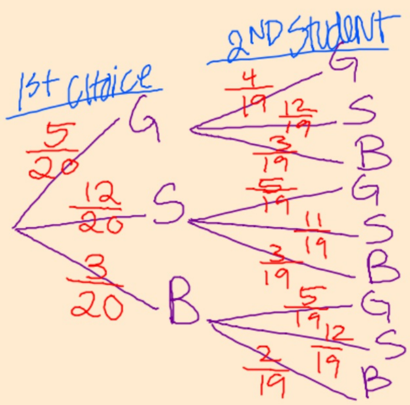
Welcome!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 7 Topic: Hidden Figures	0 1 2	
Tuesday Date: 5 - 8 Topic: Experimental vs. Theoretical Probability	0 1 2	
Wednesday Date: 5 - 9 Topic: 14G Tree Diagrams	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Warm-up:

∩ "and" ∪ "or"

Ms. Berg has a bowl of Laffy Taffy. She has **5 grape, 12 strawberry,** and **3 banana**. Each piece randomly chosen.



1) Why are probabilities different for the 2nd student?

- Each piece is eaten, so the 2nd student has different options and quantities of candy to choose from.

2) What's the probability the first 2 are banana?!

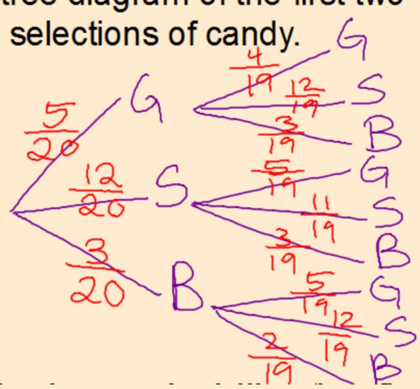
$P(B\cap B)=\frac{3}{20}\cdot\frac{2}{19}=\frac{6}{380}=\frac{3}{190}$

$\approx$ 1.6% chance!

What is the probability of choosing a **banana laffy taffy** if the first person chose **strawberry**?

P(B|S) = "Probability of banana if Strawberry was already chosen"

Ms. Berg has a bowl of Laffy Taffy. She has **5 grape, 12 strawberry,** and **3 banana**. Draw a tree diagram of the first two random selections of candy.



What is an independent event?

- Independent events are events where the outcome of one event **does not affect** the outcome of the other events
  - Example:
  - Tossing a coin and rolling a number cube are independent events.

What is a dependent event?

- If the outcome of one event **affects** the outcome of another, then the events are said to be Dependent Events.
  - Example:
  - Taking out a marble from a bag containing some marbles and not replacing it, and then taking out a second marble are dependent events.

Dependent and Conditional Events

When the probability of an event depends on the occurrence of another event, the events are **dependent.** Independent and dependent events can be described using **conditional probability.** When events A and B are dependent, the probability of A occurring given that B occurred is different from the probability of A by itself. The probability of A given B is denoted with a vertical line:

$P(A \mid B)$

In Examples B and C, the probability of a sophomore in class 2 given a sophomore in class 1 would be written as $P(S2 \mid S1)$. The fact that events S1 and S2 were dependent in Example C means that $P(S2 \mid S1) \neq P(S2)$. In Example B, however, S1 and S2 were independent, so $P(S2 \mid S1) = P(S2)$.

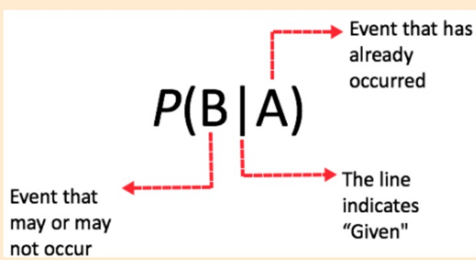
You can use tree diagrams to break up dependent events into independent ones. In the tree diagram for Example C, part b, above, the probabilities of all branches other than the first branch are actually conditional probabilities. Events S1 and (S2 given S1) are independent, as are (S2 given S1) and (S3 given S1 and S2). So, you can use the multiplication rule to find the probabilities of the paths.

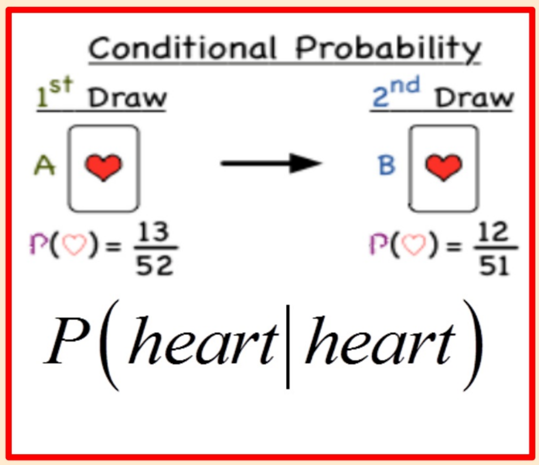
Dependent and Conditional Events

Conditional Probability

the probability of an event occurring based on a previous event already taking place

"Probability of drawing a heart, given that a heart was already drawn."





SKUNK RULES (1 is BAD!)

- Shake two dice
- **GOAL: highest score.**
- Prior to each role,choose to stand or sit. After you sit... you can't stand back up until next column.
- Only way to gain points is by standing.
- **If you are standing and a 1 is rolled...you will get 0 for the column.**
- If you are sitting you are saving the points you already have in the column

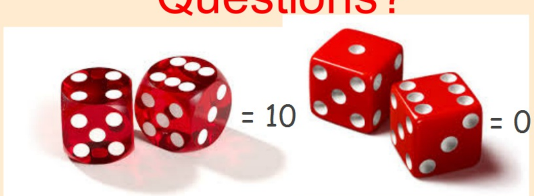
Questions?

SKUNK RULES (1 is BAD!)

Questions to consider during the game:

1) What is probability of rolling a one with 2 die?
2) What is probability of rolling ***two*** ones with 2 dice?
3) How else could this game be played?

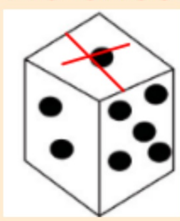
Questions?



= 0 for column

1) Draw game board.
2) Stand up and record 1st roll.
3) Choose to sit or stand.
4) ...Repeat..

No ones! :)



S	K	U	N	K

Class Plan:

1. Warm-up/ SKUNK!
2. Independent and Dependent Events/14H With or Without Replacement (using Tree Diagrams)
3. Practice

Probability of a Path

Dependent Events

The Multiplication Rule (again)

If n_1, n_2, n_3, and so on, represent events along a path, then the probability that this sequence of events will occur can be found by multiplying the probabilities of the events.

$$P(n_1 \text{ and } n_2 \text{ and } n_3 \text{ and } \ldots) = P(n_1) \cdot P(n_2 \mid n_1) \cdot P(n_3 \mid (n_1 \text{ and } n_2)) \cdot \cdots$$

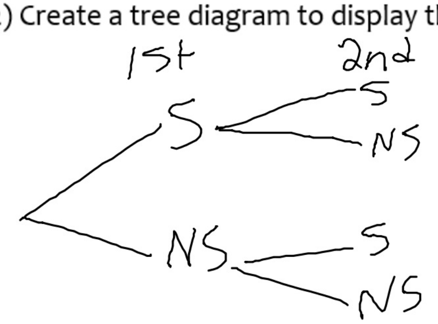
Example 1: Sophomores in Math Competition

Example: Mr. Sturm teaches three classes and each class has 20 students. 1st period has 12 sophomores, 2nd period has 8 sophomores, and his 3rd period has 10 sophomores. He randomly chooses one student from each class to compete in a math competition.

1) Determine probabilities for each class period:

1st period	P (10th grader) = 12/20 = 3/5	P (*Not* a 10th grader) = 2/5
2nd period	P (10th grader) = 8/20 = 2/5	P (*Not* a 10th grader) = 3/5
3rd period	P (10th grader) = 10/20 = 1/2	P (*Not* a 10th grader) = 1/2

2) Create a tree diagram to display the probabilities of Mr. Sturm's selection:



Solution

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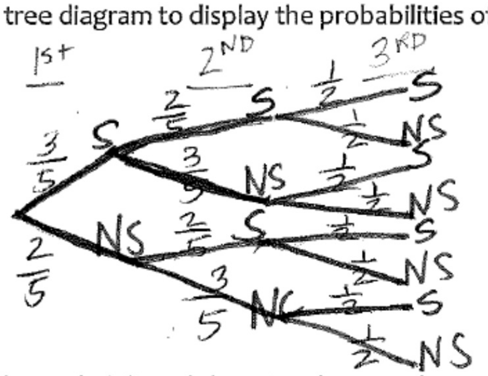
randomly chooses one student from each class to compete in a math competitior

1) Determine probabilities for each class period:

1st	P (10th grader) = $\frac{12}{20} = \frac{3}{5}$	P (*Not* a 10th grader) = $\frac{2}{5}$
2nd	P (10th grader) = $\frac{8}{20} = \frac{2}{5}$	P (*Not* a 10th grader) = $\frac{3}{6}$
3rd	P (10th grader) = $\frac{10}{20} = \frac{1}{2}$	P (*Not* a 10th grader) = $\frac{1}{2}$

2) Create a tree diagram to display the probabilities of Mr. Sturm's selection:



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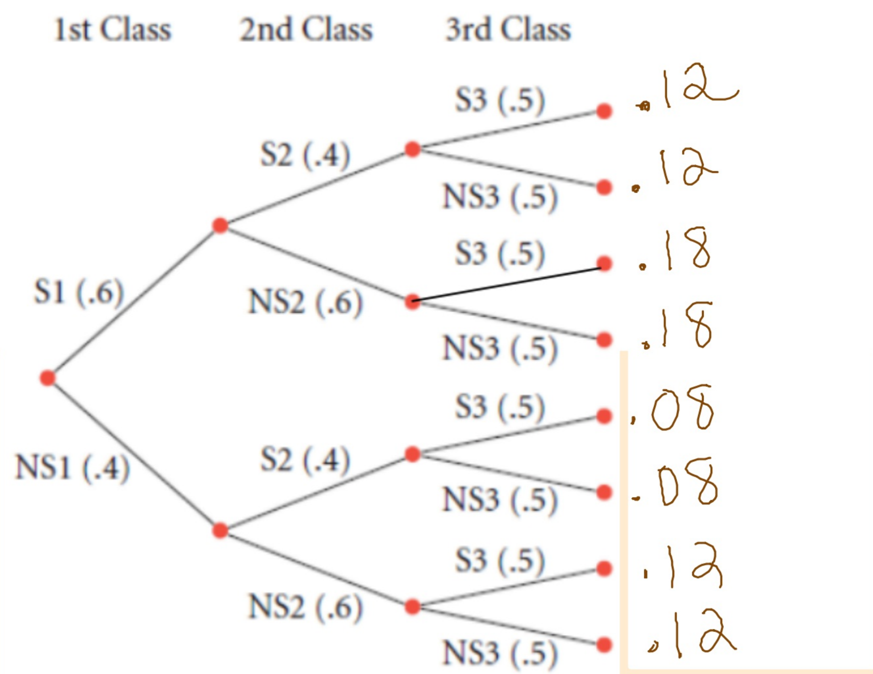
2) Create a tree diagram to display the probabilities of Mr. Sturm's selection:



Example 1: Sophomores in Math Competition

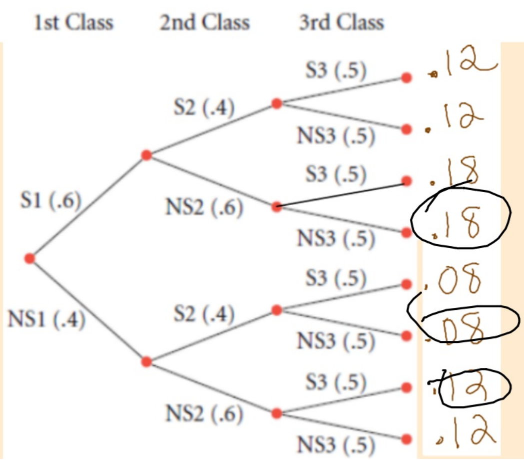
3) What is the probability of choosing three sophomores? .12 , $\frac{6}{50} = \frac{3}{25} = 12\%$

4) What is the probability that Mr. Sturm will choose only one sophomore? **Explain your answer.**

.18 + .08 + .12 = 38%

5) A sophomore is chosen in ***1st period.*** What effect does this have on the selection of a sophomore in ***2nd period***?

NONE



Solution

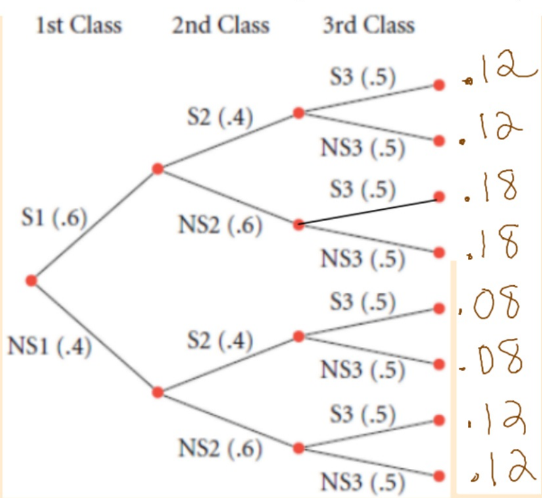
3) What is the probability of choosing three sophomores? $P(S \& S \& S) = \left(\frac{3}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) = \frac{6}{50} = \frac{3}{25}$

4) What is the probability that Mr. Sturm will choose only one sophomore? **Explain your answer.**

$$\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{5}\right)\left(\frac{3}{5}\right)\left(\frac{1}{2}\right) = \frac{9}{50} + \frac{4}{50} + \frac{6}{50} = \frac{19}{50}$$ .38

5) A sophomore is chosen in 1st ***period.*** What effect does this have on the selection of a sophomore in 2nd ***period***?

Nope. INDEPENDENT EVENTS.

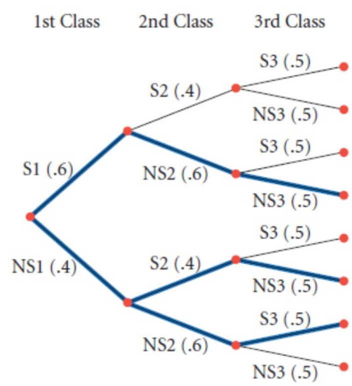


Solution

▶ *Solution*

Tree diagrams will help you determine these probabilities.

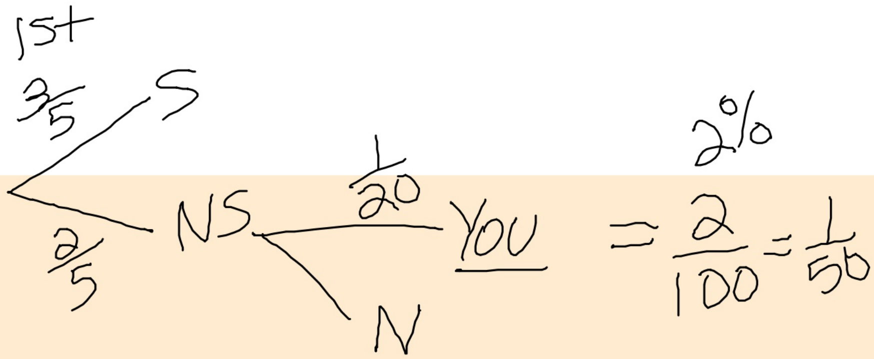
a. The three highlighted paths represent the different outcomes that include a single sophomore. The first path has probability (.6)(.6)(.5), or .18, the second path has probability (.4)(.4)(.5), or .08, and the last path has probability (.4)(.6)(.5), or .12. The probability of one of these paths occurring is .18 + .08 + .12, or .38. So, 38% of the 8000 total paths contain exactly one sophomore.



Example 1: Sophomores in Math Competition

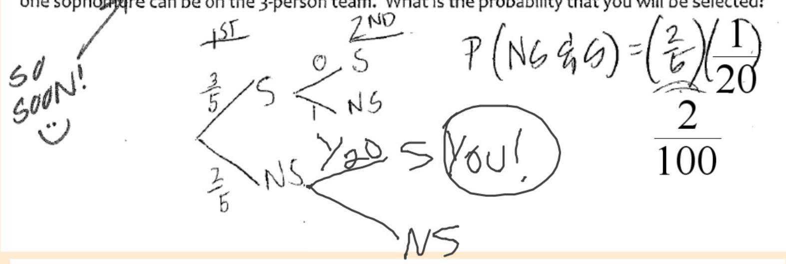
6) Suppose you are a sophomore in Mr. Sturm's 2nd period and the competition rules say that only one sophomore can be on the 3-person team. What is the probability that you will be selected?

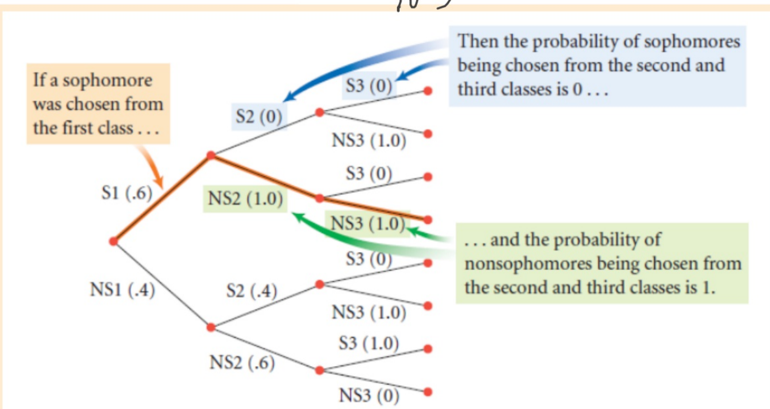
$P(YOU) = \frac{1}{20}$



Solution

6) Suppose you are a sophomore in Mr. Sturm's 2nd period and the competition rules say that only one sophomore can be on the 3-person team. What is the probability that you will be selected?





Exercises...

Tree Diagrams Day 2

(Mixture of Independent & Dependent Scenarios)

***Work together at your table!

Exercises...

1. A box of candy contains 6 grape pieces and 3 strawberry pieces. Carla takes one piece from the box and eats it. She then takes a second piece and eats the candy.

ii. Create a tree-diagram showing Carla's two pieces of candy:

ii. What is the probability that Carla will get exactly two strawberry candies? ___________________

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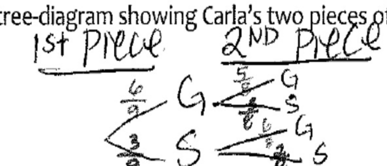

ii. What is the probability that Carla will get exactly two strawberry candies? $P(S \& S) = \left(\frac{3}{9}\right)\left(\frac{2}{8}\right) = \frac{6}{72} = \frac{1}{12}$

Exercises...

4 A cook selects an egg at random from a carton containing 7 ordinary eggs and 5 double-yolk eggs. She cracks the egg into a bowl and sees whether it has two yolks or not. She then selects another egg at random from the carton and checks it.

Let S represent a single-yolk egg, and D represent a double-yolk egg.

- **a** Draw a tree diagram to illustrate this process.
- **b** Find the probability that both eggs had:
  - **i** two yolks
  - **ii** only one yolk.

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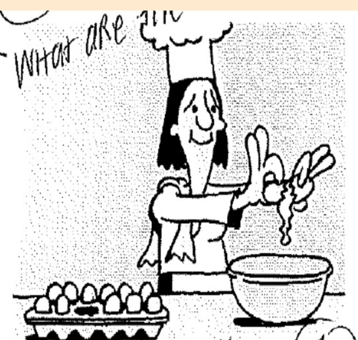

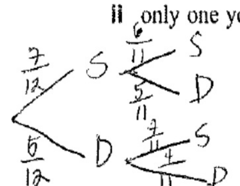

ii) $P(S \cap S) = \left(\frac{7}{12}\right)\left(\frac{6}{11}\right) = \frac{42}{132} = \frac{7}{22}$ (ii)

i) $P(D \cap D) = \left(\frac{5}{12}\right)\left(\frac{4}{11}\right) = \frac{20}{132} = \frac{5}{32}$ (i)

12. The Pistons and the Bulls are tied, and time has run out in the game. However, the

Exercises...

12. The Pistons and the Bulls are tied, and time has run out in the game. However, the Pistons have a player at the free throw line, and he has two shots to make. He generally makes 83% of the free throw shots he attempts. The shots are independent events, so each one has the same probability. Find these probabilities:

a. *P*(he misses both shots)

b. *P*(he makes at least one of the shots)

c. *P*(he makes both shots)

d. *P*(the Pistons win the game)

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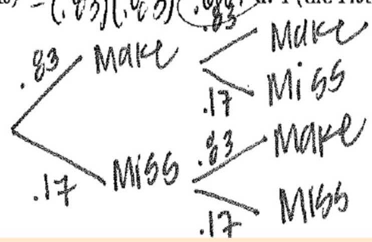
2. The Pistons and the Bulls are tied, and time has run out in the game. However, the Pistons have a player at the free throw line, and he has two shots to make. He generally makes 83% of the free throw shots he attempts. The shots are independent events, so each one has the same probability. Find these probabilities:

a. *P*(he misses both shots) $= (.17)(.17) = .0289$

b. *P*(he makes at least one of the shots) $(.83)(.17) + (.17)(.83) + (.83)(.83) = .1411 + .1411 + .6889 = .9711$

c. *P*(he makes both shots) $= (.83)(.83) = .6889$

d. *P*(the Pistons win the game) $= .9711$ (TIED! Need one shot atleast)



Exercises...

Suppose that a blue-footed booby has a 47% chance of surviving from egg to adulthood. For a nest of four eggs

a. What is the probability that all four birds will hatch and survive to adulthood?

b. What is the probability that none of the four birds will hatch and survive to adulthood?

A pair of blue-footed boobies

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A pair of blue-footed boobies

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a. What is the probability that all four birds will hatch and survive to adulthood? $(.47)^4 \approx .0488$

b. What is the probability that none of the four birds will hatch and survive to adulthood? $(.53)^4 \approx .0789$

~~c. How many birds would you expect to survive?~~

1st 2nd 3RD 4TH