

Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: <u>3/12</u> Topic: <u>4CD Distribution</u>	0 1 2	
Tuesday Date: _____ Topic: _____	0 1 2	
Wednesday Date: _____ Topic: _____	0 1 2	
Thursday Date: _____ Topic: _____	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Warm-up: Translate from factored to general form.
How could you go the other way?

Consider the expansion:

$$\underline{(x + 1)(x + 6)} = x^2 + 6x + x + 6$$

$$\begin{array}{r|cc} & x & 6 \\ x & x^2 & 6x \\ 1 & x & 6 \end{array} = x^2 + 7x + 6$$

Warm-up: Translate from factored to general form.
How could you go the other way?

Consider the expansion:

$$(x + 1)(x + 6)$$

$$x^2 + 6x + 1x + 1 \cdot 6$$

$$x^2 + \underbrace{(6+1)}_{\text{sum}} x + \underbrace{1 \cdot 6}_{\text{product}}$$

$$x^2 + \underline{7}x + \underline{6}$$

E**FACTORISING QUADRATIC TRINOMIALS**

Consider the expansion

$$\begin{aligned}(x + 1)(x + 6) &= x^2 + 6x + x + 1 \times 6 && \{\text{using FOIL}\} \\ &= x^2 + (6 + 1)x + (1 \times 6) \\ &= x^2 + (\mathbf{\text{sum of 1 and 6}})x + (\mathbf{\text{product of 1 and 6}}) \\ &= x^2 + 7x + 6\end{aligned}$$

So, to factorise $x^2 + 7x + 6$, we need two numbers with a sum of 7 and a product of 6. These numbers are 1 and 6, and so $x^2 + 7x + 6 = (x + 1)(x + 6)$.

We call this the **sum and product method**.

Class Plan:

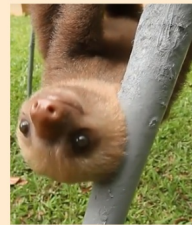
1. Warm-up

2. 9E Quadratic Factorization

- Video Break

- Examples

3. Practice



Chapter

9

Quadratic factorisation

- A** Removing common factors
- B** Difference of two squares factorisation
- C** Perfect square factorisation
- D** Factorising expressions with four terms
- E** Factorising quadratic trinomials
- F** Factorising $ax^2 + bx + c$, $a \neq 1$

Take a step back...

What is a factor?

Factors are the numbers you multiply together to get another number:

$$\text{Factor } \underbrace{2} \times \underbrace{3} = 6 \text{ Factor}$$

fac·tor

/ˈfaktər/ 

noun

plural noun: **factors**

2 values multiplied

1. a circumstance, fact, or influence that contributes to a result or outcome.
"his legal problems were not a factor in his decision"
synonyms: element, part, component, ingredient, strand, constituent, point, detail, item, feature, facet, aspect, characteristic, consideration, influence, circumstance
"this had been a key factor in his decision to withdraw"
2. a number or quantity that when multiplied with another produces a given number or expression.

Times Table - 12x12

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

$(12, 1)$ $(1, 12)$

→ Fa

$(2, 6)$

$(6, 2)$

$(3, 4)$

$(4, 3)$

Take a step back...

What is a **factor** (in Algebra)?

In Algebra, factors are what we can multiply together to get an expression.

$(x+3)$ and $(x+1)$ are factors of $x^2 + 4x + 3$:

$$(x+3)(x+1) = x^2 + 4x + 3$$

Factor Factor

Today we are solving for factors.
(**what** we can multiply together
to get an expression)

<https://www.mathsisfun.com/definitions/factor.html>

Quadratic factorisation

The process of writing an expression as a product of its factors.

A **quadratic trinomial** in x is an expression of the form $ax^2 + bx + c$ where x is the variable, and a , b , and c are constants, $a \neq 0$.

$$\begin{array}{ccccccc} ax^2 & + & bx & + & c & & \\ \uparrow & & \uparrow & & \uparrow & & \\ \text{the } x^2 \text{ term} & & \text{the } x \text{ term} & & \text{the constant term} & & \end{array}$$

For example: $x^2 + 5x + 6$, $4x^2 - 9$ and $9x^2 + 6x + 1$ are quadratic expressions.

In **Chapter 4** we studied the expansion of algebraic factors, many of which resulted in quadratic trinomials. In this chapter we will consider **factorisation**, which is the reverse process of expansion.

Factorisation is the process of writing an expression as a **product of its factors**.

For example:

$$\begin{array}{c} \text{expansion} \\ \curvearrowright \\ (x + 2)(x + 3) = x^2 + 5x + 6 \\ \curvearrowleft \\ \text{factorisation} \end{array}$$

Goal Today:

How can we translate from
general form to factored form?

The diagram illustrates the relationship between expansion and factorisation. It features the equation $(x + 2)(x + 3) = x^2 + 5x + 6$. A red curved arrow labeled "expansion" points from the factored form on the left to the general form on the right. A second red curved arrow labeled "factorisation" points from the general form on the right back to the factored form on the left. Two empty red rectangular boxes are positioned below the equation: one under the factored form and one under the general form, with arrows pointing from the boxes towards the respective parts of the equation.

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

expansion

factorisation

The *reverse* process of expansion

E**FACTORISING QUADRATIC TRINOMIALS**

Factored form

$$y = a(x - r_1)(x - r_2)$$

General form

$$y = ax^2 + bx + c$$

a: Stretch or shrink factor of the parabola

E**FACTORISING QUADRATIC TRINOMIALS**

More generally,



$$(x+a)(x+b) = x^2 + bx + ax + ab$$

$$= x^2 + \underbrace{(a+b)}_{\text{SUM}}x + \underbrace{ab}_{\text{PRODUCT}}$$

$$x^2 + px + q = (x+a)(x+b)$$

where a and b are two numbers whose sum is p , and whose product is q .

Factoring Method Introduction:

Example: $x^2 + 5x + 6$

Factors of 6

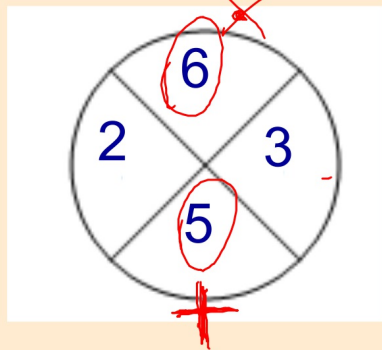
Times Table - 12x12

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4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
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10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Factoring Method Introduction:

How do the values **2 and **3** relate to the coefficients in the expression $x^2 + 5x + 6$?

***How is this digram related to the situation above?



$$\begin{aligned} (2) + (3) &= 5 \\ (2)(3) &= 6 \end{aligned}$$

E**FACTORISING QUADRATIC TRINOMIALS****Example 10**

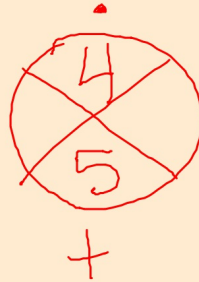
Use the sum and product method to fully factorise:

a $x^2 + 5x + 4$

$$(x+4)(x+1)$$

OR

$$(x+1)(x+4)$$



$$2 \cdot 2 = 4$$

$$4 \cdot 1 = 4$$

$$-4 \cdot -1 = 4$$

The **sum** of the numbers is the coefficient of x .
The **product** of the numbers is the constant term.



Example 10

Use the sum and product method to fully factorise:

b $x^2 - x - 12$

$$(x-4)(x+3)$$

	$x - 4$
x	$x^2 - 4x$
3	$3x + 12$

$$= x^2 - x - 12 +$$

$$-12 + 1 \neq -1$$

-12	3
-4	-1

SOLUTIONS

- a** We need two numbers with sum 5, and product 4.

The numbers are 1 and 4.

$$\therefore x^2 + 5x + 4 = (x + 1)(x + 4)$$

- b** We need two numbers with sum -1 and product -12 .

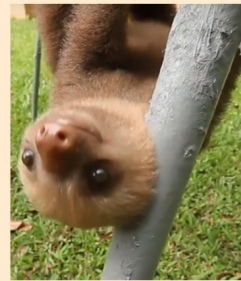
The numbers are -4 and 3 .

$$\therefore x^2 - x - 12 = (x - 4)(x + 3)$$



Video Break!

<http://www.boredpanda.com/rescued-baby-sloths-conversation/>

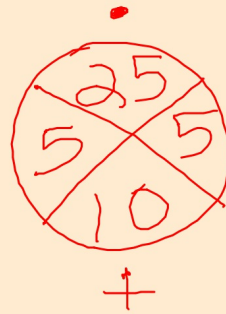


Example 7

Fully factorise:

a $x^2 + 10x + 25 = (x+5)^2$

$(x+5)(x+5)$

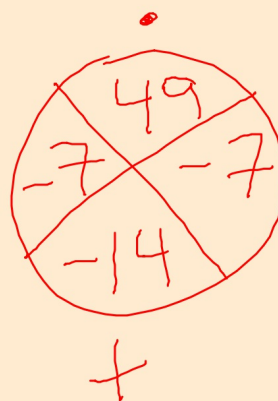


Example 7

Fully factorise:

b $x^2 - 14x + 49$

$$= (x - 7)^2$$



Do: Exit Ticket!

1) Expand - FOIL method? Box method?

2) Factor

Exit Ticket

Name _____

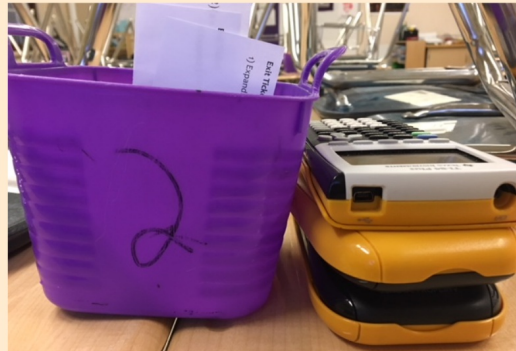
1) Expand: $(x+5)(x-3)$

2) Factor: $x^2 + 8x + 12$

Done?

1) Put in bucket or give to your teacher

2) Get homework



SOLUTIONS

Example 7



Fully factorise:

a $x^2 + 10x + 25$

b $x^2 - 14x + 49$

a $x^2 + 10x + 25$

$$= x^2 + 2 \times x \times 5 + 5^2$$

$$= (x + 5)^2$$

b $x^2 - 14x + 49$

$$= x^2 - 2 \times x \times 7 + 7^2$$

$$= (x - 7)^2$$

Exercises...Please organize your table. Thank you!

EXERCISE 9E

1 Find two numbers which have:

a product 12 and sum 7

3,4

c product 16 and sum 10

e product -36 and sum 9

-3,12

g product -12 and sum -4



2 Factorise:

a $x^2 + 4x + 3$

b $x^2 + 11x + 24$

d $x^2 + 15x + 54$

e $x^2 + 9x + 20$

EXERCISE 9E

1 Find two numbers which have:

- a product 12 and sum 7
- c product 16 and sum 10
- e product -36 and sum 9
- g product -12 and sum -4

2 Factorise:

a $x^2 + 4x + 3$

b $x^2 + 11x + 24$

d $x^2 + 15x + 54$

e $x^2 + 9x + 20$

g $x^2 + 10x + 24$

h $x^2 + 9x + 14$

3 Factorise:

a $x^2 - 3x + 2$

b $x^2 - 4x + 3$

d $x^2 - 14x + 33$

e $x^2 - 16x + 39$

g $x^2 - 11x + 28$

h $x^2 - 14x + 24$

4 Factorise:

a $x^2 - 7x - 8$

b $x^2 + 4x - 21$

d $x^2 - 2x - 8$

e $x^2 + 5x - 24$

9E Solutions

EXERCISE 9E

- 1** **a** 3 and 4 **b** 3 and 5 **c** 2 and 8
 e -3 and 12 **f** 3 and -12 **g** 2 and -6
- 2** **a** $(x + 1)(x + 3)$ **b** $(x + 3)(x + 8)$
 d $(x + 6)(x + 9)$ **e** $(x + 4)(x + 5)$
 g $(x + 4)(x + 6)$ **h** $(x + 2)(x + 7)$
- 3** **a** $(x - 1)(x - 2)$ **b** $(x - 1)(x - 3)$
 d $(x - 3)(x - 11)$ **e** $(x - 3)(x - 13)$
 g $(x - 4)(x - 7)$ **h** $(x - 2)(x - 12)$
- 4** **a** $(x - 8)(x + 1)$ **b** $(x + 7)(x - 3)$
 d $(x - 4)(x + 2)$ **e** $(x + 8)(x - 3)$