Happy Friday! :) Please reflect and turn in.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 3/5 Topic:	0 1 2	
Tuesday Date: 3/6 Topic:	0 1 2	No Homework (Test Friday)
Wednesday Date: 3/7 Topic: Polynomial Degree	0 1 2	
Thursday Date: 3/8 Topic: 4ABCDE Factored to General Form	0 1 2	
Friday Date: 3/9 Topic: 9AE General to Factored Form	0 1 2	

Warm-up: Factor completely.

$n^2 - 64$

$n^2 - 0n - 64$

$(n+8)(n-8)$

$x^2 + 0x - 9$

$3x^4 - 27x^2$

GCF = $3x^2$

$3x^2(x^2 - 9)$

$3x^2(x+3)(x-3)$

5m

$5x^2 + 15x - 50$

$5(x^2 + 3x - 10)$

$5(x+5)(x-2)$

5l

$\dfrac{-x^2}{-1} + \dfrac{4x}{-1} + \dfrac{32}{-1}$

$x^2 - 4x - 32$

$(x-8)(x+4)$

<u>Class Plan:</u>

Chapter 9

1. Warm-up
2. 9ABCD Quadratic Factorization

   Explore different factoring situations!
3. Practice

- ~~A Removing common factors~~
- B Difference of two squares factorisation
- C Perfect square factorisation
- D Factorising expressions with four terms
- ~~E Factorising quadratic trinomials~~
- F Factorising $ax^2 + bx + c, \ a \neq 1$

A REMOVING COMMON FACTORS

Some quadratic trinomials can be factorised by removing the Highest Common Factor (HCF) of the terms in the expression. In fact, we should always look to remove the HCF before proceeding with any further factorisation.

When we remove a common factor, we write it in front of a set of brackets. The expression within the brackets is found using the reverse of the distributive law.

You can check your factorisations by expansion!

Example 1 Self Tutor

Factorise by removing a common factor:

a $2x^2 + 3x$ **b** $-2x^2 - 6x$

a $2x^2 + 3x$ has HCF x

$\therefore\ 2x^2 + 3x = x(2x + 3)$

b $-2x^2 - 6x$ has HCF $-2x$

$\therefore\ -2x^2 - 6x = -2x(x + 3)$

Investigation: **DIFFERENCE OF TWO SQUARES**

Expand the following expression...

what do you notice?

Consider the product $(a+b)(a-b)$.

$a^2 - ab + ab - b^2$

$(a+b)(a-b) = a^2 - b^2$

Textbook notes...

B DIFFERENCE OF TWO SQUARES FACTORISATION

The **expansion** of $(a+b)(a-b)$ is a^2-b^2, which is the difference between the two squares a^2 and b^2.

Thus, the **factorisation** of a^2-b^2 is:

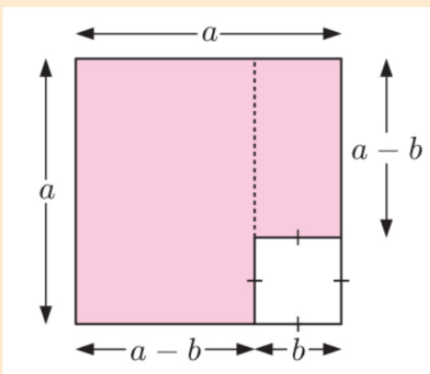
$$\boldsymbol{a^2-b^2=(a+b)(a-b)}$$

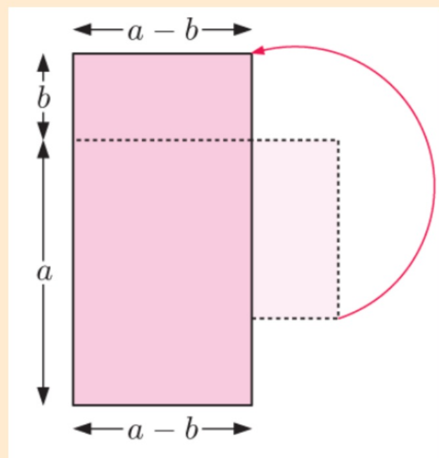
The **sum of two squares** a^2+b^2 does not factorise into two real linear factors.

Textbook notes...

shaded area = area of large square − area of small square

$= a^2 - b^2$





$\therefore$ shaded area $= (a+b)(a-b)$

$\therefore \quad (a+b)(a-b) = a^2 - b^2$ {equating areas}

Textbook notes...

C PERFECT SQUARE FACTORISATION

We know the **expansion** of $(a+b)^2$ is $a^2+2ab+b^2$,

so the **factorisation** of $a^2+2ab+b^2$ is: $\boldsymbol{a^2+2ab+b^2=(a+b)^2}$

Similarly, the expansion of $(a-b)^2$ is $a^2-2ab+b^2$,

so the factorisation of $a^2-2ab+b^2$ is: $\boldsymbol{a^2-2ab+b^2=(a-b)^2}$

Textbook notes...

D FACTORISING EXPRESSIONS WITH FOUR TERMS

Sometimes we can factorise an expression containing four terms by grouping them in two pairs.

For example, $ax^2 + 2x + 2 + ax$ can be rewritten as

$$\underbrace{ax^2 + ax} + \underbrace{2x + 2}$$

$$= ax(x+1) + 2(x+1) \quad \{\text{factorising each pair}\}$$

$$= (x+1)(ax+2) \quad \{(x+1) \text{ is a common factor}\}$$

Chapter 9 **Quadratic factorisation**

Do: Explore Factorisation Problems

Be ready to work through with the class!

9A Removing common factors ... what's in common?

9B Difference of two squares $a^2 - b^2 = (a+b)(a-b)$

9C Perfect square factorization

9D factorizing with four terms reorder terms!

Done? Keep going! Work together!

9A Removing common factors

$x^2 - 10x + 25 - 2x + 10$

$x^2 - 12x + 35$

Example 2

Fully factorise by removing a common factor:

a $(x-5)^2 - 2(x-5)$

$(x-5)$ $(x-5)$

$(x-5)[x-5-2]$

$(x-5)(x-7)$

b $(x+2)^2 + 2x + 4$

$(x+2)^2 + 2(x+2)$

$x+2$

$(x+2)[x+2+2]$

$(x+2)(x+4)$

9B Difference of two squares

Example 3

Use the rule $a^2 - b^2 = (a+b)(a-b)$ to factorise fully:

a $9 - x^2$

b $4x^2 - 25$

$(3+x)(3-x)$

$(2x-5)(2x+5)$

Check:

$9 - 3x + 3x - x^2$ ✓

Check:

$4x^2 + 10x - 10x - 25$ ✓

9B Difference of two squares

Example 4 Sc

Fully factorise: **a** $2x^2 - 8$ **b** $-3x^2 + 48$

$2(x^2 - 4)$

$2(x-2)(x+2)$

Check:

$2(x^2 + 2x - 2x - 4)$

9B Difference of two squares

Example 4

Fully factorise: **a** $2x^2 - 8$ **b** $-3x^2 + 48$

$\frac{-3x^2}{-3} + \frac{48}{-3}$

$-3(x^2 - 16)$

$-3(x-4)(x+4)$

9B Difference of two squares

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 - ab + ba - b^2$$

9B Difference of two squares

Example 6

$a^2-b^2=(a+b)(a-b)$

Factorise using the difference of two squares:

a $(3x+2)^2-9$

b $(x+2)^2-(x-1)^2$

$(3x+2)^2-3^2$

$(3x+2+3)(3x+2-3)$

$(3x+5)(3x-1)$

9B Difference of two squares

$\sqrt{(x+2)^2} = x+2$

$((x-1)^2)^{1/2}$

Example 6

Factorise using the difference of two squares:

a $(3x+2)^2 - 9$ **b** $(x+2)^2 - (x-1)^2$

x^2+4x+4

$-(x^2-2x+1)$

$6x+3$

$=$

$(x+2+x-1)(x+2-(x+1))$

$(2x+1)(3)$

9C Perfect square factorization

Example 7

Fully factorise:

a $x^2 + 10x + 25$

$(x+5)(x+5)$

$(x+5)^2$

b $x^2 - 14x + 49$

$(x-7)(x-7)$

$(x-7)^2$

9C Perfect square factorization

Example 8

Fully factorise:

a $9x^2 - 6x + 1$

b $-8x^2 - 24x - 18$

$(3x-1)(3x-1)$

$(3x-1)^2$

$-2(4x^2+12x+9)$

$-2(2x+3)(2x+3)$

9D factorizing with four terms

Example 9

Fully factorise:

a $ax + by + bx + ay$

b $2x^2 - 15 + 3x - 10x$

$ax + ay + bx + by$

$a(x+y) + b(x+y)$

$(x+y)(a+b)$

$2x^2 - 10x + 3x - 15$

$2x(x-5) + 3(x-5)$

$(2x+3)(x-5)$

9D factorizing with four terms

$2x^2 - 7x - 15$

$(\quad x \quad)$

Example 9

Fully factorise:

a $ax + by + bx + ay$ **b** $2x^2 - 15 + 3x - 10x$

$2x^2 + 3x - 10x - 15$

$x(2x+3) - 5(2x+3)$

$(2x+3)(x-5)$

Exercises... Chapter 9

Quadratic factorisation

- **A** Removing common factors
- **B** Difference of two squares factorisation
- **C** Perfect square factorisation
- **D** Factorising expressions with four terms
- **E** Factorising quadratic trinomials
- **F** Factorising $ax^2 + bx + c, \;\; a \neq 1$

EXERCISE 9A

1 Fully factorise by first removing a common factor:

a $3x^2 + 5x$

d $4x^2 - 8x$

g $-4x + 8x^2$

j $x^3 + x^2 + x$

m $ax^2 + 2ax$

Chapter 9

Exercises...

3 Fully factorise by removing a common factor:

a $(x+2)^2 - 5(x+2)$

d $(x-2)^2 + 3x - 6$

g $(x-3)^2 - x + 3$

j $3x + 6 + (x+2)^2$

m $2(x+1)^2 + x + 1$

Exercises...

Chapter 9

EXERCISE 9B

2 Fully factorise:

b $-2x^2+8$

e $8t^2-18$

h $64n^2-n^4$

4 Factorise using the difference of two squares:

a $(x+1)^2-4$

b $(2x+1)^2-9$

d $(x+3)^2-4x^2$

e $4x^2-(x+2)^2$

g $(2x+1)^2-(x-2)^2$

h $(3x-1)^2-(x+1)^2$

Chapter 9

Exercises...

EXERCISE 9C

1 Fully factorise:

a $x^2 + 6x + 9$

b $x^2 + 8x + 16$

d $x^2 - 8x + 16$

e $x^2 + 2x + 1$

g $y^2 + 18y + 81$

h $m^2 - 20m + 100$

Exercises...

2 Fully factorise:

a $9x^2 + 6x + 1$

b $4x^2 - 4x + 1$

d $25x^2 - 10x + 1$

e $16x^2 + 24x + 9$

Exercises...

3 Fully factorise:

a $-x^2 + 2x - 1$

b $2x^2 + 8x + 8$

d $32x^2 - 16x + 2$

e $36x^2 + 120x + 100$

Solutions

EXERCISE 9A

1. a $x(3x+5)$ b $x(2x-7)$ c $3x(x+2)$
   d $4x(x-2)$ e $x(9-2x)$ f $-3x(x+5)$
   g $4x(2x-1)$ h $-5x(1+2x)$ i $4x(3-x)$
   j $x(x^2+x+1)$ k $x(2x^2+11x+4)$ l $a(b+c+d)$
   m $ax(x+2)$ n $ab(b+a)$ o $ax^2(x+1)$
2. a $(x+5)(3+x)$ b $(b+3)(a-5)$ c $(x+4)(x+1)$
   d $(x+2)(2x+5)$ e $(c-d)(a+b)$ f $(y+2)(y-1)$
   g $(x-1)(ab+c)$ h $(x+2)(a-1)$
3. a $(x+2)(x-3)$ b $(x-1)(x-4)$ c $(x+1)(x+3)$
   d $(x-2)(x+1)$ e $(x+3)(x+4)$ f $(x+4)(x+6)$
   g $(x-3)(x-4)$ h $(x+4)(x+2)$ i $(x-4)(x-9)$
   j $(x+2)(x+5)$ k $(x+1)^2(x+2)$
   l $(a+b)(a^2+2ab+b^2+1)$ m $(x+1)(2x+3)$
   n $(x-2)(3x-7)$ o $2(a+b)(2a+2b-1)$

Solutions

EXERCISE 9B

1 **a** $(x+2)(x-2)$ **b** $(2+x)(2-x)$
c $(x+9)(x-9)$ **d** $(5+x)(5-x)$
e $(2x+1)(2x-1)$ **f** $(3x+4)(3x-4)$
g $(2x+3)(2x-3)$ **h** $(6+7x)(6-7x)$

2 **a** $3(x+3)(x-3)$ **b** $2(2+x)(2-x)$
c $3(k+5)(k-5)$ **d** $5(1+x)(1-x)$
e $2(2t+3)(2t-3)$ **f** $3(5+3x)(5-3x)$
g $x(x+7)(x-7)$ **h** $n^2(8+n)(8-n)$
i $7x(2x+3)(2x-3)$

3 **a** $(x+\sqrt{3})(x-\sqrt{3})$ **b** no linear factors
c $(x+\sqrt{15})(x-\sqrt{15})$ **d** $3(x+\sqrt{5})(x-\sqrt{5})$
e $(x+1+\sqrt{6})(x+1-\sqrt{6})$ **f** no linear factors
g $(x-2+\sqrt{7})(x-2-\sqrt{7})$
h $(x+3+\sqrt{17})(x+3-\sqrt{17})$ **i** no linear factors

4 **a** $(x+3)(x-1)$ **b** $4(x+2)(x-1)$ **c** $(x-5)(x+3)$
d $3(x+1)(3-x)$ **e** $(3x+2)(x-2)$
f $(2x+3)(4x-3)$ **g** $(3x-1)(x+3)$
h $8x(x-1)$ **i** $3(2x+1)(2x-3)$