Welcome Back to MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 3/12 Topic: 9BCD Factorization	0 1 2	
Tuesday Date: 3/13 Topic: 18ABC - Solving Quadratics	0 1 2	
Wednesday Date: ______ Topic: ____________	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

2

$$4x^2+72=36x$$

$$4x \quad -36x+72=0$$

$$4(x^2-9x+18)=0$$

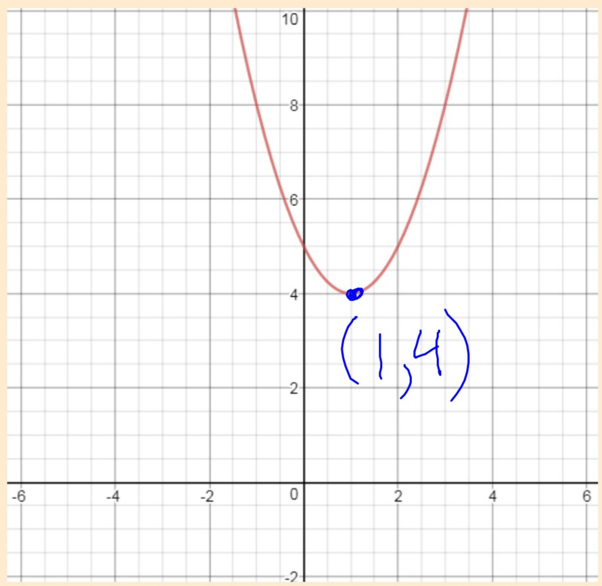
$$4(x-3)(x-6)=0$$

$$\boxed{x=3,6}$$

Nonreal Roots (Complex Numbers)

Warm-up: What are the solutions?

1 $y = (x-1)^2 + 4$



Nonreal Roots (Complex Numbers)

What are the solutions?

1 $y = (x-1)^2 + 4$

$0 = (x-1)^2 + 4$

$i = \sqrt{-1}$

$\sqrt{-4} = \sqrt{(x-1)^2}$

$\sqrt{-4} = x - 1$

$\sqrt{4}\sqrt{-1} = x - 1$

$\pm 2i = x - 1$

$+1 \quad +1$

$1 \pm 2i = x$

$-2i + 1$

$x = 1 - 2i$

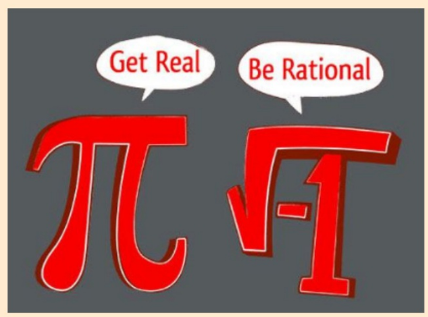
$x = 1 + 2i$

Class Plan:

1. Warm-up

2. Complex Numbers
-Nonreal solutions to quadratics

3. Practice



How do you square root a negative number?

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2}$$

How do you take the square root of a negative number? The two numbers $\frac{-4 + \sqrt{-4}}{2}$ and $\frac{-4 - \sqrt{-4}}{2}$ are unlike any of the numbers you have worked with this year—they are nonreal, but they are still numbers. In the development of mathematics, new sets of numbers have been defined in order to solve problems. Mathematicians have defined fractions and not just whole numbers, negative numbers and not just positive numbers, irrational numbers and not just fractions. For the same reasons, we also have square roots of negative numbers, not just square roots of positive numbers. Numbers that include the real numbers as well as the square roots of negative numbers are called **complex numbers.**

History Connection

History
CONNECTION

Since the 1500s, the square root of a negative number has been called an imaginary number. In the late 1700s, the Swiss mathematician Leonhard Euler (1707–1783) introduced the symbol i to represent $\sqrt{-1}$. He wrote:

> It is evident that we cannot rank the square root of a negative number amongst possible numbers, and we must therefore say that it is an impossible quantity. . . . But notwithstanding this these numbers present themselves to the mind; they exist in our imagination, and we still have a sufficient idea of them; since we know that by $\sqrt{-4}$ is meant a number which, multiplied by itself, produces -4; for this reason also, nothing prevents us from making use of these imaginary numbers, and employing them in calculation.


Leonhard Euler

Defining imaginary numbers made it possible to solve previously unsolvable problems.

Imaginary numbers were needed to solve problems.

Science & Engineering

Complex numbers are used to model many applications, particularly in science and engineering. To measure the strength of an electromagnetic field, a real number represents the amount of electricity, and an imaginary number represents the amount of magnetism. The state of a component in an electronic circuit is also measured by a complex number, where the voltage is a real number and the current is an imaginary number. The properties of calculations with complex numbers apply to these types of physical states more accurately than calculations with real numbers do. In the investigation you'll explore patterns in arithmetic with complex numbers.

Complex Numbers -

include real numbers (*a* and *b*), as well as square root of a negative number.

$$a + bi$$

$$\sqrt{-1} = i$$

$$-1 = i^2$$

imaginary number

Complex Conjugate Pair

$$a + bi \qquad a - bi$$

Why will nonreal solutions to the Quadratic formula always be in this form?

Conjugate

The conjugate is where we **change the sign in the middle** of two terms like this:

$$3x + 1$$

Conjugate: $3x - 1$

Examples of Use

The conjugate can be very useful because ...

... when we multiply something by its conjugate we get **squares** like this:

$$(a + b)(a - b) = a^2 - b^2$$

https://www.mathsisfun.com/algebra/conjugate.html

Uses of Conjugate:
-Removing Radical or Imaginary unit from the denominator.

How does that help?

It can help us move a square root from the bottom of a fraction (the *denominator*) to the top, or vice versa. Read Rationalizing the Denominator to find out more:

Example: Move the square root of 2 to the top:

$$\frac{1}{3-\sqrt{2}}$$

We can ***multiply both top and bottom by 3+√2 (the conjugate of 3−√2)***, which won't change the value of the fraction:

$$\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{3^2-(\sqrt{2})^2} = \frac{3+\sqrt{2}}{7}$$

(The denominator becomes $\mathbf{(a+b)(a-b)} = \mathbf{a^2} - \mathbf{b^2}$ which simplifies to 9−2=7)

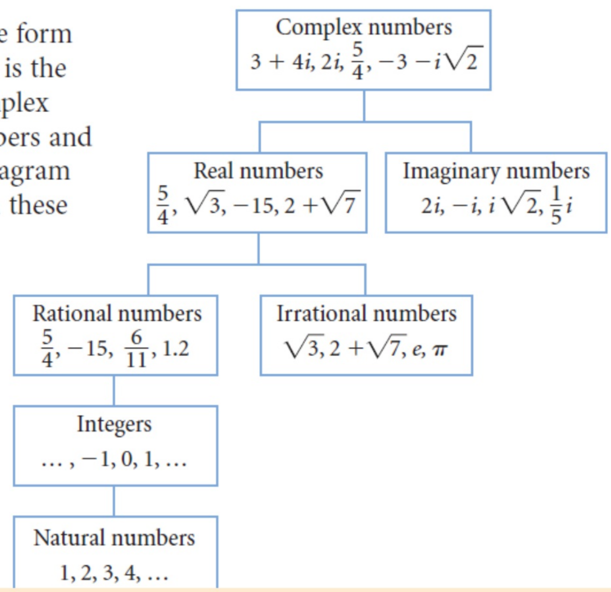
Complex Numbers

A **complex number** is a number in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$.

To express the square root of a negative number, we use an **imaginary unit** called ***i***, defined by $i^2 = -1$ or $i = \sqrt{-1}$. You can rewrite $\sqrt{-4}$ as $\sqrt{4} \cdot \sqrt{-1}$, or $2i$. Therefore, you can write the two solutions to the quadratic equation above as the complex numbers $\frac{-4 + 2i}{2}$ and $\frac{-4 - 2i}{2}$, or $-2 + i$ and $-2 - i$. These two solutions are a **conjugate pair.** That is, one is $a + bi$ and the other is $a - bi$. The two numbers in a complex pair are **complex conjugates.** Why will nonreal solutions to the quadratic formula always give answers that are a conjugate pair?

Number Sets

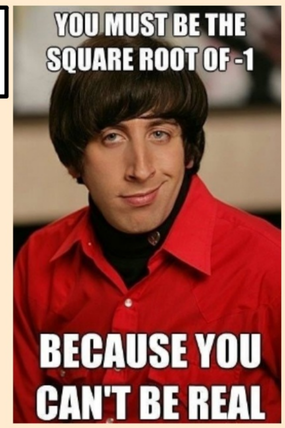
For any complex number in the form $a + bi$, a is the real part and b is the imaginary part. The set of complex numbers contains all real numbers and all imaginary numbers. This diagram shows the relationship between these numbers and some other sets you may be familiar with, as well as examples of numbers within each set.



Examples

$\sqrt{-1} = i$

$-1 = i^2$



$$x^2 + 2 = -9$$

$\sqrt{x^2} = \sqrt{-11}$

$x = \pm\sqrt{-11}$

$x = \pm\sqrt{-1}\sqrt{11}$

$x = \pm i\sqrt{11}$

$\{x = \pm 3i\}$

Examples

$$5b^2 = -315$$

$$\sqrt{-1} = i$$

$$-1 = i^2$$

$$\frac{5b^2}{5} = \frac{-315}{5}$$

$$\sqrt{b^2} = \sqrt{-63}$$

$$b = \pm\sqrt{-63}$$

$$b = \pm\sqrt{-1}\sqrt{9}\sqrt{7}$$

$$b = \pm 3i\sqrt{7}$$

Examples

$$-6r^2 - 10 = 0$$

$\sqrt{-1} = i$

$-1 = i^2$

$+10 \quad +10$

$\frac{-6r^2}{-6} = \frac{10}{-6}$

$\sqrt{r^2} = \sqrt{-\frac{5}{3}}$

$x = \pm\sqrt{\frac{-5}{3}} = \sqrt{-1}\sqrt{\frac{5}{3}}$

$x = \pm i \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \pm \frac{\sqrt{15}}{3} i$

Examples

$$-6r^2 - 10 = 0$$

$\sqrt{-1} = i$

$$\frac{-6r^2}{-6} = \frac{10}{-6}$$

$$r^2 = 5$$

$$r = i\frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$r = \frac{i\sqrt{15}}{3}$$

Exercises...

Solve each equation.

1) $x^2 + 7x = 0$

2) $k^2 - 8k = 0$

3) $p^2 - 14p + 48 = 0$

4) $v^2 - 2v - 35 = 0$

5) $x^2 - 1 = 15$

6) $6p^2 = -180$

Exercises...

7) $k^2 - 10 = -14$

8) $7k^2 = -315$

9) $12 + 8a = -a^2$

10) $x^2 = -4x$

11) $-30 = x - x^2$

12) $b^2 - 24 = -5b$

Exercises...

13) $-4r^2 + 9 = 0$

14) $-3x^2 - 11 = 0$

15) $4b^2 + 6 = 0$

16) $-4x^2 - 8 = 0$

17) $3v^2 + 9 = 0$

18) $4m^2 - 1 = 0$

Exercises...**Solutions**

1) $\{-7, 0\}$
2) $\{8, 0\}$
3) $\{8, 6\}$
4) $\{-5, 7\}$
5) $\{4, -4\}$
6) $\{i\sqrt{30}, -i\sqrt{30}\}$
7) $\{2i, -2i\}$
8) $\{3i\sqrt{5}, -3i\sqrt{5}\}$
9) $\{-6, -2\}$
10) $\{-4, 0\}$
11) $\{6, -5\}$
12) $\{-8, 3\}$
13) $\left\{-\frac{3}{2}, \frac{3}{2}\right\}$
14) $\left\{-\frac{i\sqrt{33}}{3}, \frac{i\sqrt{33}}{3}\right\}$
15) $\left\{\frac{i\sqrt{6}}{2}, -\frac{i\sqrt{6}}{2}\right\}$
16) $\{-i\sqrt{2}, i\sqrt{2}\}$
17) $\{i\sqrt{3}, -i\sqrt{3}\}$
18) $\left\{\frac{1}{2}, -\frac{1}{2}\right\}$

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