

## Welcome MYP 9 Mathematics!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
<b>Monday</b> Date: <u>4/16</u> Topic: <u>NO SCHOOL</u>	0 1 2	
<b>Tuesday</b> Date: <u>4/17</u> Topic: <u>10FG Five Number Summary &amp; Box Plots</u>	0 1 2	
<b>Wednesday</b> Date: _____ Topic: _____	0 1 2	
<b>Thursday</b> Date: _____ Topic: _____	0 1 2	
<b>Friday</b> Date: _____ Topic: _____	0 1 2	

## Class Plan

1. Warm-up

2. Investigation

-How do we interpret the different measures of spread?

-How can we verify we have outliers?

**F**

**MEASURING THE SPREAD OF A DATA SET**

3. Practice

C <sup>2</sup> O Fair	8:05-9:30
<b>Lunch A Schedule</b>	
1	9:35-10:12
2	10:17-10:54
3	10:59-11:36
LUNCH	11:41-12:11
4	12:16-12:53
5	12:58-1:35
6	1:40-2:17
7	2:22-3:00

C <sup>2</sup> O Fair	8:05-9:30
<b>Lunch B Schedule</b>	
1	9:35-10:12
2	10:17-10:54
3	10:59-11:36
4	11:41-12:18
LUNCH	12:23-12:53
5	12:58-1:35
6	1:40-2:17
7	2:22-3:00

C <sup>2</sup> O Fair	8:05-9:30
<b>Lunch C Schedule</b>	
1	9:35-10:12
2	10:17-10:54
3	10:59-11:36
4	11:41-12:18
5	12:23-1:00
LUNCH	1:05-1:35
6	1:40-2:17
7	2:22-3:00

## Warm-up:

Create a 5 # Summary.  
(Use your graphing calculator)  
Which players could be outliers?

Points Scored by Chicago Bulls  
Players Who Played over 40 Games  
(1997-98 Season)

```
1-Var Stats
↑n=10
minX=198
Q1=375
Med=524.5
Q3=841
maxX=2357
```

Chicago Bulls	Points
Michael Jordan	2357
Toni Kukoe	984
Scottie Pippen	841
Ron Harper	764
Lue Longley	633
Scott Burrell	416

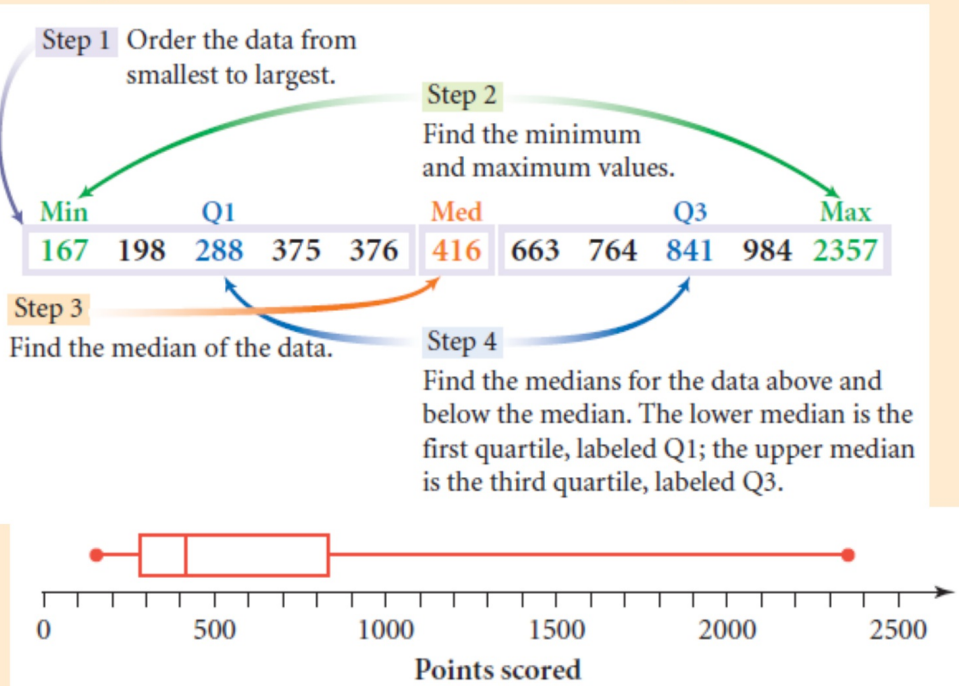
Chicago Bulls	Points
Steve Kerr	376
Dennis Rodman	375
Randy Brown	288
Jud Bucchler	198

National Basketball Association ([www.nba.com](http://www.nba.com))





## Chicago Bulls Data 5 # Summary and Box Plot



## What is an outlier?

Is there a formula to determine if a data point is an outlier?

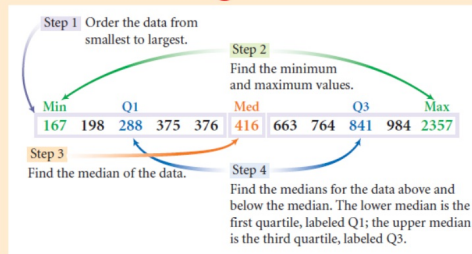
**\*\*\*A data point that is distinctly separate from the rest of the data.**

\*\*\*One definition of outlier is any data point more than 1.5 interquartile ranges (IQRs) below the first quartile or above the third quartile.

$$\text{Outlier} < Q_1 - 1.5IQR$$

$$\text{Outlier} > Q_3 + 1.5IQR$$

## Examining Outliers of Chicago Bulls



$$IQR = 841 - 288 = 553$$

$$1.5(553) = 829.5$$

$$Q1 - 1.5IQR = 288 - 829.5 = -541.5$$

$$Q3 + 1.5IQR = 841 + 829.5 = 1670.5$$

Did anyone score a number of points outside of -541.5 to 1670.5?

Michael Jordan	2357
----------------	------

## Investigation: Minnesota Lynx Data

Minnesota Lynx Players and their minutes played (2016 season)



1. Five Number Summary
2. Outliers
3. Standard Deviation (Measure of Spread)



Interpret & Analyze Measures

Maya Moore	263
Lindsay Whalen	213
Sylvia Fowles	251
Seimone Augustus	215
Rebekkah Brunson	234
Natasha Howard	104
Jia Perkins	141
Renee Montgomery	101
Keisha Hampton	8
Anna Cruz	51
Janel McCarville	19

**\*\*\*Come back together at end to go over solution & discuss standard deviation**

**Done: Help others!**

## Investigation: Minnesota Lynx Data

1) Calculate using the Lynx player's time.

Minimum 8 Maximum 263 Range 255

2) Analyze and interpret what the range says about the player's time on the basketball court.

Maya Moore	263
Lindsay Whalen	213
Sylvia Fowles	251
Seimone Augustus	215
Rebekkah Brunson	234
Natasha Howard	104
Jia Perkins	141
Renee Montgomery	101
Keisha Hampton	8
Anna Cruz	51
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## Investigation: Minnesota Lynx Data SOLUTION

Minnesota Lynx Players and their minutes played (2016 season)

Maya Moore	263
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1) Calculate using the Lynx player's time.

Minimum 8 Maximum 263 Range 255

2) Analyze and interpret what the range says about the player's time on the basketball court.

*There are 255 minutes between the players who have the most and least number of minutes played on the court.*

## Investigation: Minnesota Lynx Data

3) Identify any possible outliers and explain how they could affect the data.

Maya Moore	263
Lindsay Whalen	213
Sylvia Fowles	251
Seimone Augustus	215
Rebekkah Brunson	234
Natasha Howard	104
Jia Perkins	141
Renee Montgomery	101
Keisha Hampton	8
Anna Cruz	51
Janel McCarville	19

4) Calculate using the Lynx player's time.

Min \_\_\_\_\_ Q<sub>1</sub> \_\_\_\_\_ Median \_\_\_\_\_ Q<sub>3</sub> \_\_\_\_\_ Max \_\_\_\_\_ IQR = 183

5) Analyze and interpret what the IQR says about the player's time on the basketball court.

## Investigation: Minnesota Lynx Data

### SOLUTION

3) Identify any possible outliers and explain how they could affect the data.

Maya Moore, Keisha Hampton, and Jane McCarvill could affect the average time playing in the season

4) Calculate using the Lynx player's time.

Min 8  $Q_1$  51 Median 141  $Q_3$  234 Max 263 IQR = 234-51 = 183

$$Q_3 - Q_1 = IQR$$

5) Analyze and interpret what the **IQR** says about the player's time on the basketball court.

50% of the Lynx players have played between 51 minutes and 234 minutes in the season.  
183 minutes of time is represented in this 50%.

Spread



## Investigation: Minnesota Lynx Data

6) Verify that the possible outliers you identified in #3) *are in fact* outliers.

Outlier  $< Q_1 - 1.5IQR$

Outlier  $> Q_3 + 1.5IQR$

$$IQR = 183 \text{ min} \quad Q_3 = 234$$

$$Q_1 = 51$$

$$51 - 274.5$$

-223.5 minutes

Below

$$1.5(183) = 274.5$$

$$234 + 274.5$$

$$= 508.5 \text{ min.}$$

anyone above?

No outliers

## Calculating OUTLIERS

**“1.5IQR above  $Q_3$  or below  $Q_1$ ”**

**IQR(Interquartile Range) =  $Q_3 - Q_1$**

Any point that falls outside the interval  
calculated by

**$Q_1 - 1.5(IQR)$  and  $Q_3 + 1.5(IQR)$**

is considered an outlier.

## Investigation: Minnesota Lynx Data SOLUTION

6) Verify that the possible outliers you identified in #3) *are in fact* outliers.

$$1.5IQR = 1.5(183) = 274.5$$

$$\text{Outlier} < Q_1 - 1.5IQR$$

$$\text{Outlier} > Q_3 + 1.5IQR$$

$$Q_1 - 1.5IQR$$

$$51 - 274.5$$

$$\underline{-223.5}$$

No players below  $-223.5$

$$Q_3 + 1.5IQR$$

$$234 + 274.5$$

$$\underline{508.5}$$

No players above 508.5 minutes

Conclusion

No outliers of time played

## Investigation: Minnesota Lynx Data

### Finding the Standard Deviation

What is a standard deviation?

*Deviation just means how far from the normal*

$\sigma$

### Standard Deviation

The Standard Deviation is a measure of how spread out numbers are.

Its symbol is  $\sigma$  (the greek letter sigma)

## Investigation: Minnesota Lynx Data SOLUTION

### Finding the Standard Deviation

What is a standard deviation?

A measure of how far the data points are  
on average from the mean.  $\sigma$ : Sigma

### Investigation: Minnesota Lynx Data

How can we find the average distance from the mean ("norm")?

#### **Challenge Yourself.**

Find the algorithm for standard deviation by hand.

(average distance from the mean...)

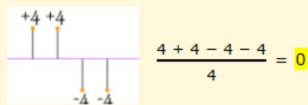
## Challenge Yourself.

### Algorithm for standard deviation.

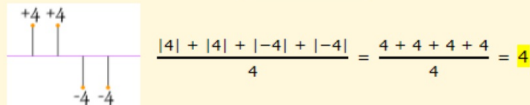
(average distance from the mean...)

#### \*Footnote: Why square the differences?

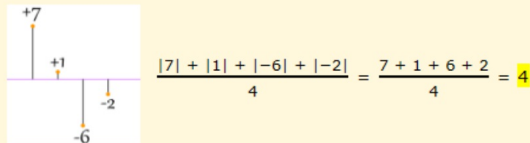
If we just add up the differences from the mean ... the negatives cancel the positives:



So that won't work. How about we use [absolute values](#)?



That looks good (and is the [Mean Deviation](#)), but what about this case:



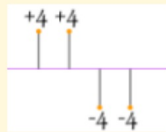
Oh No! It also gives a value of 4, Even though the differences are more spread out.

## Challenge Yourself.

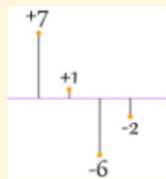
### Algorithm for standard deviation.

(average distance from the mean...)

So let us try squaring each difference (and taking the square root at the end):



$$\sqrt{\left(\frac{4^2 + 4^2 + 4^2 + 4^2}{4}\right)} = \sqrt{\left(\frac{64}{4}\right)} = 4$$



$$\sqrt{\left(\frac{7^2 + 1^2 + 6^2 + 2^2}{4}\right)} = \sqrt{\left(\frac{90}{4}\right)} = 4.74\dots$$

That is nice! The Standard Deviation is bigger when the differences are more spread out ... just what we want.

In fact this method is a similar idea to [distance between points](#), just applied in a different way.

And it is easier to use algebra on squares and square roots than absolute values, which makes the standard deviation easy to use in other areas of mathematics.



## Standard deviation by hand.

(average distance from the mean...)

Test Scores	Mean $\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
72	69	3	9
55	69	-14	196
53		-16	256
51		-18	324
74		5	25
71		2	4
83		14	196
95		26	676
67		-2	4
		Sum	

*Subtract*

$$n = 9$$

$$\bar{x} = 69$$

$$\frac{\text{sum}}{n} =$$

$$\sigma = \sqrt{\frac{\text{sum}}{n}} =$$

Standard deviation =

## Standard deviation by hand. (average distance from the mean...)

Test Scores	Mean $\bar{x}$	$x_i - \bar{x}$ Sum=0	$(x_i - \bar{x})^2$
72	69	3	9
55	69	-14	196
53	69	-16	256
51	69	-18	324
74	69	5	25
71	69	2	4
83	69	14	196
95	69	26	676
67	69	-2	4
		Sum	1690

$n = 9$   
 $\bar{x} = 69$   
 $\frac{\text{sum}}{n} = \frac{1690}{9}$   
 $\sigma = \sqrt{\frac{\text{sum}}{n}} = \sqrt{187.8}$   
 Standard deviation = 13.7

Standard deviation by calculator:

```

x̄=69
Σx=621
Σx²=44539
sx=14.53444185
σx=13.70320319
    
```

## Investigation: Minnesota Lynx Data

7) Calculate using the Lynx player's time. Mean = 145.5

8) Use the steps below to find the standard deviation of the player's time using a calculator. (TI-83/84)

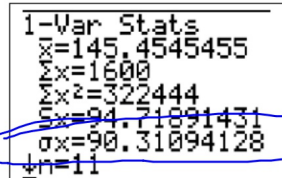
i. **STAT, 1:EDIT**, then enter the data into a list.

ii. **STAT, CALC, 1:1-var stats, ENTER**

You should see a picture like:

Find  $\sigma =$  90.3 min

9) Interpret what this measure of spread means.



```
1-Var Stats
x̄=145.4545455
Σx=1600
Σx²=322444
Sx=94.71091431
σx=90.31094128
n=11
```

## Investigation: Minnesota Lynx Data SOLUTION

7) Calculate using the Lynx player's time. Mean = 145.45 minutes

8) Use the steps below to find the standard deviation of the player's time using a calculator. (TI-83/84)

i. **STAT, 1:EDIT**, then enter the data into a list.

ii. **STAT, CALC, 1:1-var stats, ENTER**

You should see a picture like:

Find  $\sigma =$  90.3 minutes

```
1-Var Stats
x̄=145.4545455
Σx=1600
Σx²=322444
Sx=94.71891431
σx=90.31094128
↓n=11
```

9) Interpret what this measure of spread means.

On average, each player is about 90 minutes from the average time of 145.45 minutes.

## Investigation: Minnesota Lynx Data

10) Suppose an all-star team is compiled of players that are within 1 standard deviation **above** the mean. Which Lynx players would qualify for the all-star team?

$$\bar{X} \approx 145.5 \text{ min} \quad \sigma \approx 90 \text{ min}$$

$$\begin{aligned} \bar{X} + 1\sigma &= 145.5 + 90 \\ &\approx \textcircled{235.5} \end{aligned}$$

Did anyone play above??

## Investigation: Minnesota Lynx Data

### SOLUTION

10) Suppose an all-star team is compiled of players that are within 1 standard deviation **above** the mean. Which Lynx players would qualify for the all-star team?

$$1 \text{ sd} = 90.3 \quad \text{mean} + 1\sigma = 145.45 + 90.3 \approx 235.75$$

Sylvia Fowles and Maya Moore have minutes over 235.75.

## Exercises...

# Analyzing Data Spread Practice Worksheet

Analyzing Data Spread Practice

Name \_\_\_\_\_

1. The lengths in minutes of nine playlists are 45, 63, 74, 69, 72, 53, 72, 73, and 50.  $\sigma \approx 10.6$  minutes

a) Find the 5 # Summary of the data and create a box plot on the line below to show the data spread.



## Exercises...

b) Calculate and interpret the IQR. Use the IQR to verify that there are *no outliers*.

Outlier  $< Q_1 - 1.5IQR$

Outlier  $> Q_3 + 1.5IQR$

c) Suppose playlists that are 1 standard deviation above the mean require too much memory on our phones. Which playlist(s) would be too *big* for our phone?



## Exercises...

2. Apply the meaning of standard deviation.

**APPLICATION** The mean diameter of a Purdy Goode Compact Disc is 12.0 cm, with a standard deviation of 0.012 cm. No CDs can be shipped that are more than one standard deviation from the mean. How would the company's quality control engineer use those statistics?

## Exercises...

3. The students in four classes recorded their resting pulse rates in beats per minutes. The class means and standard deviations are given in the table.

**Resting Pulse Rates (beats/min)**

Class	Mean	Standard deviation
First period	79.4	3.2
Third period	74.6	5.6
Fifth period	78.2	4.1
Sixth period	80.2	7.6

a) Which class has students with pulse rates most alike?  
How can you tell?

b) Which class has the students with the fastest pulse rates? How could you show this information?

c) Which class had the biggest range of pulse rates? How can you tell?

## Exercises...

4. Thirty students were given an English vocabulary test out of 100 points. The number of questions the students **got wrong** are listed below.

{5, 2, 3, 45, 13, 12, 10, 4, 4, 5, 6, 7, 9, 3, 0, 2, 11, 23, 29, 11, 23, 22, 23, 24, 25, 6, 17, 18, 9, 2}

a) Find the 5 # Summary and IQR of the data.

b) Calculate and verify whether there are **any outliers**.  
Outlier  $< Q_1 - 1.5IQR$   
Outlier  $> Q_3 + 1.5IQR$

c) Students who were upper outliers received an award. How many students won an award?

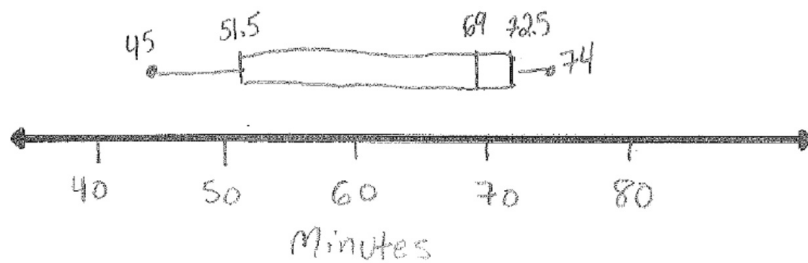
## Exercise Solutions...

1. The lengths in minutes of nine playlists are 45, 63, 74, 69, 72, 53, 72, 73, and 50.  $\sigma \approx 10.6$  minutes

a) Find the 5 # Summary of the data and create a box plot on the line below to show the data spread.

$\{45, 51.5, 69, 72.5, 74\}$

### Playlist of Music



## Exercise Solutions...

b) Calculate and interpret the IQR. Use the IQR to verify that there are **no outliers**.  $IQR = Q_3 - Q_1$

Outlier  $< Q_1 - 1.5IQR$   $1.5IQR = 1.5(21) = 31.5$   $IQR = 72.5 - 51.5$

Outlier  $> Q_3 + 1.5IQR$   $IQR = 21 \text{ min}$

$Q_1 - 31.5$   $Q_3 + 31.5$

$51.5 - 31.5 = 20 \text{ min}$   $72 + 31.5 = 103.5 \text{ min}$

No playlist under 20 min } No playlist above 103.5 min

c) Suppose playlists that are 1 standard deviation **above** the mean require too much memory on our phones. Which playlist(s) would be too **big** for our phone?

$$1\sigma \approx 10.6 \text{ min}$$

$$1\sigma + \bar{x} \approx 10.6 + 63 \approx 73.6 \text{ minutes}$$

$$\bar{x} \approx 63.4 \text{ min}$$

The 74 min playlist is slightly too big for the available memory.

2. Apply the meaning of standard deviation

**Note:** Answer could be different for **c**, depending how you rounded the mean.

## Exercise Solutions...

2. Apply the meaning of standard deviation.

**APPLICATION** The mean diameter of a Purdy Goode Compact Disc is 12.0 cm, with a standard deviation of 0.012 cm. No CDs can be shipped that are more than one standard deviation from the mean. How would the company's quality control engineer use those statistics?

$1\sigma + \bar{X} \approx .012 + 12 \approx 12.012 \text{ cm}$   
The control engineer should NOT ship CD's with diameters larger than 12.012 cm. The engineer should make sure all packaged CD's diameter's are measured.

# Exercise Solutions...

3. The students in four classes recorded their resting pulse rates in beats per minutes. The class means and standard deviations are given in the table.

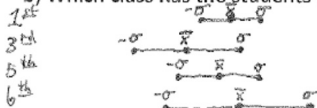
Resting Pulse Rates (beats/min)

Class	Mean	Standard deviation
First period	79.4	3.2
Third period	74.6	5.6
Fifth period	78.2	4.1
Sixth period	80.2	7.6

a) Which class has students with pulse rates most alike?  
How can you tell?

First period! Their SD is the smallest, so they have less deviation from the mean beats/min.

b) Which class has the students with the fastest pulse rates? How could you show this information?



Periods 3 and 5 would NOT have the fastest pulse rates.

- Period 6 could have the fastest pulse rates because their mean is the highest and their positive deviation ↑.
- Period 1 could have the fastest rates, because a student chosen at random will consistently have a high rate. ↓ deviation

c) Which class had the biggest range of pulse rates? How can you tell?

6<sup>th</sup> period, On average, each student has the highest difference from their average pulse rate. Each student is about 7.6 beats/min above or below 80 beats/min.

## Exercise Solutions...

4. Thirty students were given an English vocabulary test out of 100 points. The number of questions the students **got wrong** are listed below.  $\bar{x} = 12.933$   $\sigma = 10.266864$

{5, 2, 3, 45, 13, 12, 10, 4, 4, 5, 6, 7, 9, 3, 0, 2, 11, 23, 29, 11, 23, 22, 23, 24, 25, 6, 17, 18, 9, 2}

a) Find the 5 # Summary and IQR of the data.

$$\begin{array}{l} \text{Min} = 0 \\ Q_1 = 4 \\ \text{Median} = 9.5 \\ Q_3 = 22 \\ \text{Max} = 45 \end{array} \quad ] \text{IQR} = 18$$

$$4 - 1.5(18) = 4 - 27 = -23$$

Lower  
Outlier  
Boundary

b) Calculate and verify whether there are **any outliers**.

$$\text{Outlier} < Q_1 - 1.5\text{IQR}$$

$$\text{Outlier} > Q_3 + 1.5\text{IQR}$$

Although 45 appears much higher than the rest of the data, it does NOT fall outside the upper outlier boundary (as defined  $Q_3 + 1.5\text{IQR}$ ).

$$22 + 1.5(18) = 22 + 27 = 49$$

Upper  
Outlier  
Boundary

There are no lower or upper outliers in the data set.

c) Students who were **upper** outliers received an award. How many students won an award?

None. No students received the award. Additionally, this doesn't make sense in the context of questions answered **WRONG**. An award should be given to students who got a lower number of questions wrong. I would suggest giving awards to students in the lowest quartile or the 25% of students getting the least wrong.