

Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: <u>3/5</u> Topic: _____	0 1 2	
Tuesday Date: <u>3/6</u> Topic: _____	0 1 2	No Homework (Test Friday)
Wednesday Date: <u>3/7</u> Topic: <u>Polynomial Degree</u>	0 1 2	
Thursday Date: <u>3/8</u> Topic: <u>4ABCDE Factored to General Form</u>	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Warm-up: Translate from factored to general form.
How could you go the other way?

Consider the expansion:

$$2x(x - 3)$$

$$2x^2 - 6x$$

$$(x + 1)(x + 6)$$

$$x^2 + 6x + x + 6$$

$$x^2 + 7x + 6$$

Warm-up: Translate from factored to general form.
How could you go the other way?

Consider the expansion:

$$2x(x - 3)$$

$$2x^2 - 6x$$

$$(x + 1)(x + 6)$$

$$x^2 + 6x + x + 6$$
$$= x^2 + 7x + 6$$

Warm-up: Translate from factored to general form.
How could you go the other way?

Consider the expansion:

$$(x + 1)(x + 6)$$

$$x^2 + 6x + 1x + 1 \cdot 6$$

$$x^2 + \underbrace{(6+1)}_{\text{sum}} x + \underbrace{1 \cdot 6}_{\text{PRODUCT}}$$

$$x^2 + \underline{7}x + \underline{6}$$

E**FACTORISING QUADRATIC TRINOMIALS**

Consider this expansion

$$\begin{aligned}(x + 1)(x + 6) &= x^2 + 6x + x + 1 \times 6 && \{\text{using FOIL}\} \\ &= x^2 + (6 + 1)x + (1 \times 6) \\ &= x^2 + (\text{sum of 1 and 6})x + (\text{product of 1 and 6}) \\ &= x^2 + 7x + 6\end{aligned}$$

To factor: We need two numbers that **sum** to 7 and **multiply** to 6.... **1 and 6!**

So, to factorise $x^2 + 7x + 6$, we need two numbers with a sum of 7 and a product of 6. These numbers are 1 and 6, and so $x^2 + 7x + 6 = (x + 1)(x + 6)$.

We call this the **sum and product method**.

Class Plan:

1. Warm-up

2. 9AE Quadratic Factorization

3. Practice

$$5x^2 + 10x$$
$$5x^2 + 10x = 5x(x+2)$$

a $5x!$

Chapter

9

Quadratic factorisation

- A** Removing common factors
- B** Difference of two squares factorisation
- C** Perfect square factorisation
- D** Factorising expressions with four terms
- E** Factorising quadratic trinomials
- F** Factorising $ax^2 + bx + c$, $a \neq 1$

Take a step back...

What is a factor?

Factors are numbers you multiply together to get another number.

$$\text{Factor } \boxed{2} \times \boxed{3} = 6$$

fac·tor

/ˈfaktər/ 

noun

plural noun: **factors**

1. a circumstance, fact, or influence that contributes to a result or outcome.
"his legal problems were not a factor in his decision"
synonyms: element, part, component, ingredient, strand, constituent, point, detail, item, feature, facet, aspect, characteristic, consideration, influence, circumstance
"this had been a key factor in his decision to withdraw"
2. a number or quantity that when multiplied with another produces a given number or expression.

Take a step back...

What is a **factor** (in Algebra)?

In Algebra, factors are what we can multiply together to get an expression.

$(x+3)$ and $(x+1)$ are factors of $x^2 + 4x + 3$:

$$\underbrace{(x+3)}_{\text{Factor}} \underbrace{(x+1)}_{\text{Factor}} = x^2 + 4x + 3$$

Today we are solving for factors.
(**what** we can multiply together
to get an expression)

<https://www.mathsisfun.com/definitions/factor.html>

Quadratic factorisation

The process of writing an expression as a product of its factors.

A **quadratic trinomial** in x is an expression of the form $ax^2 + bx + c$ where x is the variable, and a , b , and c are constants, $a \neq 0$.

$$\begin{array}{ccccccc} ax^2 & + & bx & + & c & & \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{the } x^2 \text{ term} & & \text{the } x \text{ term} & & \text{the constant term} & & \end{array}$$

For example: $x^2 + 5x + 6$, $4x^2 - 9$ and $9x^2 + 6x + 1$ are quadratic expressions.

In **Chapter 4** we studied the expansion of algebraic factors, many of which resulted in quadratic trinomials. In this chapter we will consider **factorisation**, which is the reverse process of expansion.

Factorisation is the process of writing an expression as a **product** of its **factors**.

For example:

$$\begin{array}{c} \text{expansion} \\ \curvearrowright \\ (x + 2)(x + 3) = x^2 + 5x + 6 \\ \curvearrowleft \\ \text{factorisation} \end{array}$$

Goal Today:

How can we translate from
general form to factored form?

The diagram illustrates the relationship between expansion and factorisation. It features a central equation: $(\square)(\square) = x^2 + 5x + 6$. Above the equation, the word "expansion" is written in bold black text, with a red curved arrow pointing from the factored form to the general form. Below the equation, the word "factorisation" is written in bold black text, with a red curved arrow pointing from the general form back to the factored form.

$$(\square)(\square) = x^2 + 5x + 6$$

The *reverse* process of expansion

E**FACTORISING QUADRATIC TRINOMIALS**

More generally, $(x + a)(x + b) = x^2 + bx + ax + ab$

$$= x^2 + (a + b)x + ab$$



SUM

PRODUCT

$$x^2 + px + q = (x + a)(x + b)$$

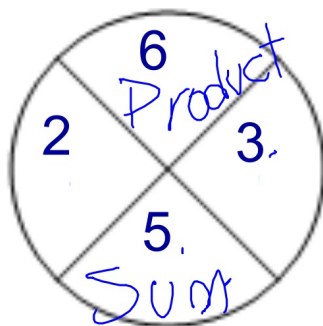
where a and b are two numbers whose sum is p , and whose product is q .

Factoring Method Introduction:

How do the values **2 and **3** relate to the coefficients in the expression $x^2 + 5x + 6$?

$$2 + 3 = 5 \quad 2 \cdot 3 = 6$$

***How is this digram related to the situation above?



A**REMOVING COMMON FACTORS**

What are the common factors?

Factorise by removing a common factor:

$$\text{GCF} = 2x$$

a $\frac{2x^2}{x} + \frac{3x}{x}$ GCF

b $\frac{-2x^2}{-2x} - \frac{6x}{-2x}$

$$x(2x + 3)$$

$$\begin{aligned} & -2x(x + 3) \\ & -x(2x + 6) \end{aligned}$$

A**REMOVING COMMON FACTORS**

What are the common factors?

Factorise by removing a common factor: $GCF = -2x$

a $2x^2 + 3x$ $GCF = x$

$$x(2x + 3)$$

b $\frac{-2x^2 - 6x}{-2x}$

$$-2x(x + 3)$$

$$x(x+1) \Leftrightarrow x^2 + x$$

$$(x+2)(x+3) \Leftrightarrow x^2 + 5x + 6$$

a $2x^2 + 3x$ has HCF x
 $\therefore 2x^2 + 3x = x(2x + 3)$

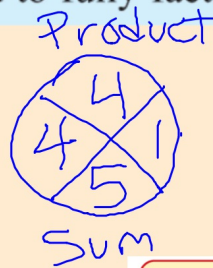
b $-2x^2 - 6x$ has HCF $-2x$
 $\therefore -2x^2 - 6x = -2x(x + 3)$

E**FACTORISING QUADRATIC TRINOMIALS****Example 10**

Use the sum and product method to fully factorise:

a $x^2 + 5x + 4$

$(x+4)(x+1)$



The **sum** of the numbers is the coefficient of x .
The **product** of the numbers is the constant term.

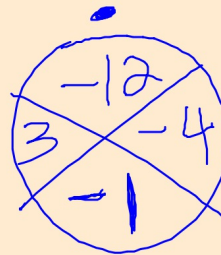


Example 10

Use the sum and product method to fully factorise:

b $x^2 - x - 12$

$$(x+3)(x-4)$$



+

$$3 + -4 = -1$$

$$3 \cdot -4 = -12$$

SOLUTIONS

- a** We need two numbers with sum 5, and product 4.

The numbers are 1 and 4.

$$\therefore x^2 + 5x + 4 = (x + 1)(x + 4)$$

- b** We need two numbers with sum -1 and product -12 .

The numbers are -4 and 3 .

$$\therefore x^2 - x - 12 = (x - 4)(x + 3)$$

Example 7

Fully factorise:

a $x^2 + 10x + 25$

$$(x+5)(x+5)$$
$$(x+5)^2$$

$$\begin{array}{r} \cancel{25} \\ 5 \times 5 \\ \cancel{10} \end{array}$$

Example 7

Fully factorise:

b $x^2 - 14x + 49$

$$(x-7)^2$$

Handwritten diagram illustrating the factoring process. It shows a large 'X' shape. At the top vertex is '49', at the bottom vertex is '14', at the left vertex is '-7', and at the right vertex is '-7'. Two diagonal lines cross at the center, forming the 'X'.

SOLUTIONS

Example 7



Fully factorise:

a $x^2 + 10x + 25$

b $x^2 - 14x + 49$

a $x^2 + 10x + 25$

$$= x^2 + 2 \times x \times 5 + 5^2$$

$$= (x + 5)^2$$

b $x^2 - 14x + 49$

$$= x^2 - 2 \times x \times 7 + 7^2$$

$$= (x - 7)^2$$

Exercises...

9E: first column

1 aceg

2adg

3adg

3adg

4adgjm

5adgjm

If you need additional
practice, do more!!

Exercises... DO: First column of 9E

Chapter 9E: Quadratic Factorizations

EXERCISE 9E

1 Find two numbers which have:

- | | |
|------------------------------|------------------------------|
| a product 12 and sum 7 | b product 15 and sum 8 |
| c product 16 and sum 10 | d product 18 and sum 11 |
| e product -36 and sum 9 | f product -36 and sum -9 |
| g product -12 and sum -4 | h product -30 and sum 13 |

2 Factorise:

- | | | |
|--------------------|--------------------|--------------------|
| a $x^2 + 4x + 3$ | b $x^2 + 11x + 24$ | c $x^2 + 10x + 21$ |
| d $x^2 + 15x + 54$ | e $x^2 + 9x + 20$ | f $x^2 + 8x + 15$ |
| g $x^2 + 10x + 24$ | h $x^2 + 9x + 14$ | i $x^2 + 6x + 8$ |

3 Factorise:

- | | | |
|--------------------|--------------------|--------------------|
| a $x^2 - 3x + 2$ | b $x^2 - 4x + 3$ | c $x^2 - 5x + 6$ |
| d $x^2 - 14x + 33$ | e $x^2 - 16x + 39$ | f $x^2 - 19x + 48$ |
| g $x^2 - 11x + 28$ | h $x^2 - 14x + 24$ | i $x^2 - 20x + 36$ |

4 Factorise:

- | | | |
|-------------------|-------------------|--------------------|
| a $x^2 - 7x - 8$ | b $x^2 + 4x - 21$ | c $x^2 - x - 2$ |
| d $x^2 - 2x - 8$ | e $x^2 + 5x - 24$ | f $x^2 - 3x - 10$ |
| g $x^2 + 3x - 54$ | h $x^2 + x - 72$ | i $x^2 - 4x - 21$ |
| j $x^2 - x - 6$ | k $x^2 - 7x - 60$ | l $x^2 + 7x - 60$ |
| m $x^2 + 3x - 18$ | n $x^2 - 7x - 18$ | o $x^2 - 12x + 35$ |

5 Fully factorise by first removing a common factor:

- | | | |
|---------------------|----------------------|----------------------|
| a $2x^2 - 6x - 8$ | b $3x^2 + 9x - 12$ | c $5x^2 + 10x - 15$ |
| d $4x^2 + 4x - 80$ | e $2x^2 - 4x - 30$ | f $3x^2 + 12x - 63$ |
| g $-2x^2 + 2x + 40$ | h $-3x^2 + 12x - 12$ | i $-7x^2 - 21x + 28$ |
| j $-x^2 - 3x - 2$ | k $-x^2 + 5x - 6$ | l $-x^2 + 9x - 18$ |
| m $5x^2 + 15x - 50$ | n $-2x^2 - 8x + 42$ | o $4x - x^2 + 32$ |

Late Start Schedule:

<u>Lunch A</u>			<u>Lunch B</u>			<u>Lunch C</u>		
1st Hour	10:05-10:38	33 minutes	1st Hour	10:05-10:38	33 minutes	1st Hour	10:05-10:38	33 minutes
2nd Hour	10:43-11:16	33 minutes	2nd Hour	10:43-11:16	33 minutes	2nd Hour	10:43-11:16	33 minutes
3rd Hour	11:21-11:54	33 minutes	3rd Hour	11:21-11:54	33 minutes	3rd Hour	11:21-11:54	33 minutes
Lunch A	11:59-12:29	30 minutes	4th Hour	11:59-12:32	33 minutes	4th Hour	11:59-12:32	33 minutes
4th Hour	12:34-1:07	33 minutes	Lunch B	12:37-1:07	30 minutes	5th Hour	12:37-1:12	33 minutes
5th Hour	1:12-1:45	33 minutes	5th Hour	1:12-1:45	33 minutes	Lunch C	1:17-1:45	30 minutes
6th Hour	1:50-2:23	33 minutes	6th Hour	1:50-2:23	33 minutes	6th Hour	1:50-2:23	33 minutes
7th Hour	2:28-3:00	32 minutes	7th Hour	2:28-3:00	32 minutes	7th Hour	2:28-3:00	32 minutes

Exercises: 9E, page 180

EXERCISE 9E

1 Find two numbers which have:

a product 12 and sum 7

c product 16 and sum 10

e product -36 and sum 9

g product -12 and sum -4

b product 15 and sum 8

d product 18 and sum 11

f product -36 and sum -9

h product -30 and sum 13

2 Factorise:

a $x^2 + 4x + 3$

d $x^2 + 15x + 54$

g $x^2 + 10x + 24$

b $x^2 + 11x + 24$

e $x^2 + 9x + 20$

h $x^2 + 9x + 14$

c $x^2 + 10x + 21$

f $x^2 + 8x + 15$

i $x^2 + 6x + 8$

3 Factorise:

a $x^2 - 3x + 2$

b $x^2 - 4x + 3$

c $x^2 - 5x + 6$

d $x^2 - 14x + 33$

e $x^2 - 16x + 39$

f $x^2 - 19x + 48$

g $x^2 - 11x + 28$

h $x^2 - 14x + 24$

i $x^2 - 20x + 36$

4 Factorise:

a $x^2 - 7x - 8$

d $x^2 - 2x - 8$

g $x^2 + 3x - 54$

j $x^2 - x - 6$

m $x^2 + 3x - 18$

b $x^2 + 4x - 21$

e $x^2 + 5x - 24$

h $x^2 + x - 72$

k $x^2 - 7x - 60$

n $x^2 - 7x - 18$

c $x^2 - x - 2$

f $x^2 - 3x - 10$

i $x^2 - 4x - 21$

l $x^2 + 7x - 60$

o $x^2 - 12x + 35$

5 Fully factorise by first removing a common factor:

a $2x^2 - 6x - 8$

b $3x^2 + 9x - 12$

c $5x^2 + 10x - 15$

d $4x^2 + 4x - 80$

e $2x^2 - 4x - 30$

f $3x^2 + 12x - 63$

g $-2x^2 + 2x + 40$

h $-3x^2 + 12x - 12$

i $-7x^2 - 21x + 28$

j $-x^2 - 3x - 2$

k $-x^2 + 5x - 6$

l $-x^2 + 9x - 18$

m $5x^2 + 15x - 50$

n $-2x^2 - 8x + 42$

o $4x - x^2 + 32$

g
$$\frac{-2x^2 + 2x + 40}{-2 \quad -2 \quad -2}$$
$$-2(x^2 - x - 20)$$
$$\boxed{-2(x-5)(x+4)}$$

~~$$\begin{array}{r} -20 \\ -5 \quad 4 \\ -1 \\ + \end{array}$$~~

5 Fully factorise by first removing a common factor:

a $2x^2 - 6x - 8$

b $3x^2 + 9x - 12$

c $5x^2 + 10x - 15$

d $4x^2 + 4x - 80$

e $2x^2 - 4x - 30$

f $3x^2 + 12x - 63$

g $-2x^2 + 2x + 40$

h $-3x^2 + 12x - 12$

i $-7x^2 - 21x + 28$

j $-x^2 - 3x - 2$

k $-x^2 + 5x - 6$

l $-x^2 + 9x - 18$

m $5x^2 + 15x - 50$

n $-2x^2 - 8x + 42$

o $4x - x^2 + 32$

a) $2x^2 - 6x - 8$
 $2(x^2 - 3x - 4)$
 $2(x+1)(x-4)$

~~$\begin{array}{r} -4 \\ 1 \quad -4 \\ \hline -3 \end{array}$~~

Solutions to Factorization:

EXERCISE 9E

- 1** **a** 3 and 4 **b** 3 and 5 **c** 2 and 8 **d** 2 and 9
 e -3 and 12 **f** 3 and -12 **g** 2 and -6 **h** -2 and 15
- 2** **a** $(x + 1)(x + 3)$ **b** $(x + 3)(x + 8)$ **c** $(x + 3)(x + 7)$
 d $(x + 6)(x + 9)$ **e** $(x + 4)(x + 5)$ **f** $(x + 3)(x + 5)$
 g $(x + 4)(x + 6)$ **h** $(x + 2)(x + 7)$ **i** $(x + 2)(x + 4)$
- 3** **a** $(x - 1)(x - 2)$ **b** $(x - 1)(x - 3)$ **c** $(x - 2)(x - 3)$
 d $(x - 3)(x - 11)$ **e** $(x - 3)(x - 13)$ **f** $(x - 3)(x - 16)$

Solutions to Factorization: #3 continued, and 4 & 5

- | | | | | | |
|------------|--------------------|----------|--------------------|----------|-------------------|
| g | $(x - 4)(x - 7)$ | h | $(x - 2)(x - 12)$ | i | $(x - 2)(x - 18)$ |
| 4 a | $(x - 8)(x + 1)$ | b | $(x + 7)(x - 3)$ | c | $(x - 2)(x + 1)$ |
| d | $(x - 4)(x + 2)$ | e | $(x + 8)(x - 3)$ | f | $(x - 5)(x + 2)$ |
| g | $(x + 9)(x - 6)$ | h | $(x + 9)(x - 8)$ | i | $(x - 7)(x + 3)$ |
| j | $(x - 3)(x + 2)$ | k | $(x - 12)(x + 5)$ | l | $(x + 12)(x - 5)$ |
| m | $(x + 6)(x - 3)$ | n | $(x + 2)(x - 9)$ | o | $(x - 5)(x - 7)$ |
| 5 a | $2(x - 4)(x + 1)$ | b | $3(x + 4)(x - 1)$ | | |
| c | $5(x + 3)(x - 1)$ | d | $4(x + 5)(x - 4)$ | | |
| e | $2(x - 5)(x + 3)$ | f | $3(x + 7)(x - 3)$ | | |
| g | $-2(x - 5)(x + 4)$ | h | $-3(x - 2)^2$ | | |
| i | $-7(x + 4)(x - 1)$ | j | $-(x + 2)(x + 1)$ | | |
| k | $-(x - 3)(x - 2)$ | l | $-(x - 3)(x - 6)$ | | |
| m | $5(x + 5)(x - 2)$ | n | $-2(x + 7)(x - 3)$ | | |
| o | $-(x - 8)(x + 4)$ | | | | |