Welcome!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 7 Topic: Hidden Figures	0 1 2	
Tuesday Date: ______ Topic: _____________	0 1 2	
Wednesday Date: ______ Topic: _____________	0 1 2	
Thursday Date: ______ Topic: _____________	0 1 2	
Friday Date: ______ Topic: _____________	0 1 2	

Warm-up:

P(prime) = 5/8

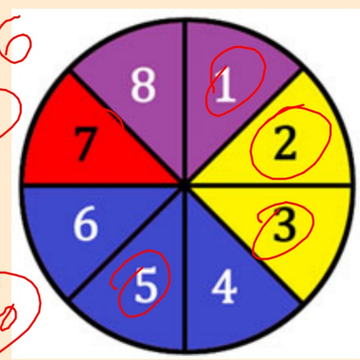
Carlos takes one turn spinning the spinner.

What are the chances he will land on a multiple of 3?

multiples of 3 = 3, 6

8 spaces

$P(\text{mult } 3) = \frac{2}{8} = \frac{1}{4} = 0.25 = 25\%$



Done...odd number? $\frac{4}{8} = \frac{1}{2}$

Class Plan:

1. Warm-up
2. What is Probability?
3. Dice Game

Experimental vs. Theoretical Probability (Ch. 14)

A EXPERIMENTAL PROBABILITY

D THEORETICAL PROBABILITY

4. Practice

Chapter 14

Probability

Contents:

A Experimental probability
B Probabilities from tabled data
C Sample space
D Theoretical probability
E Using 2-dimensional grids
F Compound events
G Using tree diagrams
H Sampling with and without replacement
I Probabilities from Venn diagrams
J Expectation

What is probability?

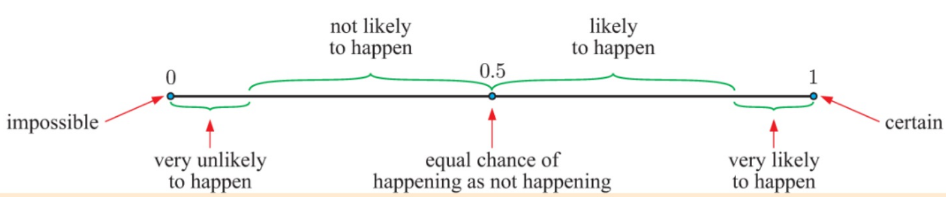
The study of **probability** deals with the **chance** or **likelihood** of an event happening. For every event we can assign a number which lies between 0 and 1 inclusive.

An **impossible** event has 0% chance of happening, and is assigned the probability 0.

A **certain** event has 100% chance of happening, and is assigned the probability 1.

All other events between these two extremes can be assigned a probability between 0 and 1.

The number line below shows how we could interpret different probabilities:



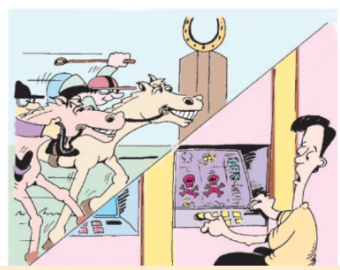
Experimental & Theoretical Probability

To each event we assign either:

- an **experimental probability** by observing the results of an experiment, or
- a **theoretical probability** by using arguments of symmetry.

The study of chance has important applications in physical and biological sciences, economics, politics, sport, life insurance, quality control, production planning, and many other areas.

Probability theory can be applied to card and dice games to try to increase our chances of success. It may therefore appear that an understanding of probability encourages gambling. However, a better knowledge of probability theory actually helps us to understand why the majority of habitual gamblers lose in the long term.



Historical Note

HISTORICAL NOTE

Chevalier de Méré was a French aristocrat and gambler in the 17th century. He wanted to know the answer to this question:

"Should I bet even money on the occurrence of at least one 'double six' when rolling a pair of dice 25 times?"

De Méré's experience of playing dice games convinced him that the answer was yes, but he did not know how to prove it. He therefore asked his friend, the French mathematician **Blaise Pascal**, for help.

In a series of letters between Pascal and fellow mathematician **Pierre de Fermat**, the problem was solved. In the process, they became interested in solving other questions of this kind, and together they laid the foundations of a new branch of mathematics called **theoretical probability**.

Pierre de Fermat

Dice Game:

Dice Game

Directions: Place chips on your board, and remove one when that sum is rolled. The goal is to clear your game board.

1. Place 12 beans on your gameboard
2. Remove **1** bean when the sum is rolled
3. GOA

Record the

1	2	3	4	5	6	7	8	9	10	11	12
	1	2	4	1	4	7	7	6	1	1	2

11	12
1	2

<u>Experimental Probability</u> 1st hour

Based on our experiment: Rolls : 40

P (Sum of 1) = $\frac{0}{40}$

P (Sum of 2) = $\frac{2}{40} = \frac{1}{20}$

P (Sum of 3) = $\frac{0}{40}$

P (Sum of 4) = $\frac{3}{40}$

P (Sum of 5) = $\frac{5}{40} = \frac{1}{8}$

P (Sum of 6) = $\frac{5}{40} = \frac{1}{8}$

P (Sum of 7) = $\frac{8}{40} = \frac{1}{5}$

P (Sum of 8) = $\frac{6}{40} = \frac{3}{20}$

P (Sum of 9) = $\frac{8}{40} = \frac{1}{5}$

P (Sum of 10) = $\frac{3}{40}$

P (Sum of 11) = $\frac{0}{40}$

P (Sum of 12) = $\frac{0}{40}$

Experimental P

2nd hour

1	2	3	4	5	6	7	8	9	10	11	12
	1	2	4	1	4	7	7	6	1	1	2

Based on our experiment: Rolls = 36

P (Sum of 1) = $\frac{0}{36}$

P (Sum of 2) = $\frac{1}{36}$

P (Sum of 3) = $\frac{2}{36}$

P (Sum of 4) = $\frac{4}{36}$

P (Sum of 5) = $\frac{1}{36}$

P (Sum of 6) = $\frac{4}{36}$

Sum = $\frac{36}{36}$ = 1 = $\boxed{100\%}$

P (Sum of 7) = $\frac{7}{36}$

P (Sum of 8) = $\frac{7}{36}$

P (Sum of 9) = $\frac{6}{36}$

P (Sum of 10) = $\frac{1}{36}$

P (Sum of 11) = $\frac{1}{36}$

P (Sum of 12) = $\frac{2}{36}$

Experimental Probability 4th hour

Based on our experiment:

P (Sum of 1) =

P (Sum of 2) =

P (Sum of 3) =

P (Sum of 4) =

P (Sum of 5) =

P (Sum of 6) =

P (Sum of 7) =

P (Sum of 8) =

P (Sum of 9) =

P (Sum of 10) =

P (Sum of 11) =

P (Sum of 12) =

Experimental Probability 5th hour

Based on our experiment:

P (Sum of 1) =

P (Sum of 2) =

P (Sum of 3) =

P (Sum of 4) =

P (Sum of 5) =

P (Sum of 6) =

P (Sum of 7) =

P (Sum of 8) =

P (Sum of 9) =

P (Sum of 10) =

P (Sum of 11) =

P (Sum of 12) =

Experimental Probability 7th hour

Based on our experiment:

P (Sum of 1) =

P (Sum of 2) =

P (Sum of 3) =

P (Sum of 4) =

P (Sum of 5) =

P (Sum of 6) =

P (Sum of 7) =

P (Sum of 8) =

P (Sum of 9) =

P (Sum of 10) =

P (Sum of 11) =

P (Sum of 12) =

What is strategy for this game? Why? Explain in the space below.

- more toward center (7)
- No sum of 1 ...

(Normal Distribution)

What is strategy for this game? Why? Explain in the space below.

- Sum of 1 is impossible to happen
- 2 and 12 occur rarely because there is only 1 possible outcome to produce these sums.
- 7 occurs most frequently as there are more possible outcomes that produce this sum (1+6, 2+5, 3+4)

Notation

P(sum of 7)

"the probability of the sum of 7 occurring"

Did you expect these results? Why or why not?

KINDA!

· more 7, Less 12

<u>Example:</u> Experimental Probability

Examples: I am going to continue rolling the two dice 100 times!
a) Using our experiment results, how many times should I roll a sum of 7?

$$\frac{7}{36} = \frac{x}{100} \text{ (sum of 7)}$$

$$36x = 7(100)$$

$$\frac{36x}{36} = \frac{700}{36}$$

$x \approx 19.4$ times

Possible Outcomes

$\boxed{6} \cdot \boxed{6}$

= 36 possible ways to sum 2 dice

Based on what *should* happen (mathematical theory):

Show all possible sums in the table below.

Blue

Orange

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	
4	5	6	7	8		
5	6	7	8			
6	7	8				

Possible Outcomes

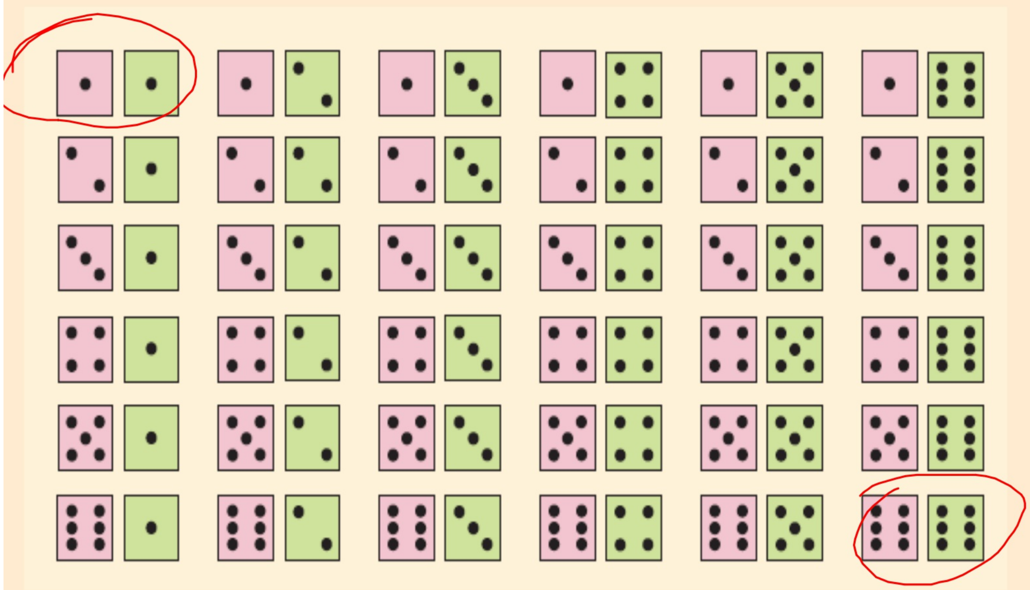
Based on what *should* happen (mathematical theory):

$\frac{P}{6} \cdot \frac{G}{6} = 36$

Show all possible sums in the table below.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Possible Outcomes



Theoretical Probability

P (Sum of 1) = 0/36

P (Sum of 2) = 1/36

P (Sum of 3) = 2/36

P (Sum of 4) = 3/36

P (Sum of 5) = 4/36

P (Sum of 6) = 5/36

P (Sum of 7) = 6/36

P (Sum of 8) = 5/36

P (Sum of 9) = 4/36

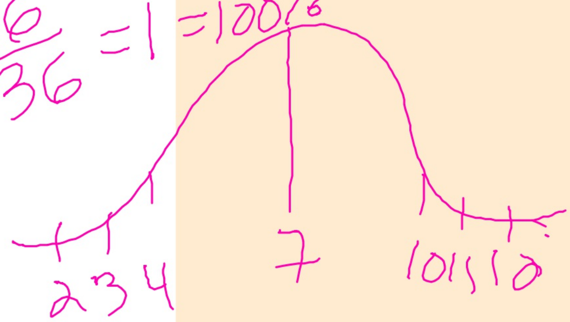
P (Sum of 10) = 3/36

P (Sum of 11) = 2/36

P (Sum of 12) = 1/36

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$\frac{36}{36} = 1 = 100\%$



Theoretical Probability

P (Sum of 1) = 0

P (Sum of 2) = $\boxed{\frac{1}{36}}$

P (Sum of 3) = $2/36 = \boxed{\frac{1}{18}}$

P (Sum of 4) = $3/36 = \boxed{\frac{1}{12}}$

P (Sum of 5) = $4/36 = \boxed{\frac{1}{9}}$

P (Sum of 6) = $\boxed{\frac{5}{36}}$

P (Sum of 7) = $6/36 = \boxed{\frac{1}{6}}$

P (Sum of 8) = $\boxed{\frac{5}{36}}$

P (Sum of 9) = $4/36 = \boxed{\frac{1}{9}}$

P (Sum of 10) = $3/36 = \boxed{\frac{1}{12}}$

P (Sum of 11) = $2/36 = \boxed{\frac{1}{18}}$

P (Sum of 12) = $\boxed{\frac{1}{36}}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

PINK GREEN

6 outcomes · 6 outcomes

= 36 sums

Example: Theoretical Probability

Examples: I am going to continue rolling the two dice 100 times!
b) Using theoretical probability, how many times should I roll a sum of 7?

$$\frac{6}{36} = \frac{x}{100}$$

$$\frac{36x}{36} = \frac{600}{36}$$

$$\boxed{x \approx 16.67}$$

times a sum of 7 is rolled

A EXPERIMENTAL PROBABILITY

The **experimental probability** is the **relative frequency** of the outcome.

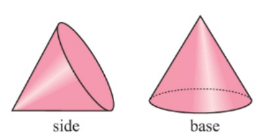
In experiments involving chance, we use the following terms to describe what we are doing and the results we are obtaining.

- The **number of trials** is the total number of times the experiment is repeated.
- The **outcomes** are the different results possible for one trial of the experiment.
- The **frequency** of a particular outcome is the number of times that this outcome is observed.
- The **relative frequency** of an outcome is the frequency of that outcome expressed as a fraction or percentage of the total number of trials.

$$\textbf{relative frequency} = \frac{\textbf{frequency}}{\textbf{number of trials}}$$

For example, suppose a small plastic cone was tossed into the air 300 times. It fell on its *side* 203 times and on its *base* 97 times. We say that:

- the number of trials is 300
- the possible outcomes are *side* and *base*
- the frequency of *side* is 203, and the frequency of *base* is 97
- the relative frequency of *side* $= \frac{203}{300} \approx 0.677$
- the relative frequency of *base* $= \frac{97}{300} \approx 0.323$.



D THEORETICAL PROBABILITY

The sample space when rolling a single die is $\{1, 2, 3, 4, 5, 6\}$.

Since the die is a cube and therefore symmetrical, we expect that each of the six outcomes will be **equally likely** to occur. We say that the **theoretical probability** of any given outcome occurring is 1 in 6, or $\frac{1}{6}$.

For example, $\text{P(rolling a 2)} = \frac{1}{6}$.

If a sample space has n outcomes which are **equally likely** to occur when the experiment is performed once, then each outcome has probability $\frac{1}{n}$ of occurring.

EVENTS

An **event** occurs when we obtain an outcome with a particular property or feature.

Consider the event of *rolling a prime number* with an ordinary die. Of the 6 possible outcomes, the three outcomes 2, 3, and 5 all correspond to this event. So, the probability of rolling a prime number is 3 in 6, or $\frac{3}{6}$.

When the outcomes of an experiment are equally likely, the probability of an event E occurring is given by:

$$\mathrm{P}(E) = \frac{\textbf{number of outcomes corresponding to } \boldsymbol{E}}{\textbf{number of outcomes in the sample space}}$$

C SAMPLE SPACE

The **sample space** of an experiment is the set of its possible outcomes.

We can represent sample spaces in a number of ways, including:

- lists
- grids
- tree diagrams.

Exercises...

Experimental Probabilities: Based on the results experiment (frequencies)

Theoretical Probabilities: Based on what we expect (equally expected outcomes)

$$\text{relative frequency} = \frac{\text{frequency}}{\text{number of trials}}$$

$$P(E) = \frac{\text{number of outcomes corresponding to } E}{\text{number of outcomes in the sample space}}$$

1. Something to consider... When would experimental probabilities and theoretical probabilities become more similar?

- More rolls
- Bigger sample space

Exercises...

Experimental Probabilities: Based on the results experiments. (relative frequency of occurrences).

Theoretical Probabilities: Based on mathematical theory – equally likely outcomes.

$$\text{relative frequency} = \frac{\text{frequency}}{\text{number of trials}}$$

$$P(E) = \frac{\text{number of outcomes corresponding to } E}{\text{number of outcomes in the sample space}}$$

1. *Something to consider... When would experimental probabilities and theoretical probabilities become more similar?*

- more trials
- bigger sample size

Exercises...

2. Identify if the following situations represent Experimental or Theoretical Probability.

a. The chance of flipping a heads is ½ ______________________________

b. Bryn flipped a coin 8 times and landed on heads 6 times, so the probability of heads is ¾ (6/8 reduced).

____

c. Probability is 1/12 that the month you were born in is May ______________________________

d. Nathaniel got 3 "hits" in the last 7 times at bat, so his probability of a hit is 3/7

____

Exercises...

3. If you rolled two dice 50 times, theoretically, how many times should you roll the following sums?

Show all solving.

a. P(sum of 5)	b. P(sum of 2)	c. P(sum of 10)	d. P(sum of 12)

Exercises...

Find each probability. Write answers as fractions.

4. A bowl contains 3 red chips, 6 green chips, 5 yellow chips, and 8 orange chips. A chip is drawn randomly.

a. P(green) = ________ b. P(yellow) = ________ c. P(white) = ________

d. P(not yellow) = ________ e. P(yellow or orange) = ________ f. P(red or green) = ________

5. You roll a die with six sides.

a.P(4) = ________ b. P(odd number) = ________ c. P(even number) = ________

d. P(2 or 3) = ________ e. P(number greater than 6)= ________ f. P(prime number) = ________

Exercises...

2 José recorded the length of TV commercials in seconds. His results are summarised in the table.
Estimate, to 3 decimal places, the probability that a randomly chosen TV commercial will last:

a between 20 and 40 seconds
b at least a minute
c between 20 seconds and a minute.

Length	*Frequency*
$0 \leqslant t < 20$	17
$20 \leqslant t < 40$	38
$40 \leqslant t < 60$	19
$t \geqslant 60$	4

3

Hours slept	*Frequency*
$5 \leqslant h < 6$	7
$6 \leqslant h < 7$	29
$7 \leqslant h < 8$	46
$8 \leqslant h < 9$	39

This table shows how long Nathan has slept each night recently.
Estimate the probability that tonight he will sleep for:

a between 6 and 7 hours
b at least 7 hours
c between 5 and 8 hours.

Exercise Solutions...

2. Identify if the following situations represent Experimental or Theoretical Probability.

a. The chance of flipping a heads is ½ Theoretical

b. Bryn flipped a coin 8 times and landed on heads 6 times, so the probability of heads is ¾ (6/8 reduced).
experimental

c. Probability is 1/12 that the month you were born in is May theoretical

d. Nathaniel got 3 "hits" in the last 7 times at bat, so his probability of a hit is 3/7
experimental

Exercise Solutions...

3. If you rolled two dice 50 times, theoretically, how many times should you roll the following sums?

Show all solving.

a. P(sum of 5)	b. P(sum of 2)	c. P(sum of 10)	d. P(sum of 12)
$P(2+3, 1+4, 4+1, 3+2)$ $= \frac{4}{36}$ $\frac{4}{36} = \frac{x}{50}$ About 5.6 times	$P(1+1) = \frac{1}{36}$ $\frac{1}{36} = \frac{x}{50}$ $x \approx 1.39$ times	$P(5+5, 6+4, 4+6)$ $= \frac{3}{36}$ $\frac{3}{36} = \frac{x}{50}$ $36x = 150$ $x \approx 4.2$ times	$P(6+6) = \frac{1}{36}$ $\frac{1}{36} = \frac{x}{50}$ $x \approx 1.39$ times

Exercise Solutions...

Find each probability. Write answers as fractions.

4. A bowl contains 3 red chips, 6 green chips, 5 yellow chips, and 8 orange chips. A chip is drawn randomly. (22 total Chips)

a. P(green) = $\frac{6}{22} = \frac{3}{11}$ b. P(yellow) = $\frac{5}{22}$ c. P(white) = $\frac{0}{22}$

d. P(not yellow) = $\frac{17}{22}$ e. P(yellow or orange) = $\frac{13}{22}$ f. P(red or green) = $\frac{9}{22}$

5. You roll a die with six sides.

a. P(4) = $\frac{1}{6}$ b. P(odd number) = P(1,3,5) $\frac{3}{6} = \frac{1}{2}$ c. P(even number) = P(2,4,6) $\frac{3}{6} = \frac{1}{2}$

d. P(2 or 3) = $\frac{2}{6} = \frac{1}{3}$ e. P(number greater than 6)= $\frac{0}{6}$ f. P(prime number) = P(1,3,5) $\frac{3}{6} = \frac{1}{2}$

Exercise Solutions...

2 José recorded the length of TV commercials in seconds. His results are summarised in the table.

Estimate, to 3 decimal places, the probability that a randomly chosen TV commercial will last:

a between 20 and 40 seconds $38/78 \approx 0.487$

b at least a minute $4/78 \approx 0.051$

c between 20 seconds and a minute. $\frac{38+19}{78} = \frac{57}{78} \approx 0.731$

Length	*Frequency*
$0 \leqslant t < 20$	17
$20 \leqslant t < 40$	38
$40 \leqslant t < 60$	19
$t \geqslant 60$	4
Total	78

3 This table shows how long Nathan has slept each night recently. Estimate the probability that tonight he will sleep for:

Hours slept	*Frequency*
$5 \leqslant h < 6$	7
$6 \leqslant h < 7$	29
$7 \leqslant h < 8$	46
$8 \leqslant h < 9$	39
Total nights	121

a between 6 and 7 hours $\frac{29}{121} \approx 0.24$

b at least 7 hours

c between 5 and 8 hours.

b $\frac{46+39}{121} = \frac{85}{121} \approx 0.70$

c $\frac{7+29+46}{121} = \frac{82}{121} \approx 0.68$