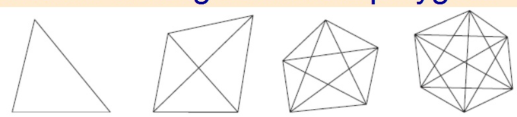
- $y = 3x^2 - 2x + 1$ is a quadratic function with $a = 3$, $b = -2$, $c = 1$
- $y = 2x^2 + 3x$ is a quadratic function with $a = 2$, $b = 3$, $c = 0$.

How many equations are needed to find 3 unknown coefficients (*a, b, c*)?

From the warm-up:

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.

Step 1: Find the degree of the polynomial.



Number of sides x	Number of diagonals y
3	0
4	2
5	5
6	9
7	14
8	20

D_1	D_2
2	
	1
3	
	1
4	
	1
5	
	1
6	

From the warm-up:

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.

Goal: Find coefficients a, b, c, in general form.

$$y = ax^2 + bx + c$$

Step 2: Write the general form of the quadratic. Choose three pairs of values.

We will choose....

Number of sides x	Number of diagonals y
3	0
4	2
5	5
6	9
7	14
8	20

(4, 2), (6, 9), and (8, 20)

From the warm-up:

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.

Goal: Find coefficients a, b, c, in general form.

Step 2: Write the general form of the quadratic.

Choose three pairs of values.

$y = ax^2 + bx + c$ **(4, 2), (6, 9), and (8, 20)**

$2 = a(4)^2 + b(4) + c$

$2 = 16a + 4b + c$

$9 = a(6)^2 + b(6) + c$

$9 = 36a + 6b + c$

$20 = a(8)^2 + 8b + c$

$20 = 64a + 8b + c$

From the warm-up:

$y = \frac{1}{2}x^2 - \frac{3}{2}x$

Step 3: Use the three equations to solve for *a, b,* and *c*.

$2 = 16a + 4b + c \quad (1)$

$9 = 36a + 6b + c \quad (2)$

$20 = 64a + 8b + c \quad (3)$

$(3) - (2)$

$20 = 64a + 8b + c$

$-(9 = 36a + 6b + c)$

$11 = 28a + 2b$

$(2) - (1)$

$9 = 36a + 6b + c$

$-(2 = 16a + 4b + c)$

$7 = 20a + 2b$

$11 = 28a + 2b$

$-(7 = 20a + 2b)$

$\frac{4}{8} = \frac{8a}{8}$

$a = \frac{1}{2}$

$7 = 20\left(\frac{1}{2}\right) + 2b$

$7 = 10 + 2b$

$\frac{-3}{2} = \frac{2b}{2}$

$b = -\frac{3}{2}$

$2 = 16\left(\frac{1}{2}\right) + 4\left(-\frac{3}{2}\right) + c$

$2 = 8 - 6 + c$

$2 = 2 + c$

$0 = c$

From the warm-up: Solution

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.

Step 3: Use the three equations to solve for *a, b,* and *c*.

$$y = ax^2 + bx + c$$

$2 = 16a + 4b + c$ (1)
$9 = 36a + 6b + c$ (2)
$20 = 64a + 8b + c$ (3)

$a = \frac{1}{2}$ | $b = -\frac{3}{2}$

$2 = 16\left(\frac{1}{2}\right) + 4\left(\frac{-3}{2}\right) + c$

$2 = 8 - 6 + c$

$2 = 2 + c$

$0 = c$

$y = \frac{1}{2}x^2 - \frac{3}{2}x$

<u>**From the warm-up:**</u> Work condensed

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.

Step 3: Use the three equations to solve for *a, b,* and *c*.

$36a + 6b + c = 9$

$-(16a + 4b + c = 2)$

$20a + 2b = 7$

(Substitute to find b)

$20\left(\frac{1}{2}\right) + 2b = 7$

$10 + 2b = 7$

$2b = -3$

$\boxed{b = -\frac{3}{2}}$

$64a + 8b + c = 20$

$-(36a + 6b + c = 9)$

$28a + 2b = 11$

(Eliminate b to find a)

$28a + 2b = 11$

$-(20a + 2b = 7)$

$8a = 4$

$\boxed{a = \frac{1}{2}}$

(Use a and b to fin c)

$16\left(\frac{1}{2}\right) + 4\left(\frac{-3}{2}\right) + c = 2$

$8 - 6 + c = 2$

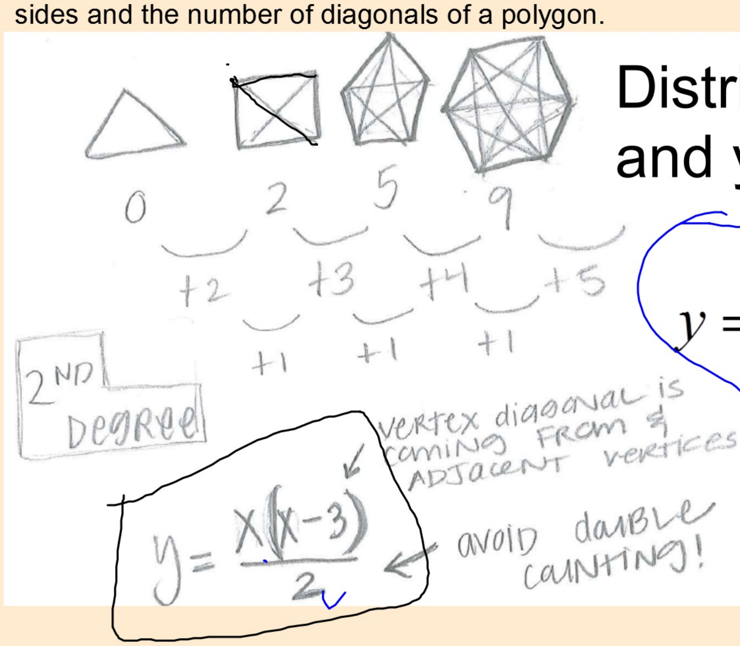
$2 + c = 2$

$\boxed{c = 0}$

$$y = \frac{1}{2}x^2 - \frac{3}{2}x$$

From the geometry perspective:

Find a polynomial function that models the relationship between the number of sides and the number of diagonals of a polygon.



Distribute the ***x*** and you get:

$$y = \frac{1}{2}x^2 - \frac{3}{2}x$$

Ms. Paulson's favorite meme :)

Exercises...

$-\frac{4}{p^2} = -4p^{-2}$

1. Identify the degree of each polynomial.

a. $x^3 + 9x^2 + 26x + 24$

b. $7x^2 - 5x$

c. $x^7 + 3x^6 - 5x^5 + 24x^4 + 17x^3 - 6x^2 + 2x + 40$

d. $16 - 5x^2 + 9x^5 + 36x^3 + 44x$

2. Determine which of these expressions are polynomials. For each polynomial, state its degree and write it in general form. If it is not a polynomial, explain why not.

a. $-3 + 4x - 3.5x^2 + \frac{5}{9}x^3$

b. $5p^4 + 3.5p - \frac{4}{p^2} + 16$

c. $4\sqrt{x^3} + 12$

d. $x^2\sqrt{15} - x - 4^{-2}$

$\sqrt{x^3} = x^{3/2}$

Exercises...

4. Find the degree of the polynomial function that models these data.

x	0	2	4	6	8	10	12
y	12	−4	−164	−612	−1492	−2948	−5124

3. For each data set, decide whether the last column shows constant values. If it does not, calculate the next set of finite differences.

a.

x	y
2	4.4
3	6.6
4	9.2
5	11.0
6	10.8
7	7.4

b.

x	y	D_1
3.7	−8.449	
		−0.257
3.8	−8.706	
		−0.250
3.9	−8.956	
		−0.244
4.0	−9.200	
		−0.236
4.1	−9.436	
		−0.227
4.2	−9.662	

c.

x	y	D_1	D_2
−5	−101		
		95	
0	−6		−100
		−5	
5	−11		50
		45	
10	34		200
		245	
15	279		350
		595	
20	874		

Exercises...

5. Consider the data at right.

n	1	2	3	4	5	6
s	1	3	6	10	15	21

a. Calculate finite differences to find the degree of the polynomial function that models these data.

b. Describe how the degree of this polynomial function is related to the finite differences you calculated.

c. What is the minimum number of data points required to determine the degree of this polynomial function? Why?

d. Find the polynomial function that models these data and use it to find s when n is 12.

e. The values in the s row are called triangular numbers. Why do you think they are called triangular? (*Hint:* Find some pennies and try to arrange each number of pennies into a triangle.)

Exercises...**Solutions**

1. Identify the degree of each polynomial.

a. $x^3 + 9x^2 + 26x + 24$ Degree 3

b. $7x^2 - 5x$ Degree 2

c. $x^7 + 3x^6 - 5x^5 + 24x^4 + 17x^3 - 6x^2 + 2x + 40$ Degree 7

d. $16 - 5x^2 + 9x^5 + 36x^3 + 44x$ Degree 5

2. Determine which of these expressions are polynomials. For each polynomial, state its degree and write it in general form. If it is not a polynomial, explain why not.

a. $-3 + 4x - 3.5x^2 + \frac{5}{9}x^3$ $\frac{5}{9}x^3 - 3.5x^2 + 4x - 3$ Degree 3

b. $5p^4 + 3.5p - \frac{4}{p^2} + 16$ Not polynomial. ← Negative Exponent.

c. $4\sqrt{x^3} + 12$ Not Polynomial. → $4x^{\frac{3}{2}} + 12$ has a non-integer power.

d. $x^2\sqrt{15} - x - 4^{-2}$ Polynomial. Degree 2. Aready in general form.

Exercises...**Solutions**

3. For each data set, decide whether the last column shows constant values. If it does not, calculate the next set of finite differences.

a.

x	y
2	4.4
3	6.6
4	9.2
5	11.0
6	10.8
7	7.4

D_1: 2.2, 2.6, 1.8, −0.2, −3.4

D_2: 0.4, −0.8, −2, −3.2

D_3: −1.2, −1.2, −1.2

Cubic Polynomial
Degree 3

b.

x	y
3.7	−8.449
3.8	−8.706
3.9	−8.956
4.0	−9.200
4.1	−9.436
4.2	−9.662

D_1: −0.257, −0.250, −0.244, −0.236, −0.227

D_2: 0.007, 0.006, 0.008, 0.009

D_3: −0.001, 0.002, 0.001

D_4: 0.004, −0.001

−0.005

Greater Than Degree 4

c.

x	y
−5	−101
0	−6
5	−11
10	34
15	279
20	874

D_1: 95, −5, 45, 245, 595

D_2: −100, 50, 200, 350

D_3: 150, 150, 150

Cubic Polynomial
Degree 3

Exercises...**Solutions**

4. Find the degree of the polynomial function that models these data.

x	0	2	4	6	8	10	12
y	12	−4	−164	−612	−1492	−2948	−5124

D1 −16 −160 −448 −880 −1456 −2176

D2 −144 −288 −432 −576 −720

D3 −144 −144 −144 −144

Degree 3

Exercises...**Solutions**

5. a. $D_1 = \{2, 3, 4, 5, 6\}$, $D_2 = \{1, 1, 1, 1\}$; 2nd degree

n	1	2	3	4	5	6
s	1	3	6	10	15	21

D_1 2 3 4 5 6

D_2 1 1 1 1

b. The polynomial is 2nd degree, and the D_2 values are constant.

c. 4 points. You have to find the finite differences twice, so you need at least four data points to calculate two D_2 values that can be compared.

d. $s = 0.5n^2 + 0.5n$; $s = 78$ when $n = 12$.
Because the function is quadratic, start with $s = an^2 + bn + c$.

Use three data points to get three equations and solve for the constants a, b, and c.

$$\begin{cases} a + b + c = 1 & \text{Using the data point (1, 1).} \\ 4a + 2b + c = 3 & \text{Using the data point (2, 3).} \\ 9a + 3b + c = 6 & \text{Using the data point (3, 6).} \end{cases}$$

Solve this system to get $a = 0.5$, $b = 0.5$, and $c = 0$. The equation is $s = 0.5n^2 + 0.5n$.

Now substitute 12 for n, and solve for s:
$s = 0.5(12)^2 + 0.5(12) = 72 + 6 = 78$.