Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 3/5 Topic:	0 1 2	No Homework (Test Friday)
Tuesday Date: 3/6 Topic:	0 1 2	
Wednesday Date: 3/7 Topic: Polynomial Degree	0 1 2	
Thursday Date: Topic:	0 1 2	
Friday Date: Topic:	0 1 2	

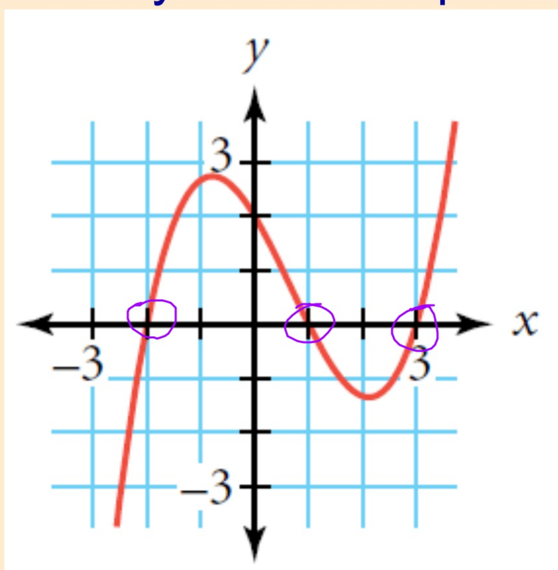
Warm-up: Identify the intercepts.

X-intercept(s)

(-2,0) (1,0)
(3,0)

Y-intercept

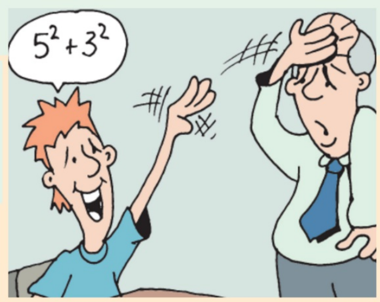
(0,2)



What do you think the order (degree) is of the polynomial? 3

OPENING PROBLEM

Anton thinks that to find the square of the sum of two numbers, you can just square each of the numbers, then add the results.



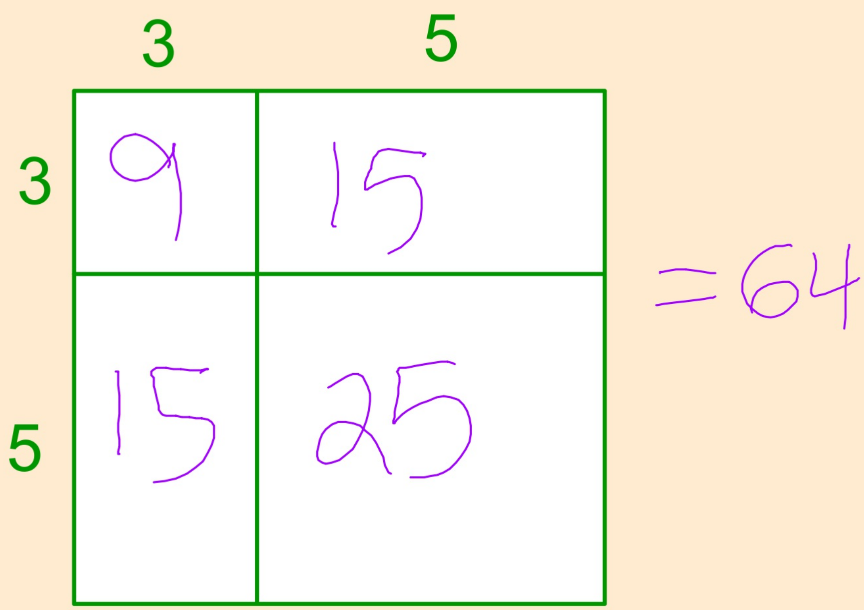
Things to think about:

- **a** Does $(5+3)^2 = 5^2 + 3^2$?
- **b** Can you explain why Anton is incorrect?

PEMDAS

$8^2 = 64 \neq 25 + 9 = 34$

OPENING PROBLEM



Class Plan:

1. Warm-up
2. Factored Form of Polynomials

$$y = a(x - r_1)(x - r_2)(x - r_3)\cdots$$

3. Expanding Polynomials to general form.
4. Practice

Factored form of polynomials

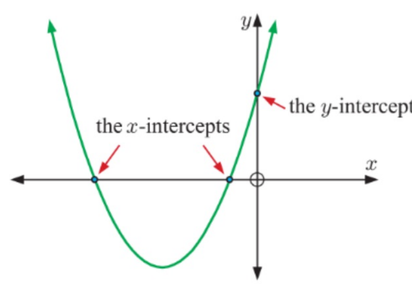
What are roots of a function?

The x-intercepts! (aka: "zeros")

C Chapter 21 **AXES INTERCEPTS**

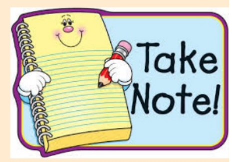
Suppose we are given a function and its graph.

- An **x-intercept** is a value of x where the graph meets the x-axis.
  x-intercepts are found by letting y be 0 in the equation of the function.
- A **y-intercept** is a value of y where the graph meets the y-axis.
  y-intercepts are found by letting x be 0 in the equation of the function.



Factored form of polynomials

Quadratic function (deg. 2):



$$y = a(x - r_1)(x - r_2)$$

Function of higher order:

$$y = a(x - r_1)(x - r_2)(x - r_3) \cdots$$

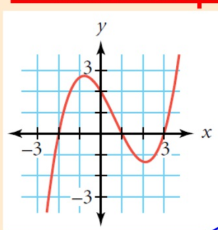
a: stretch/shrink factor

r_1, r_2, r_3, ...: the roots of the graph

Building the equation from the warm-up.

How could we model the graph?

x-intercepts: (-2,0) (1,0) (3,0) Y-intercept: (0,2)



$$y = a(x - r_1)(x - r_2)(x - r_3)\cdots$$

$y = a(x - -2)(x-1)(x-3)$

$y = \frac{1}{3}(x+2)(x-1)(x-3)$

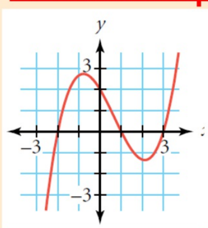
$2 = a(2)(-1)(-3)$

$\frac{2}{6} = \frac{6a}{6}$ $\boxed{a = \frac{1}{3}}$

Building the equation from the warm-up.

How could we model the graph?

x-intercepts: (-2,0) (1,0) (3,0) Y-intercept: (0,2)



c. $y = \frac{1}{3}(x + 2)(x - 1)(x - 3)$. The x-intercepts are -2, 1, and 3, so start with the equation $y = a(x + 2)(x - 1)(x - 3)$. Use the y-intercept point, (0, 2), to solve for a.

$$2 = a(0 + 2)(0 - 1)(0 - 3)$$

$$2 = 6a$$

$$a = \frac{1}{3}$$

The equation is $y = \frac{1}{3}(x + 2)(x - 1)(x - 3)$.

B THE PRODUCT $(a+b)(c+d)$

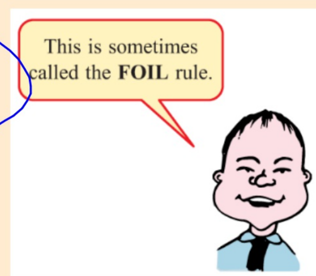
The product $(a+b)(c+d)$ has two **factors**, $(a+b)$ and $(c+d)$.

We can evaluate this product by using the distributive law several times.

The final result contains four terms:

ac	is the product of the	**F**irst	terms of each bracket.
ad	is the product of the	**O**uter	terms of each bracket.
bc	is the product of the	**I**nner	terms of each bracket.
bd	is the product of the	**L**ast	terms of each bracket.

$$\begin{aligned}(a+b)(c+d) &= a(c+d)+b(c+d)\\ &= ac+ad+bc+bd\end{aligned}$$



B Chapter 4 THE PRODUCT $(a+b)(c+d)$

How do we translate to general from factored?:

Example 3

Factored form

$$y = a(x - r_1)(x - r_2)$$

General form

$$y = ax^2 + bx + c$$

a $(x+3)(x+2)$

F: x^2
O: $2x$
I: $3x$
L: 6

$\longrightarrow$

	x	2
x	x^2	$2x$
3	$3x$	6

$= x^2 + 5x + 6$

B Chapter 4 **THE PRODUCT** $(a+b)(c+d)$

How do we translate to general from factored?:

Example 3

a $(x+3)(x+2)$

x^2+5x+6

	x	3
x	x^2	$3x$
2	$2x$	6

Method: Area model

B Chapter 4 THE PRODUCT $(a+b)(c+d)$

Example 3

b $(2x+1)(3x-2)$

	$2x$	1
$3x$	$6x^2$	$3x$
-2	$-4x$	-2

$6x^2 - x - 2$

Joke break! Distribute: P + L

$$(P + L)(A + N)$$

$$PA + PN + LA + LN$$

Uh oh, I just ***FOIL***ed your plan!

Higher order factored equations...

Factored form $y = a(x - r_1)(x - r_2)(x - r_3)\cdots$

General form $y = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$

E Chapter 4 FURTHER EXPANSION

In this Section we expand more complicated expressions by repeated use of the expansion laws.

Consider the expansion of $(a+b)(c+d+e)$.

Now $(a+b)(c+d+e)$ {Compare with $\square(c+d+e)$

$= (a+b)c + (a+b)d + (a+b)e$ $= \square c + \square d + \square e$}

$= ac + bc + ad + bd + ae + be$

Notice that there are 6 terms in this expansion. Each term in the first bracket is multiplied by each tern in the second bracket.

2 terms in the first bracket $\times$ 3 terms in the second bracket $\longrightarrow$ 6 terms in the expansion.

Example 13 Expand and simplify:

$$(2x+3)(x^2+4x+5)$$

$$2x^3 + 8x^2 + 10x + 3x^2 + 12x + 15$$

$$2x^3 + 11x^2 + 22x + 15$$

Example 13 Expand and simplify:

$(2x+3)(x^2+4x+5)$

	$2x$	3
x^2	$2x^3$	$3x^2$
$4x$	$8x^2$	$12x$
5	$10x$	15

$2x^3+11x^2+22x+15$

Method: Area model

Example 13 Expand and simplify:

$$(2x+3)(x^2+4x+5)$$

$2x^3 + 8x^2 + 10x + 3x^2 + 12x + 15$

$$\boxed{2x^3 + 11x^2 + 22x + 15}$$

Exercises...Select problems
from chapter 4, page 68 - 75

B THE PRODUCT $(a+b)(c+d)$

C DIFFERENCE OF TWO SQUARES

D PERFECT SQUARE EXPANSION

E FURTHER EXPANSION

<u>Challenge:</u> 17 - 19

Need additional practice?
Examine more exercises in chapter 4.

Exercises...

Write 1-16 randomly on your game board! Draw a 4x4 Grid

Four in a Row

1	8	7	~~5~~
~~15~~	2	6	~~16~~
14	9	3	~~4~~
13	~~10~~	11	~~12~~

Exercises...

1) $2x(x-5)-3x(2-x)$

2) $(x+2)(x+5)$

Exercises...

3) $(x-3)(x+7)$

4) $(x+5)(x-5)$

	x	5
x	x^2	$5x$
-5	$-5x$	-25

$\boxed{x^2-25}$

Exercises...

5) $(3x+2)(4x+1)$

6) $(k+2)(k-5)-(2k+1)(k-3)$

	$3x$	2
$4x$	$12x^2$	$8x$
1	$3x$	2

$12x^2+11x+2$

Exercises...

7) $(x+2)(x^2+x+1)$

8) $2x(x+1)(7-x)$

Exercises...

$x^3-4x^2-7x+10$

9) $(x+1)(x+2)(x+4)$

10) $(x-5)(x+2)(x-1)$

	x	-5
x	x^2	$-5x$
2	$2x$	-10

$= x^2-3x-10$

	x	-1
x^2	x^3	$-x^2$
$-3x$	$-3x^2$	$3x$
-10	$-10x$	10

Exercises...

11) $(3+x)(3-x)$

12) $(p-q)(p+q)$

$p^2 + \cancel{pq} \cancel{-qp} - q^2$

$\boxed{p^2 - q^2}$

Exercises...

13) $(5-y)^2$

14) $(6-5x)^2$

Exercises...

15) $(x-1)^4$

16) $(x-1)(x^2+x+1)$

$(x-1)(x-1)=x^2-2x+1$

$(x-1)(x-1)=x^2-2x+1$

	x^2	$-2x$	1
x^2	x^4	$-2x^3$	x^2
$-2x$	$-2x^3$	$4x^2$	$-2x$
1	x^2	$-2x$	1

$x^4-4x^3+6x^2-4x+1$

	x	-1
x^2	x^3	$-x^2$
x	x^2	$-x$
1	x	-1

x^3-1

Exercises...Additional Challenges!

17) $\left(x-1\right)\left(x^3+x^2+x+1\right)$

18) Predict the simplification of $\left(x-1\right)\left(x^n+x^{n-1}+x^{n-2}+....+x^2+x+1\right)$ where $n \in \mathbb{Z}^+$.

19) The difference between the cubes of two consecutive integers is 127. Find the integers.

Solutions

Solutions

1. $2x(x-5) - 3x(2-x) = 2x^2 - 10x - 6x + 3x^2 = \boxed{5x^2 - 16x}$
2. $(x+2)(x+5) = x^2 + 5x + 2x + 10 = \boxed{x^2 + 7x + 10}$
3. $(x-3)(x+7) = x^2 + 7x - 3x - 21 = \boxed{x^2 - 4x - 21}$
4. $(x+5)(x-5) = x^2 - 5x + 5x - 25 = \boxed{x^2 - 25}$
5. $(3x+2)(4x+1) = 12x^2 + 3x + 8x + 2 = \boxed{12x^2 + 11x + 2}$

Solutions

6. $(k+2)(k-5) - (2k+1)(k-3) = k^2 - 5x + 2k - 10 - (2k^2 - 6k + k - 3) = \boxed{-k^2 + 2k - 7}$

7. $(x+2)(x^2+x+1)$ $\quad \begin{array}{c|ccc} & x^2 & +x & +1 \\ \hline x & x^3 & x^2 & x \\ +2 & 2x^2 & 2x & 2 \end{array}$ $\quad \boxed{x^3 + 3x^2 + 3x + 2}$

8. $2x(x+1)(7-x)$ $\quad 2x(7x - x^2 + 7 - x) = 2x(-x^2 + 6x + 7) = \boxed{-2x^3 + 2x^2 + 14x}$

9. $(x+1)(x+2)(x+4) = (x+1)(x^2 + 4x + 2x + 8)$ $\quad \begin{array}{c|ccc} & x^2 & +6x & +8 \\ \hline x & x^3 & 6x^2 & 8x \\ +1 & x^2 & 6x & 8 \end{array}$ $\quad \boxed{x^3 + 7x^2 + 14x + 8}$

10. $(x-5)(x+2)(x-1) = (x-5)(x^2 - x + 2x - 2)$ $\quad \begin{array}{c|ccc} & x^2 & +x & -2 \\ \hline x & x^3 & x^2 & -2x \\ -5 & -5x^2 & -5x & 10 \end{array}$ $\quad \boxed{x^3 - 4x^2 - 7x + 10}$

11. $(3+x)(3-x) = 9 - 3x + 3x - x^2 = \boxed{-x^2 + 9}$

Solutions

12. $(p-q)(p+q) = p^2 + pq - pq - q^2 = \boxed{p^2 - q^2}$

13. $(5-y)^2 = (5-y)(5-y) = 25 - 5y - 5y + y^2 = \boxed{y^2 - 10y + 25}$

14. $(6-5x)^2 = (6-5x)(6-5x) = 36 - 30x - 30x + 25x^2 = \boxed{25x^2 - 60x + 36}$

15. $(x-1)^4 = ((x-1)^2)^2 = ((x-1)(x-1))^2 = (x^2 - x - x + 1)^2 = (x^2 - 2x + 1)^2$

$$\begin{array}{c|c|c|c} & x^2 & -2x & +1 \\ \hline x^2 & x^4 & -2x^3 & x^2 \\ \hline -2x & -2x^3 & 4x^2 & -2x \\ \hline +1 & x^2 & -2x & 1 \end{array}$$

$= \boxed{x^4 - 4x^3 + 6x^2 - 4x + 1}$

16. $(x-1)(x^2+x+1)$

$$\begin{array}{c|c|c|c} & x^2 & +x & +1 \\ \hline x & x^3 & x^2 & x \\ \hline -1 & -x^2 & -x & -1 \end{array}$$

$\boxed{x^3 - 1}$

17. $(x-1)(x^3+x^2+x+1)$

$$\begin{array}{c|c|c|c|c} & x^3 & +x^2 & +x & +1 \\ \hline x & x^4 & x^3 & x^2 & x \\ \hline -1 & -x^3 & -x^2 & -x & -1 \end{array}$$

$\boxed{x^4 - 1}$

Solutions

18. Predict the simplification of $(x-1)(x^n + x^{n-1} + x^{n-2} + ... + x^2 + x + 1)$ where $n \in \mathbb{Z}^+$. $\boxed{x^{n+1} - 1}$

19. The difference between the cubes of two consecutive integers is 127. Find the integers.

$$127 = x^3 - (x-1)^3$$

	x^2	$-2x$	$+1$
x	x^3	$-2x^2$	x
-1	$-x^2$	$2x$	-1

$\boxed{6 \text{ and } 7}$

$$= x^3 - (x-1)(x-1)(x-1)$$

$$= x^3 - (x-1)(x^2 - x - x + 1) = x^3 - (x^3 - 3x^2 + 3x - 1) = 3x^2 - 3x + 1$$

$$3x^2 - 3x + 1 = 127$$
$$-1 \qquad -1$$
$$3x^2 - 3x = 126$$
$$3(x^2 - x) = 126$$
$$x^2 - x = 42 \rightarrow x(x-1) = 42$$
$$x = 7,\ x - 1 = 6$$

Additional Examples from Chapter 4

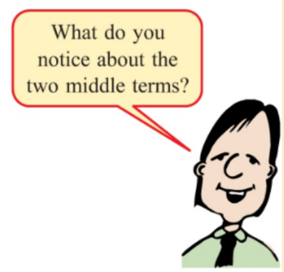
Example 4 — Self Tutor

Expand and simplify:

a $(x+3)(x-3)$ **b** $(3x-5)(3x+5)$

a
$$\begin{aligned}&(x+3)(x-3)\\&=x^2-3x+3x-9\\&=x^2-9\end{aligned}$$

b
$$\begin{aligned}&(3x-5)(3x+5)\\&=9x^2+15x-15x-25\\&=9x^2-25\end{aligned}$$



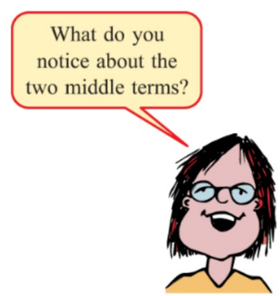
Example 5 — Self Tutor

Expand and simplify:

a $(3x+1)^2$ **b** $(2x-3)^2$

a
$$\begin{aligned}&(3x+1)^2\\&=(3x+1)(3x+1)\\&=9x^2+3x+3x+1\\&=9x^2+6x+1\end{aligned}$$

b
$$\begin{aligned}&(2x-3)^2\\&=(2x-3)(2x-3)\\&=4x^2-6x-6x+9\\&=4x^2-12x+9\end{aligned}$$



Additional Examples from Chapter 4

Example 6 Self Tutor

Expand and simplify: $(x+2)(x-3)+5(x+4)$

$$\begin{aligned} &(x+2)(x-3)+5(x+4) \\ &= x^2-3x+2x-6+5x+20 \\ &= x^2+4x+14 \end{aligned}$$

Example 7 Self Tutor

Expand and simplify:

a $(x+5)(x-5)$ **b** $(3-y)(3+y)$

a
$$\begin{aligned} &(x+5)(x-5) \\ &= x^2-5^2 \\ &= x^2-25 \end{aligned}$$

b
$$\begin{aligned} &(3-y)(3+y) \\ &= 3^2-y^2 \\ &= 9-y^2 \end{aligned}$$

Additional Examples from Chapter 4

Example 8 — Self Tutor

Expand and simplify:

a $(2x-3)(2x+3)$ **b** $(5-3y)(5+3y)$

a
$$\begin{aligned}&(2x-3)(2x+3)\\&=(2x)^2-3^2\\&=4x^2-9\end{aligned}$$

b
$$\begin{aligned}&(5-3y)(5+3y)\\&=5^2-(3y)^2\\&=25-9y^2\end{aligned}$$

Example 10 — Self Tutor

Expand and simplify:

a $(x+3)^2$ **b** $(x-5)^2$

a
$$\begin{aligned}&(x+3)^2\\&=x^2+2\times x\times 3+3^2\\&=x^2+6x+9\end{aligned}$$

b
$$\begin{aligned}&(x-5)^2\\&=x^2+2\times x\times(-5)+(-5)^2\\&=x^2-10x+25\end{aligned}$$

Additional Examples from Chapter 4

Example 11 — Self Tutor

Expand and simplify using the perfect square expansion rule:

a $(5x+1)^2$ **b** $(4-3x)^2$

a
$$\begin{aligned} &(5x+1)^2 \\ &= (5x)^2 + 2\times 5x \times 1 + 1^2 \\ &= 25x^2 + 10x + 1 \end{aligned}$$

b
$$\begin{aligned} &(4-3x)^2 \\ &= 4^2 + 2\times 4\times(-3x) + (-3x)^2 \\ &= 16 - 24x + 9x^2 \end{aligned}$$

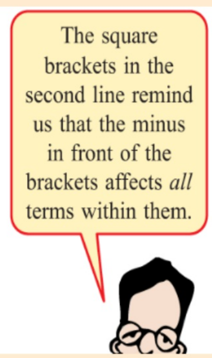
Example 12 — Self Tutor

Expand and simplify:

a $(2x^2+3)^2$ **b** $5-(x+2)^2$

a
$$\begin{aligned} &(2x^2+3)^2 \\ &= (2x^2)^2 + 2\times 2x^2\times 3 + 3^2 \\ &= 4x^4 + 12x^2 + 9 \end{aligned}$$

b
$$\begin{aligned} &5-(x+2)^2 \\ &= 5 - [x^2+4x+4] \\ &= 5 - x^2 - 4x - 4 \\ &= 1 - x^2 - 4x \end{aligned}$$



Additional Examples from Chapter 4

Example 13 **Self Tutor**

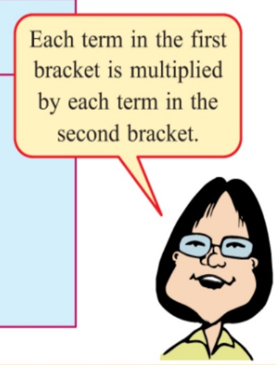
Expand and simplify: $(2x+3)(x^2+4x+5)$

$$(2x+3)(x^2+4x+5)$$

$= 2x^3 + 8x^2 + 10x$ {all terms in 2nd bracket $\times\, 2x$}

$\quad + 3x^2 + 12x + 15$ {all terms in 2nd bracket $\times\, 3$}

$= 2x^3 + 11x^2 + 22x + 15$ {collecting like terms}



Additional Examples from Chapter 4

Example 14 — Self Tutor

Expand and simplify: $(x+2)^3$

$$
\begin{aligned}
(x+2)^3 &= (x+2)\times(x+2)^2 \\
&= (x+2)(x^2+4x+4) && \{\text{perfect square expansion}\} \\
&= x^3+4x^2+4x && \{\text{all terms in 2nd bracket} \times x\} \\
&\quad\; +2x^2+8x+8 && \{\text{all terms in 2nd bracket} \times 2\} \\
&= x^3+6x^2+12x+8 && \{\text{collecting like terms}\}
\end{aligned}
$$

Additional Examples from Chapter 4

Example 15 **Self Tutor**

Expand and simplify:

a $x(x+1)(x+2)$ **b** $(x+1)(x-2)(x+2)$

a $x(x+1)(x+2)$

$= (x^2+x)(x+2)$ {all terms in first bracket $\times x$}

$= x^3+2x^2+x^2+2x$ {expanding remaining factors}

$= x^3+3x^2+2x$ {collecting like terms}

b $(x+1)(x-2)(x+2)$

$= (x+1)(x^2-4)$ {difference of two squares}

$= x^3-4x+x^2-4$ {expanding factors}

$= x^3+x^2-4x-4$

Always look for ways to make your expansions simpler. In **b** we can use the difference of two squares.