Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 3/12 Topic: 9BCD Factorization	0 1 2	
Tuesday Date: _____ Topic:_____	0 1 2	
Wednesday Date: _____ Topic:_____	0 1 2	
Thursday Date: _____ Topic:_____	0 1 2	
Friday Date: _____ Topic:_____	0 1 2	

9A (3g)

$(x-3)(x-4)$

$(x-3)^2 - 1(x+3)$

$\frac{(x-3)^2}{(x-3)^1} - \frac{1(x-3)^1}{(x-3)}$

$(x-3)[x-3-1]$

9B 4b $3^2 = 9$ $(2x+1)(2x+1)$

$(2x+1)^2 - 9$

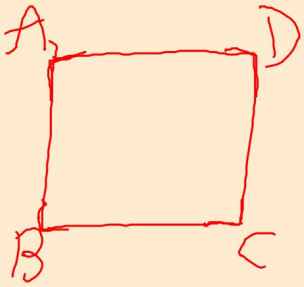
$(2x+1)^2 - 3^2$

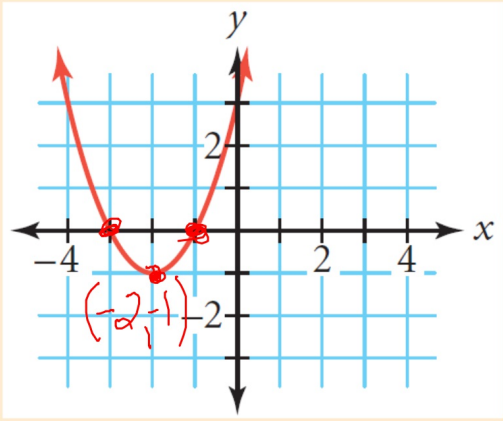
$(2x+1-3)(2x+1+3)$

$(2x-2)(2x+4)$

Warm-up: What are the intercepts of the graph below?

(-3, 0) (-1, 0)





At what point does the graph change direction?

(-2, -1)

Factored form

$y = a(x - r_1)(x - r_2)$

General form

$y = ax^2 + bx + c$

Another form... that allows us to see the change in the graph.

Vertex Form

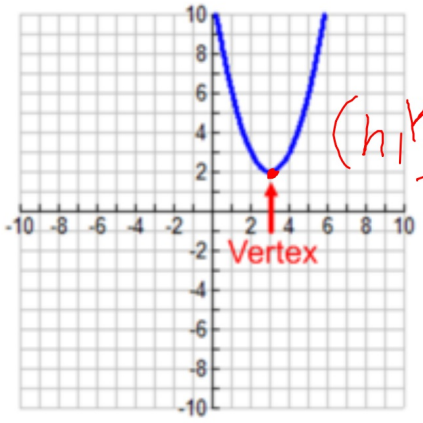
$y = a(x - h)^2 + k$

Vertex Form

$y = (x - 3)^2 + 2$

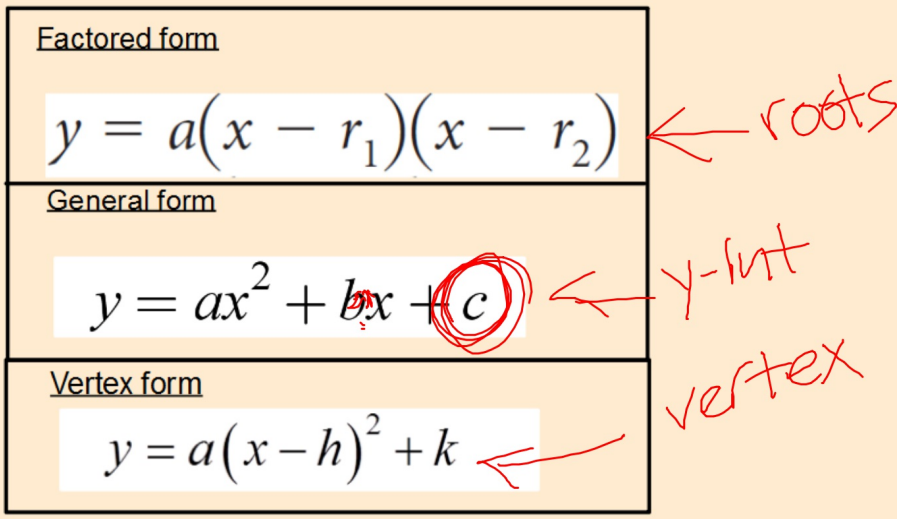
h k

Vertex: (h, k)
Vertex: (3, 2)



(h, k) = (3, 2)

Forms of a Quadratic:



What critical points are found easily with each form?

Class Plan:

1. Warm-up
2. Solving Quadratics

A	EQUATIONS OF THE FORM $x^2 = k$
B	THE NULL FACTOR LAW
C	SOLUTION BY FACTORISATION

4. Practice

http://www.boredpanda.com/rescued-baby-sloths-conversation/

Take a step back... ***Before writing anything...***

What could the value of *x* be...?

Now show using solving methods.

$(x-2)^2+3=39$

$-3 \quad -3$

$\sqrt{(x-2)^2}=\sqrt{36}$

$x-2=-6$ and 6

$x-2=6$ | $x-2=-6$

$x=8$ | $x=-4$

$(-6)^2=36$

$(6)^2=36$

Solving from Vertex Form...

Take a step back... ***Before writing anything...***

What could the value of *x* be...?

Now show using solving methods.

$$(x-2)^2+3=39$$

$$-3 \quad -3$$

$$(x-2)^2=36$$

$$x-2=6 \quad | \quad x-2=-6$$

$$\boxed{x=8} \quad | \quad \boxed{x=-4}$$

Solving from Vertex Form...

A EQUATIONS OF THE FORM $x^2 = k$

Example 1 Self Tutor

Solve for x:

a $x^2 + 3 = 6$

$-3 \quad -3$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \sqrt{3}$ and $x = -\sqrt{3}$

b $3 - 2x^2 = 7$

$-3 \quad -3$

$-2x^2 = 4$

$\frac{-2x^2}{-2} = \frac{4}{-2}$

$x^2 = -2$

SOLUTIONS

Example 1 — Self Tutor

Solve for x:

a $x^2 + 3 = 6$ **b** $3 - 2x^2 = 7$

a $x^2 + 3 = 6$

$\therefore \quad x^2 = 3$ {subtracting 3 from both sides}

$\therefore \quad x = \pm\sqrt{3}$

nonreal ans.

b $3 - 2x^2 = 7$

$\therefore \quad -2x^2 = 4$ {subtracting 3 from both sides}

$\therefore \quad x^2 = -2$ {dividing both sides by -2}

which has no real solutions as x^2 cannot be < 0.

Solving from Vertex Form... $y = a(x-h)^2 + k$

Example 2 — Self Tutor

Solve for x:

a $(x-2)^2 = 25$

b $(x+1)^2 = 7$

$x+1 = -\sqrt{7}$ $\quad$ $x+1 = \sqrt{7}$

$-1 \quad -1 \qquad -1 \quad -1$

$x = -1 \pm \sqrt{7}$

SOLUTIONS

Example 2 — Self Tutor

Solve for x:

a $(x-2)^2 = 25$ **b** $(x+1)^2 = 7$

a
$(x-2)^2 = 25$
$\therefore\ x-2 = \pm\sqrt{25}$
$\therefore\ x-2 = \pm 5$
$\therefore\ x = 2 \pm 5$
$\therefore\ x = 7$ or -3

b
$(x+1)^2 = 7$
$\therefore\ x+1 = \pm\sqrt{7}$
$\therefore\ x = -1 \pm \sqrt{7}$

The two solutions to **b** are $x = -1+\sqrt{7}$ and $x = -1-\sqrt{7}$.

$x-2=5$ and $x-2=-5$
$+2\ +2$ $+2\ +2$
$x=7$ $x=-3$

What do we know about the product of zero?

In other words...how can two factors multiply to equal zero?

$$a \cdot b = 0$$

$a = 0$ and/or $b = 0$

One ***or*** both factors **MUST** be zero!

Take a step back... ***Before writing anything...***

What could the value of *x* be...?

Now show using solving methods.

$$(x-2)(x-1)=0$$

$(0)(0)=0$

$x=2,1$

Solving from Factored Form...

Take a step back... ***Before writing anything...***

What could the value of *x* be...?

Now show using solving methods.

$$(x-2)(x-1)=0$$

$x-2=0$ | $x-1=0$

$x=2$ and $x=1$

Solving from Factored Form...

B THE NULL FACTOR LAW

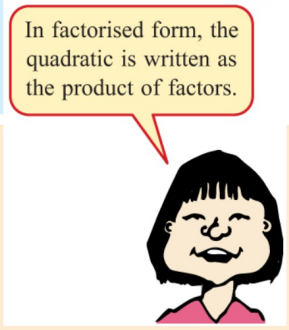
For quadratic equations which are not of the form $x^2 = k$, we need an alternative method of solution.

If a quadratic equation is given in **factorised form** then we can use the **Null Factor law**.

The **Null Factor law** states that:

When the product of two or more numbers is zero, then *at least one* of them must be zero.

So, if $ab = 0$ then $a = 0$ or $b = 0$.



B THE NULL FACTOR LAW

Example 3

Solve for x using the Null Factor law:

a $2x(x-4)=0$

b $(x+3)(2x-5)=0$

$\frac{2x}{2}=\frac{0}{2}$

$x=0$

$x-4=0$

$+4 \quad +4$

$x=4$

B THE NULL FACTOR LAW

Example 3 Self

Solve for x using the Null Factor law:

a $2x(x-4)=0$

$\therefore\ 2x=0$ or $x-4=0$

$\therefore\ x=0$ or 4

b $(x+3)(2x-5)=0$

$\therefore\ x+3=0$ or $2x-5=0$

$\therefore\ x=-3$ or $2x=5$

$\therefore\ x=-3$ or $\frac{5}{2}$

C SOLUTION BY FACTORISATION

To solve quadratic equations which are not in factorised form, we use the following procedure:

Step 1: If necessary, rearrange the equation so one side is **zero**.
Step 2: **Fully factorise** the other side (usually the LHS).
Step 3: Use the **Null Factor law**: if $ab = 0$ then $a = 0$ or $b = 0$.
Step 4: **Solve** the resulting linear equations.

To factorise the quadratic, we use the techniques we learnt in **Chapter 9**. We first take out any common factors, then recognise the type of factorisation required. We look for:

- **Difference of two squares** $x^2 - a^2 = (x + a)(x - a)$
- **Perfect squares** $x^2 + 2ax + a^2 = (x + a)^2$ or $x^2 - 2ax + a^2 = (x - a)^2$
- **Sum and product method** $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$
- **Splitting the x-term** $ax^2 + bx + c, \quad a \neq 0$
  - Find numbers p and q whose sum is b and whose product is ac.
  - Replace bx by $px + qx$.
  - Complete the factorisation.

Take a step back... ***Before writing anything...***

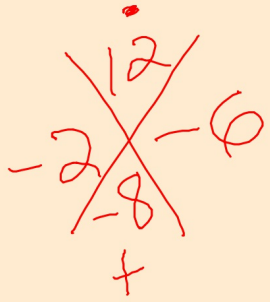
What could the value of *x* be...?

Now show using solving methods.

$$x^2 - 8x + 12 = 0$$

$(x-2)(x-6) = 0$

$x = 2, 6$

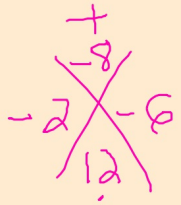


Solving from General Form...

Take a step back... ***Before writing anything...***

What could the value of *x* be...?

Now show using solving methods.



$$x^2 - 8x + 12 = 0$$

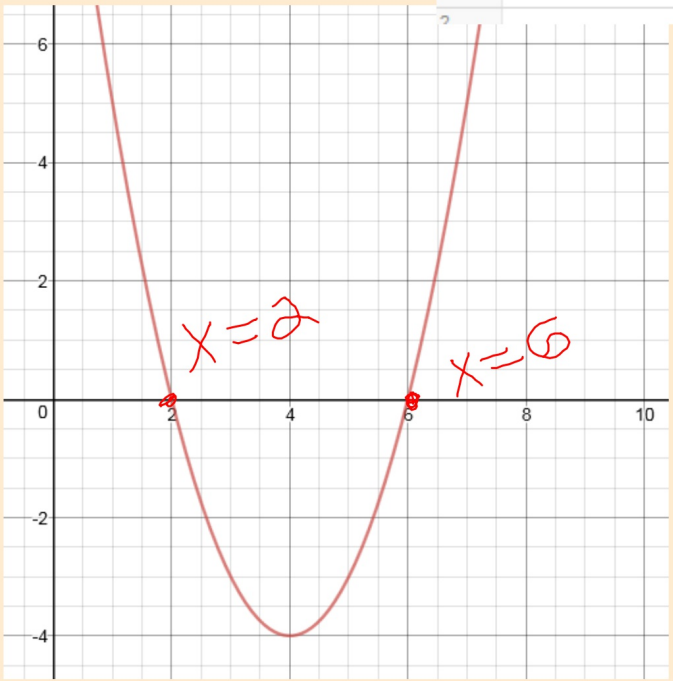
$$(x-2)(x-6) = 0$$

$$x = 2 \text{ and } 6$$

Solving from General Form...

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$



$y = 0$

From Desmos...

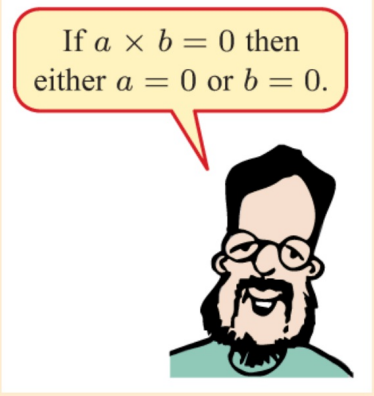
Video Break!

http://www.boredpanda.com/rescued-baby-sloths-conversation/

C SOLUTION BY FACTORISATION

Example 4

Solve for x: $x^2 = 7x$



SOLUTIONS

$$
\begin{aligned}
x^2 &= 7x \\
\therefore\ x^2 - 7x &= 0 && \{\text{making the RHS} = 0\} \\
\therefore\ x(x-7) &= 0 && \{\text{factorising the LHS}\} \\
\therefore\ x = 0 \ \text{ or } \ x - 7 &= 0 && \{\text{Null Factor law}\} \\
\therefore\ x &= 0 \ \text{ or } \ 7
\end{aligned}
$$

C SOLUTION BY FACTORISATION

Example 5

...verify your solution!

Solve for x: $x^2 + 2x = 8$

$-8 \quad -8$

$x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$

$x = 2, -4$

$-8 = -2 \times 4$

$-2 + 4 = 2$

$+$

$(2)^2 + 2(2) = 8$

$(2-2)(2+4)$

$0(6) = 0$ ✓

$(-4)^2 + 2(-4) = 8$

$(-4-2)(-4+4)$

$(-6)(0) = 0$ ✓

C SOLUTION BY FACTORISATION

Example 5

Solve for x: $x^2 + 2x = 8$

$x^2 + 2x = 8$

$\therefore\ x^2 + 2x - 8 = 0$ {making the RHS $= 0$}

$\therefore\ (x+4)(x-2) = 0$ {the numbers 4 and -2 have sum 2 and product -8}

$\therefore\ x + 4 = 0$ or $x - 2 = 0$ {Null Factor law}

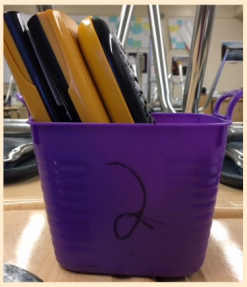
$\therefore\ x = -4$ or 2

Solution verified

Check: If $x = -4$, then $x^2 + 2x = (-4)^2 + 2(-4) = 16 - 8 = 8$ ✓

If $x = 2$, then $x^2 + 2x = 2^2 + 2(2) = 4 + 4 = 8$ ✓

Exercises...Please organize your table. Thank you!



EXERCISE 18A

1 Solve for x:

- **a** $x^2 = 16$
- **d** $12x^2 = 72$
- **g** $2x^2 + 1 = 51$

2 Solve for x:

- **a** $(x-2)^2 = 9$
- **d** $(x-4)^2 = 2$
- **g** $(x+5)^2 = 3$
- **j** $(3x+5)^2 = 0$

EXERCISE 18B

2 Solve for x using the Null Factor law:

- **a** $x(x+3) = 0$
- **d** $4x(2-x) = 0$
- **g** $(2x+3)(2x+1) = 0$
- **j** $2(x-3)^2 = 0$
- **m** $2x(3x-5) = 0$
- **p** $8(3x+1)^2 = 0$

Exercises...

EXERCISE 18C

1 Solve for x:

- **a** $x^2 + 12x = 0$
- **d** $3x^2 = 21x$
- **g** $3x^2 = 7x$

2 Solve for x:

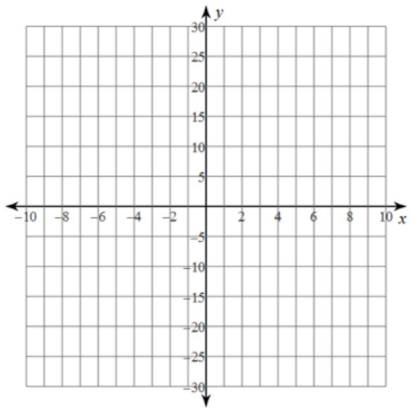
- **a** $x^2 + 6x + 8 = 0$
- **d** $x^2 - 49 = 0$
- **g** $x^2 = 2x + 15$
- **j** $x^2 + 60 = 17x$

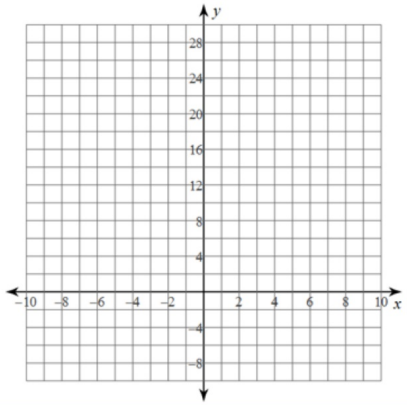
3 Solve for x:

- **a** $-x^2 - 5x + 24 = 0$
- **d** $2x^2 + 8x = 42$
- **g** $4x^2 + 72 = 36x$

Exercises...

4. Graph the quadratic functions of #3a and #2a. Label critical points.





Exercises...**Solutions**

EXERCISE 18A

1 **a** $x = \pm 4$ **b** $x = \pm\sqrt{19}$ **c** $x = \pm 3$

d $x = \pm\sqrt{6}$ **e** no real solutions **f** $x = 0$

g $x = \pm 5$ **h** no real solutions **i** $x = \pm\sqrt{3}$

2 **a** $x = 5$ or -1 **b** $x = 1$ or -9

c no real solutions **d** $x = 4 \pm \sqrt{2}$

e no real solutions **f** $x = 2$ **g** $x = -5 \pm \sqrt{3}$

h $x = 6 \pm \sqrt{2}$ **i** $x = -8 \pm \sqrt{13}$ **j** $x = -\frac{5}{3}$

k $x = \frac{1}{5}$ or $-\frac{3}{5}$ **l** $x = \frac{1+\sqrt{2}}{2}$ or $\frac{1-\sqrt{2}}{2}$

Exercises...

EXERCISE 18B

1 **a** $p = 0$ or $q = 0$ **b** $a = 0$ **c** $p = 0$

d $x = 0$ or $y = 0$ **e** $m = 0$ or $n = 0$

f $a = 0$ or $c = 0$ **g** $w = 0$ or $x = 0$

h $w = 0$ or $x = 0$ or $y = 0$ or $z = 0$
{at least one of them is 0}

2 **a** $x = 0$ or -3 **b** $x = 0$ or 5 **c** $x = 1$ or 3

d $x = 0$ or 2 **e** $x = 0$ or $-\frac{1}{2}$ **f** $x = -2$ or $\frac{1}{2}$

g $x = -\frac{3}{2}$ or $-\frac{1}{2}$ **h** $x = -2$ or 7 **i** $x = 5$ or $-\frac{2}{3}$

j $x = 3$ **k** $x = -\frac{1}{2}$ **l** $x = 0$

m $x = 0$ or $\frac{5}{3}$ **n** $x = 0$ or $\frac{6}{5}$ **o** $x = -\frac{1}{3}$ or $\frac{5}{2}$

p $x = -\frac{1}{3}$ **q** $x = \frac{3}{4}$ **r** $x = 0$ or $\frac{4}{3}$

Exercises...Solutions

EXERCISE 18C

1	**a**	$x = 0$ or -12	**b** $x = 0$ or -3	**c** $x = 0$ or 4
	d	$x = 0$ or 7	**e** $x = 0$ or $-\frac{7}{2}$	**f** $x = 0$ or 9
	g	$x = 0$ or $\frac{7}{3}$	**h** $x = 0$ or $\frac{9}{4}$	**i** $x = 0$ or $-\frac{8}{3}$
2	**a**	$x = -2$ or -4	**b** $x = -3$ or -8	**c** $x = -2$
	d	$x = \pm 7$	**e** $x = -8$ or 6	**f** $x = 5$
	g	$x = 5$ or -3	**h** $x = 7$ or -2	**i** $x = 3$ or 4
	j	$x = 5$ or 12	**k** $x = 11$ or -2	**l** $x = 6$ or -3
3	**a**	$x = -8$ or 3	**b** $x = -2$ or -6	**c** $x = 5$ or -4
	d	$x = -7$ or 3	**e** $x = 6$ or -1	**f** $x = 1$
	g	$x = 3$ or 6	**h** $x = -3$	**i** $x = -15$ or 2

SOLUTIONS

18ABC Solutions to Quadratic Equations Name Key

A EQUATIONS OF THE FORM $x^2 = k$

EXERCISE 18A

1 Solve for x:

a $\sqrt{x^2} = \sqrt{16}$ $\quad x = \pm\sqrt{16} \Rightarrow \boxed{x = \pm 4}$

d $\frac{12x^2}{12} = \frac{72}{12} \Rightarrow x^2 = 6 \Rightarrow \sqrt{x^2} = \pm\sqrt{6} \Rightarrow \boxed{x = \pm\sqrt{6}}$

g $2x^2 + 1 = 51$ $\quad \Rightarrow \frac{2x^2}{2} = \frac{50}{2} \Rightarrow x^2 = 25 \Rightarrow \sqrt{x^2} = \pm\sqrt{25} \Rightarrow \boxed{x = \pm 5}$

2 Solve for x:

a $\sqrt{(x-2)^2} = \pm\sqrt{9}$ $\quad x - 2 = \pm\sqrt{9} \Rightarrow x = 2 \pm 3 \Rightarrow \boxed{x = -1, x = 5}$

d $\sqrt{(x-4)^2} = \pm\sqrt{2}$ $\quad x = 4 \pm \sqrt{2} \Rightarrow \boxed{x = 4 - \sqrt{2},\ x = 4 + \sqrt{2}}$

g $\sqrt{(x+5)^2} = \pm\sqrt{3}$ $\quad x = -5 \pm \sqrt{3} \Rightarrow \boxed{x = -5 - \sqrt{3},\ x = -5 + \sqrt{3}}$

j $\sqrt{(3x+5)^2} = \pm\sqrt{0}$ $\quad 3x + 5 = \pm\sqrt{0} \Rightarrow 3x = -5 \Rightarrow \frac{3x}{3} = \frac{-5}{3} \Rightarrow \boxed{x = -\frac{5}{3}}$

SOLUTIONS

B THE NULL FACTOR LAW

EXERCISE 18B

2 Solve for x using the Null Factor law:

a $x(x+3)=0 \Rightarrow$ $x=0,\ x=-3$

d $4x(2-x)=0 \Rightarrow$ $x=0,\ x=2$

g $(2x+3)(2x+1)=0 \Rightarrow$ $2x+3=0$, $\frac{2x}{2}=\frac{-3}{2}$, $x=-\frac{3}{2}$ and $2x+1=0$, $\frac{2x}{2}=\frac{-1}{2}$, $x=-\frac{1}{2}$

j $2(x-3)^2=0 \Rightarrow$ $x=3$

m $2x(3x-5)=0 \Rightarrow$ $x=0$ and $3x-5=0 \Rightarrow \frac{3x}{3}=\frac{5}{3}$, $x=\frac{5}{3}$

p $8(3x+1)^2=0$

$3x+1=0 \Rightarrow \frac{3x}{3}=\frac{-1}{3} \Rightarrow x=-\frac{1}{3}$

SOLUTIONS

EXERCISE 18C

1 Solve for x:

a $x^2 + 12x = 0$ $\quad x(x+12) = 0$ $\quad x = 0,\ x = -12$

d $3x^2 = 21x$ $\quad 3x^2 - 21x = 0$ $\quad 3x(x-7) = 0$ $\quad x = 0,\ x = 7$

g $3x^2 = 7x$ $\quad 3x^2 - 7x = 0$ $\quad x(3x-7) = 0$ $\quad x = 0$ or $3x - 7 = 0$, $3x = 7$, $x = \frac{7}{3}$

2 Solve for x:

a $x^2 + 6x + 8 = 0$ $\quad (x+4)(x+2)$ $\quad x = -4$ OR $x = -2$

d $x^2 - 49 = 0$ $\quad (x+7)(x-7) = 0$ $\quad x = 7$ OR $x = -7$

g $x^2 = 2x + 15$ $\quad x^2 - 2x - 15 = 0$ $\quad (x-5)(x+3)$ $\quad x = 5$ OR $x = -3$

j $x^2 + 60 = 17x$ $\quad x^2 - 17x + 60 = 0$ $\quad (x-5)(x-12)$ $\quad x = 5$ OR $x = 12$

3 Solve for x:

a $-x^2 - 5x + 24 = 0$ $\quad -(x^2 + 5x - 24) = 0 \Rightarrow -(x+8)(x-3) = 0 \Rightarrow x = -8,\ x = 3$

d $2x^2 + 8x = 42$ $\quad \Rightarrow 2x^2 + 8x - 42 = 0 \Rightarrow 2(x^2 + 4x - 21) = 0 \Rightarrow 2(x+7)(x-3) = 0$ $\quad x = -7,\ x = 3$

g $4x^2 + 72 = 36x \Rightarrow 4x^2 - 36x + 72 = 0 \Rightarrow 4(x^2 - 9x + 18) = 0 \Rightarrow 4(x-18)(x+9) = 0$ $\quad x = 18$, $x = -9$

4. Graph the quadratic functions of #3a and #2a (create a table of values). Label critical points.

SOLUTIONS

$y = -x^2 - 5x + 24$

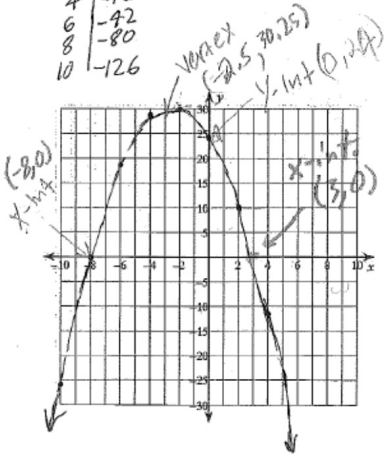
X	Y
-10	-26
-8	0
-6	18
-4	28
-2	30
0	24
2	10
4	-12
6	-42
8	-80
10	-126

$-(x^2+5x-24)=0$

$-(x+8)(x-3)=0$

$x=-8,\ x=3$

$-(-2.5)^2-5(-2.5)+24=30.25$



$y = x^2 + 6x + 8$

X	Y
-10	48
-8	24
-6	8
-4	0
-2	0
0	8
2	24
4	48
6	80
8	120
10	168

$(-3)^2+6(-3)+8$

$=9-18+8$

$=-1$