Welcome!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 7 Topic: Hidden Figures	0 1 2	
Tuesday Date: 5 - 8 Topic: Experimental vs. Theoretical Probability	0 1 2	
Wednesday Date: ______ Topic: ____________	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

1. Warm-up
2. Chapter 14 **TREE DIAGRAMS**

C **SAMPLE SPACE**

3. Investigation:

Will you get a prize in your cereal box?

Joke Break!

4. Practice

Warm-up:

$\frac{3}{E} \cdot \frac{2}{V} \cdot \frac{2}{D} = 12$

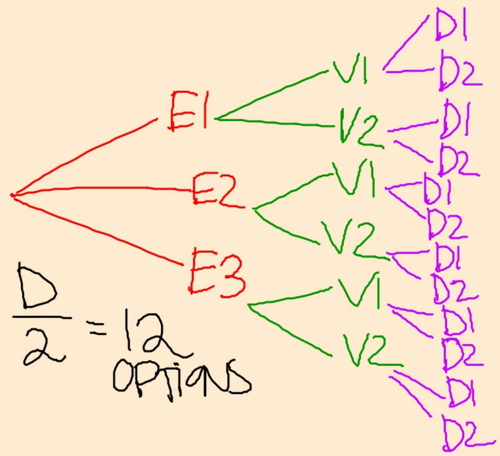
Create a tree diagram showing the different outcomes if the cafeteria has three main entree choices, two vegetable choices, and two dessert choices.

E1 V1 D1
E1 V2 D1
E1 V1 D2

How else could we visualize these meal options?

The cafeteria has three main entree choices, two vegetable choices, and two dessert choices.

How else could we visualize these meal options?



$$\frac{E}{3} \cdot \frac{V}{2} \cdot \frac{D}{2} = 12$$

OPTIONS

LIST

E1V1D1
E1V1D2
E1V2D1
E1V2D2

E2V1D1
E2V1D2
E2V2D1
E2V2D2

E3V1D1
E3V1D2
E3V2D1
E3V2D2

C SAMPLE SPACE

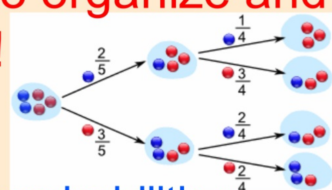
The **sample space** of an experiment is the set of its possible outcomes.

We can represent sample spaces in a number of ways, including:

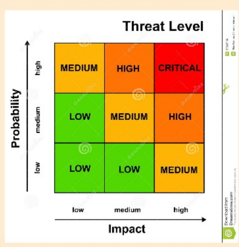
- lists
- grids
- tree diagrams.
- Venn Diagrams

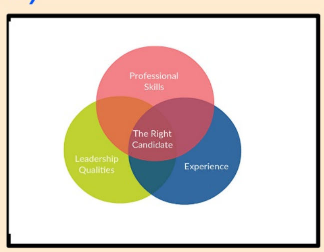
Caitlin	Dave
Aero	Aero
Aero	Bounty
Aero	Crunchie
Aero	Dime
Bounty	Aero
Bounty	Bounty
Bounty	Crunchie
Bounty	Dime
Crunchie	Aero
Crunchie	Bounty
Crunchie	Crunchie
Crunchie	Dime
Dime	Aero
Dime	Bounty
Dime	Crunchie
Dime	Dime

Trees - Method to organize and count outcomes!



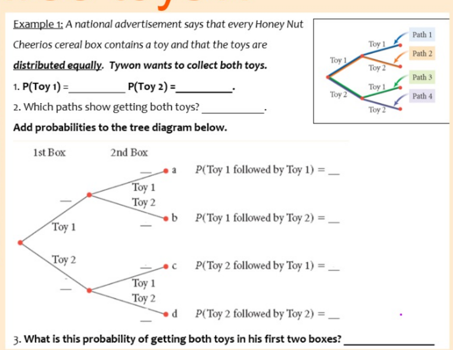
(Very useful when probabilities of branches are different)





Investigation: Cereal Box Toys

If I buy 3 boxes...what are the chances I'll get all 3 different free toys?!



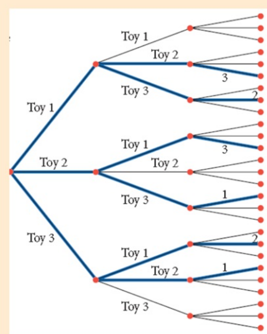
Example 1: *A national advertisement says that every Honey Nut Cheerios cereal box contains a toy and that the toys are* ***distributed equally****.* ***Tywon wants to collect both toys.***

1. P(Toy 1) =__________ P(Toy 2) =__________.

2. Which paths show getting both toys? ___________.

Add probabilities to the tree diagram below.

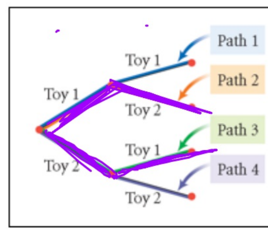
3. What is this probability of getting both toys in his first two boxes? ______________



Done? Help others -
We will come back together.

Investigation: Cereal Box Toys

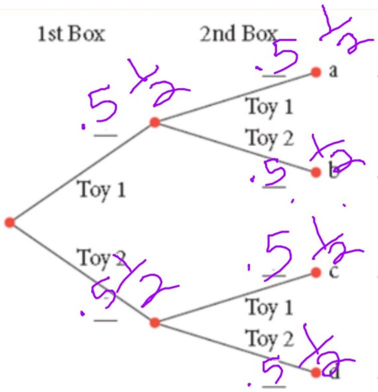
Example 1: *A national advertisement says that every Honey Nut Cheerios cereal box contains a toy and that the toys are* ***distributed equally. Tywon wants to collect both toys.***



1. **P(Toy 1) =** 1/2 **P(Toy 2) =** .1/2.

2. Which paths at right show getting both toys? 2, 3.

Add probabilities to the tree diagram below.



P(Toy 1 followed by Toy 1) = 1/4

P(Toy 1 followed by Toy 2) = 1/4

P(Toy 2 followed by Toy 1) = 1/4

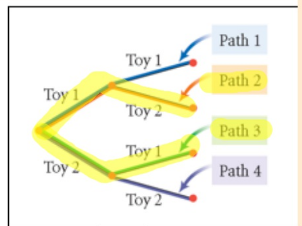
P(Toy 2 followed by Toy 2) = 1/4

sum to
1 = 100%

$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

3. **What is this probability of getting both toys in his first two boxes?** ______

Example 1: *A national advertisement says that every Honey Nut Cheerios cereal box contains a toy and that the toys are* ***distributed equally*****. *Tywon wants to collect both toys.***

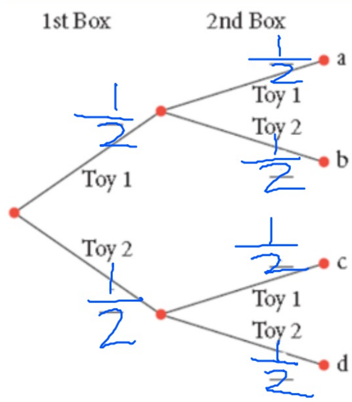


1. **P(Toy 1)** = $\frac{1}{2}$ **P(Toy 2) =** $\frac{1}{2}$.

2. Which paths show getting both toys? Paths 2 & 3

Add probabilities to the tree diagram below.

MULTIPLICATION RULE



$P(\text{Toy 1 followed by Toy 1}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(\text{Toy 1 followed by Toy 2}) = \frac{1}{4}$

$P(\text{Toy 2 followed by Toy 1}) = \frac{1}{4}$

$P(\text{Toy 2 followed by Toy 2}) = \frac{1}{4}$

3. **What is this probability of getting both toys in his first two boxes?** $P_2 + P_3 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Probability of a Path

Independent Events

Probability of a Path
(The Multiplication Rule for Independent Events)

If n_1, n_2, n_3, and so, on represent events along a path, then the probability that this sequence of events will occur can be found by multiplying the probabilities of the events.

$$P(n_1 \text{ and } n_2 \text{ and } n_3 \text{ and } \ldots) = P(n_1) \cdot P(n_2) \cdot P(n_3) \cdot \cdots$$

Investigation:

$$\frac{3}{\text{Box 1}}\cdot\frac{3}{B2}\cdot\frac{3}{B3}$$

Suppose there are now 3 toys Tywon would like for his collection.

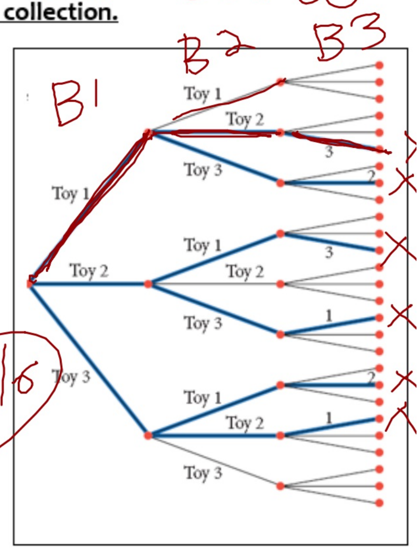
a. How many outcomes (paths) are there? 27

b. How many paths allow Tywon to get toy 1? 19

c. How many paths allow Tywon to get toy 2? 19

d. How many paths allow Tywon to get toy 3? 19

e. **P(T 1)** = $\frac{19}{27}$ **P(T 2)** = $\frac{19}{27}$ **P(T 3)** = $\frac{19}{27}$



4. **What is this probability of getting all three toys in his first three boxes?**

$$\frac{6}{27} = \frac{2}{9}$$

22.2%

$\underline{1^{st}}$ $\underline{2^{nd}}$ $\underline{3^{rd}}$

$3 \cdot 3 \cdot 3 = 27$

Suppose there are now 3 toys Tywon would like for his collection.

a. How many outcomes (paths) are there? 27

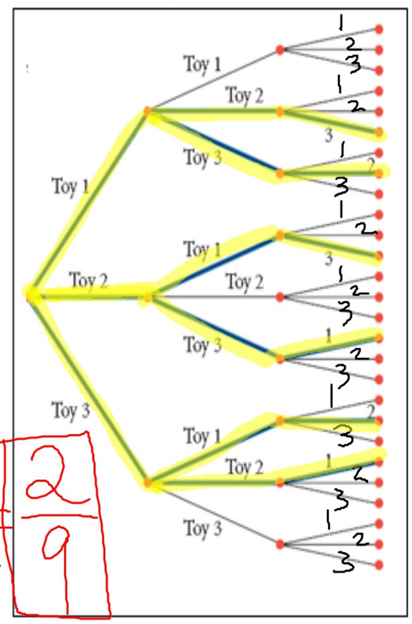
b. How many paths allow Tywon to get toy 1? 19

$9 + 5 + 5$

c. How many paths allow Tywon to get toy 2? 19

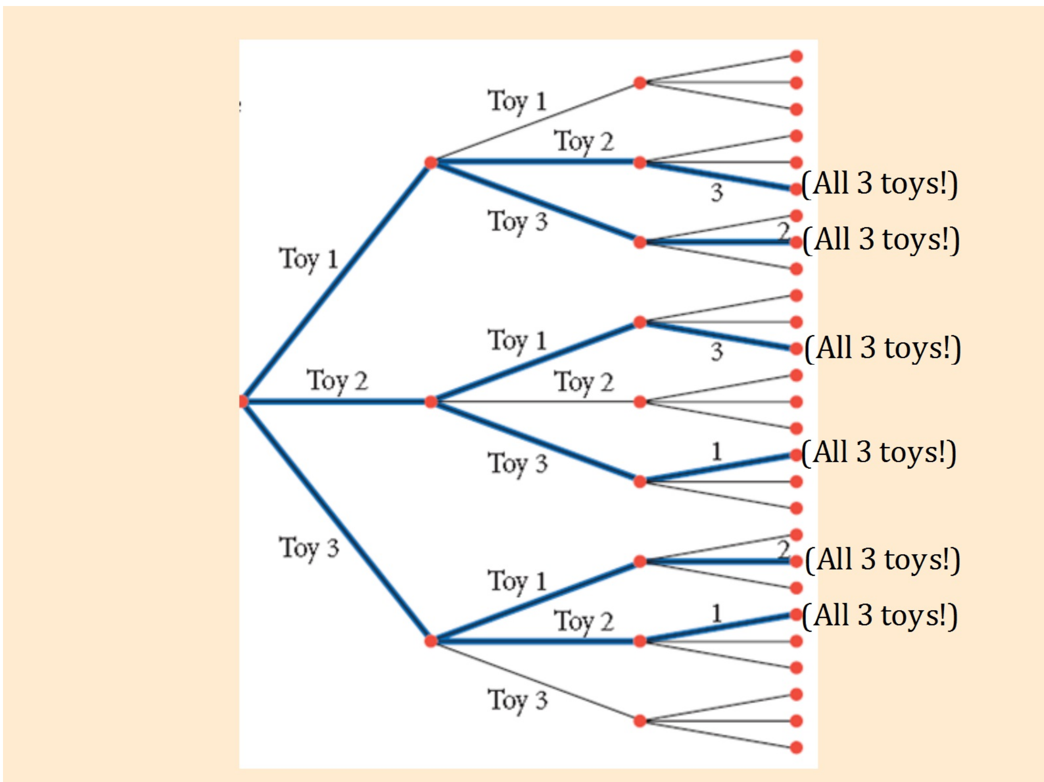
d. How many paths allow Tywon to get toy 3? 19

e. **P(T 1)** = $\frac{19}{27}$ **P(T 2)** = $\frac{19}{27}$ **P(T 3)** = $\frac{19}{27}$



4. **What is this probability of getting all three toys in his first three boxes?**

P(ALL 3 TOYS IN 1st 3 BOXES) $= \frac{6}{27} = \boxed{\frac{2}{9}}$



Toy 1
Toy 2
Toy 3
Toy 1
Toy 2
Toy 3
3
(All 3 toys!)
2
(All 3 toys!)
Toy 1
Toy 2
Toy 3
3
(All 3 toys!)
1
(All 3 toys!)
Toy 1
Toy 2
Toy 3
2
(All 3 toys!)
1
(All 3 toys!)

Investigation: Cereal Box Toys

If I buy 3 boxes...what are the chances I'll get all 3 different free toys?!

(You may miss an outcome using a list... and you NEED EQUAL OUTCOMES)

111	211	311
112	212	312
113	213	313
121	221	321
122	222	322
123	223	323
131	231	331
132	232	332
133	233	333

3 * 3 * 3 =

27 outcomes

6/27 chance

G USING TREE DIAGRAMS

Example 11 — Self Tutor

Bag A contains 4 red jelly beans and 1 yellow jelly bean. Bag B contains 2 red and 3 yellow jelly beans. A bag is randomly selected by tossing a coin, and one jelly bean is removed from it. Determine the probability that it is yellow.

1^{st}: Choose Bag

2^{nd}: Choose beans

A	B
$Y=\frac{1}{5}$	$Y=\frac{3}{5}$
$R=\frac{4}{5}$	$R=\frac{2}{5}$

$\frac{1}{2}$ A: $\frac{4}{5}$ R $=\frac{4}{10}$; $\frac{1}{5}$ Y $=\frac{1}{10}$

$\frac{1}{2}$ B: $\frac{2}{5}$ R $=\frac{2}{10}$; $\frac{3}{5}$ Y $=\frac{3}{10}$

$\frac{1}{10}+\frac{3}{10}=\frac{4}{10}=\frac{2}{5}$

40%

Bag A

Bag B

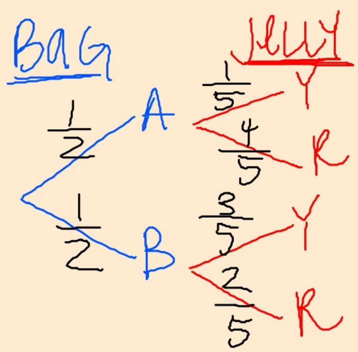
G USING TREE DIAGRAMS

Example 11 Self Tutor

Bag A contains 4 red jelly beans and 1 yellow jelly bean. Bag B contains 2 red and 3 yellow jelly beans. A bag is randomly selected by tossing a coin, and one jelly bean is removed from it. Determine the probability that it is yellow.

$P(\text{Yellow}) = \frac{1}{10} + \frac{3}{10} = \frac{4}{10} = \frac{2}{5}$

1. CHOOSE BAG
2. CHOOSE JELLY

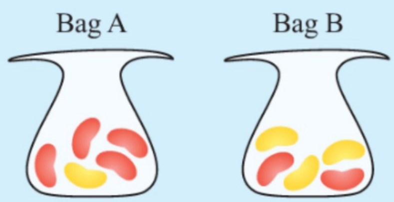


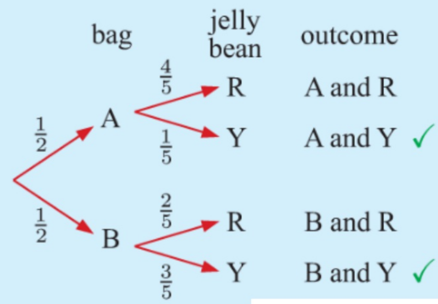
$P(A \,\&\, Y) = \left(\frac{1}{2}\right)\left(\frac{1}{5}\right) = \frac{1}{10}$ *

$P(A \,\&\, R) = \left(\frac{1}{2}\right)\left(\frac{4}{5}\right) = \frac{4}{10} = \frac{2}{5}$

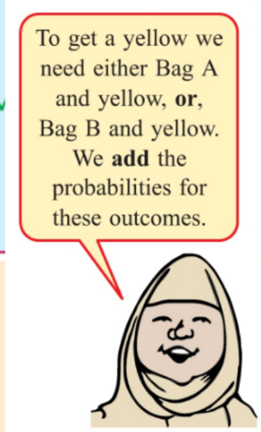
$P(B \,\&\, Y) = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{3}{10}$ *

$P(B \,\&\, R) = \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) = \frac{2}{10} = \frac{1}{5}$





$$\begin{aligned} P(\text{yellow}) &= P(\text{A and Y}) + P(\text{B and Y}) \\ &= \tfrac{1}{2} \times \tfrac{1}{5} + \tfrac{1}{2} \times \tfrac{3}{5} \qquad \{\text{branches marked} \\ &= \tfrac{4}{10} \\ &= \tfrac{2}{5} \end{aligned}$$



Additional Example to look at...

G USING TREE DIAGRAMS

Example 10 Se

Stephano is having computer problems. His desktop computer will only boot up 90% of th and his laptop will only boot up 70% of the time. Stephano attempts to boot both machine

a Draw a tree diagram to illustrate this situation.

b ~~Use the tree diagram~~ to determine the chance that:

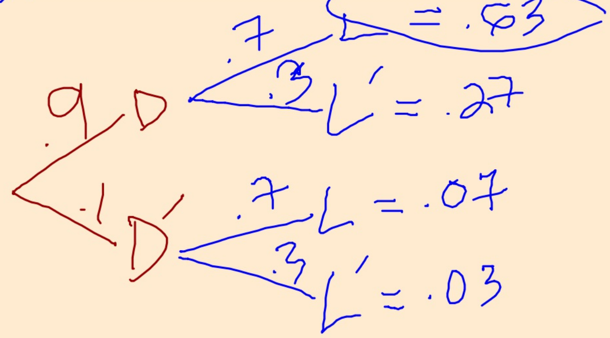
i both will boot up 63%   **ii** only the desktop computer boots up. or 27% .27

D(Boot) = .9

D′(Not boot) = .1

L(Boot) = .7

L′(Boot) = .3



Additional Example to look at...

G USING TREE DIAGRAMS

Example 10 Self Tutor

Stephano is having computer problems. His desktop computer will only boot up 90% of the time, and his laptop will only boot up 70% of the time. Stephano attempts to boot both machines.

a Draw a tree diagram to illustrate this situation.

b Use the tree diagram to determine the chance that:

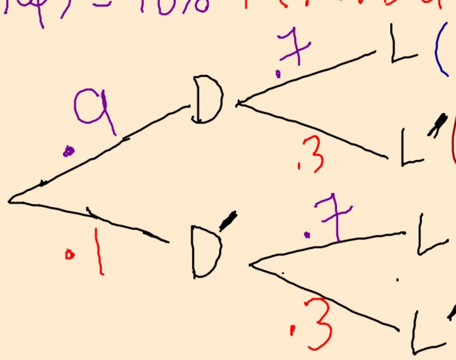
i both will boot up 63% **ii** only the desktop computer boots up. 27%

D′ : Doesn't boot up

L′ : Doesn't boot up

P(Desk) = 90% P(Not Boot D) = 10%

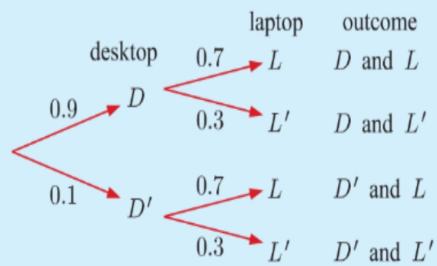
P(laptop) = 70% P(Not Boot L) = 30%



L (Both bootup!) = .63

L′ (Desktop, not laptop) = .27

a D = desktop computer boots up
L = laptop boots up

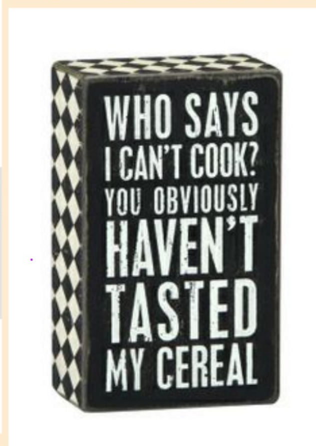


b **i** P(both boot up)
$= \mathrm{P}(D \text{ and } L)$
$= \mathrm{P}(D) \times \mathrm{P}(L)$
$= 0.9 \times 0.7$
$= 0.63$

ii P(desktop boots up but laptop does not)
$= \mathrm{P}(D \text{ and } L')$
$= \mathrm{P}(D) \times \mathrm{P}(L')$
$= 0.9 \times 0.3$
$= 0.27$

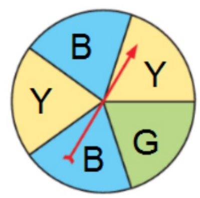
Jokes for the day

"I taped his homework to the back of the cereal box."



EXERCISE 14G

1 Suppose this spinner is spun twice.



a Draw a tree diagram to illustrate the sample space.

b Determine the probability that:

i blue appears on both spins

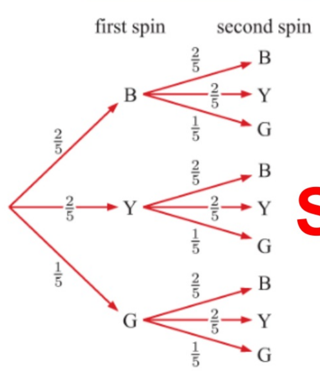
ii green appears on both spins

iii different colours appear on the two spins

iv blue appears on *either* spin.

EXERCISE 14G

1 **a**



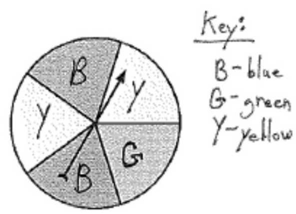
b **i** $\frac{4}{25}$ **ii** $\frac{1}{25}$ **iii** $\frac{16}{25}$ **iv** $\frac{16}{25}$

Solution

Solution

EXERCISE 14G

1 Suppose this spinner is spun twice.



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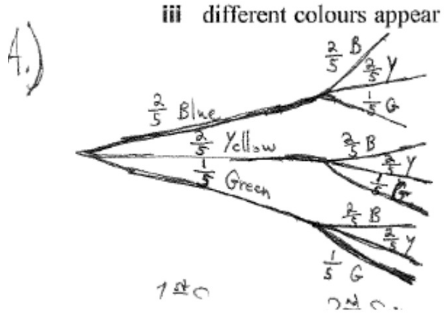
A.)



B.) i. $\frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

ii. $\frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$

iii. $\frac{2}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{1}{5} + \frac{2}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{1}{5} + \frac{1}{5} \cdot \frac{2}{5} + \frac{1}{5} \cdot \frac{2}{5} = \frac{16}{25}$

$P(BY) + P(BG) + P(YB) + P(YG) + P(GB) + P(GY) = P(\text{different colors})$

iv.

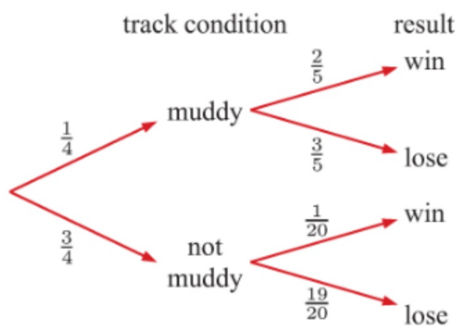
$\frac{2}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{1}{5} + \frac{2}{5} \cdot \frac{2}{5} + \frac{1}{5} \cdot \frac{2}{5} = \frac{16}{25}$

s $P(BB) + P(BY) + P(BG) + P(YB) + P(GB) = P(B)$

2 The probability of the race track being muddy next week is estimated to be $\frac{1}{4}$. If it is muddy, the horse Rising Tide will start favourite with probability $\frac{2}{5}$ of winning. If it is dry, Rising Tide has a $\frac{1}{20}$ chance of winning.

- **a** Display the sample space of possible results on a tree diagram.
- **b** Determine the probability that Rising Tide will win next week.

2 **a**

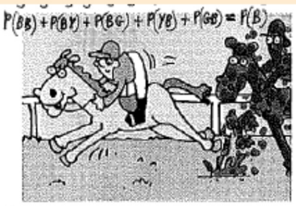


b $\frac{11}{80}$

Solution

Solution

$P(BB)+P(BY)+P(BG)+P(YB)+P(GB) = P(B)$



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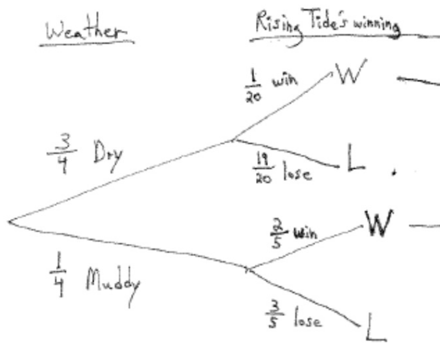
a Display the sample space of possible results on a tree diagram.

b Determine the probability that Rising Tide will win next week.

A.)



B.) Probability

$\frac{3}{4}\cdot\frac{1}{20} = \frac{3}{80}$

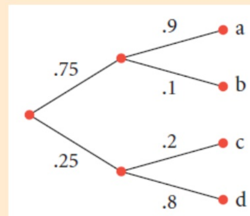
and

$\frac{1}{4}\cdot\frac{2}{5} = \frac{2}{20} = \frac{8}{80}$

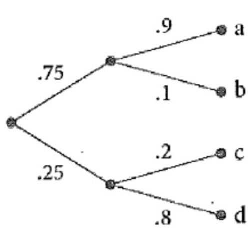
$P(\text{dry, win}) + P(\text{muddy, win}) = P(\text{win})$

$\frac{3}{80} + \frac{8}{80} = \frac{11}{80}$

2. Find the probability of each path, a–d, in the tree diagram at right. What is the sum of the values of a, b, c, and d?



2. Find the probability of each path, a–d, in the tree diagram at right. What is the sum of the values of a, b, c, and d?



Solution

$P(a) = .75 \times .9 = 0.675$

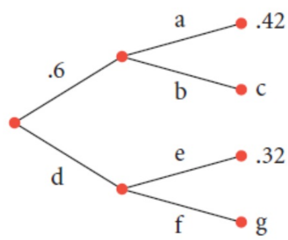
$P(b) = .75 \cdot .1 = 0.075$

$P(c) = .25(.2) = 0.05$

$P(d) = .25(.8) = 0.2$

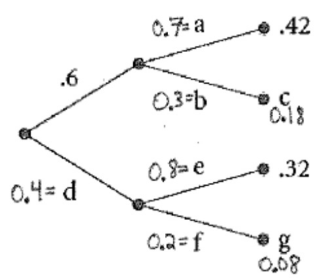
$1 = 0.675 + 0.075 + 0.05 + 0.2$

3. Find the probability of each path, a–g, in the tree diagram below.



3. Find the probability of each path, a–g, in the tree diagram below.

Solution



$.6(a) = .42$ so $a = \frac{.42}{.6} = 0.7$

$1 - 0.7 = 0.3 = b$ $\quad c = 0.6(.3) = 0.18$

$0.4(e) = 0.32$ so $e = \frac{.32}{.4} = 0.8$

$1 - 0.8 = 0.2$ $\quad g = 0.4(.2) = 0.08$

10. You are totally unprepared for a true-false quiz, so you decide to guess randomly at the answers. There are four questions. Find the probabilities described in 10a–e.

a. P(none correct)

b. P(exactly one correct)

c. P(exactly two correct)

d. P(exactly three correct)

e. P(all four correct)

f. What should be the sum of the five probabilities in 10a–e?

g. If a passing grade means you get at least three correct answers, what is the probability that you passed the quiz?

Exercises...**Solutions**

10. You are totally unprepared for a true-false quiz, so you decide to guess randomly at the answers. There are four questions. Find the probabilities described in 10a–e.

a. P(none correct) $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$ or 1 outcome (F,F,F,F) out of 16 possible and equally likely outcomes

b. P(exactly one correct) $= \frac{4}{16} = \frac{1}{4}$ T,F,F,F F,T,F,F F,F,T,F F,F,F,T

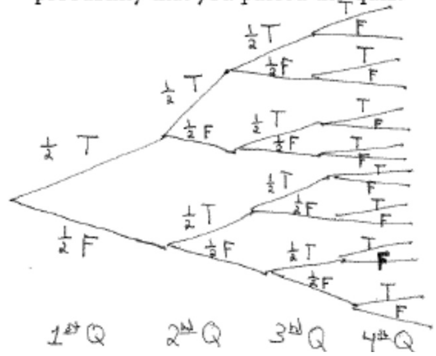
c. P(exactly two correct) $= \frac{6}{16} = \frac{3}{8}$ TTFF, TFTF, TFFT, FTTF, FTFT, FFTT

d. P(exactly three correct) $= \frac{4}{16} = \frac{1}{4}$ TTTF, TTFT, TFTT, FTTT

e. P(all four correct) $= \frac{1}{16}$

f. What should be the sum of the five probabilities in 10a–e? 1 $= \frac{16}{16} = \frac{1}{16} + \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16}$

g. If a passing grade means you get at least three correct answers, what is the probability that you passed the quiz?



$P(\text{pass}) = P(3 \text{ correct}) + P(4 \text{ correct})$

$P(\text{pass}) = \frac{4}{16} + \frac{1}{16}$

$P(\text{pass}) = \frac{5}{16}$