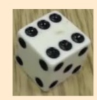
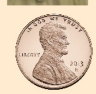
Friday! Please reflect....Don't turn in.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 7 Topic: Hidden Figures	0 1 2	
Tuesday Date: 5 - 8 Topic: Experimental vs. Theoretical Probability	0 1 2	
Wednesday Date: 5 - 9 Topic: Tree Diagrams - **Independent Events**	0 1 2	
Thursday Date: 5 - 10 Topic: Tree Diagrams - **Dependent Events**	0 1 2	
Friday Date: 5 - 11 Topic: Sampling Populations	0 1 2	

Warm-up:





How many possible outcomes are there for rolling a six-sided die and flipping a coin? ___12___

List the outcomes of the sample space.

1H 2H 3H 4H 5H 6H

1T 2T 3T 4T 5T 6T

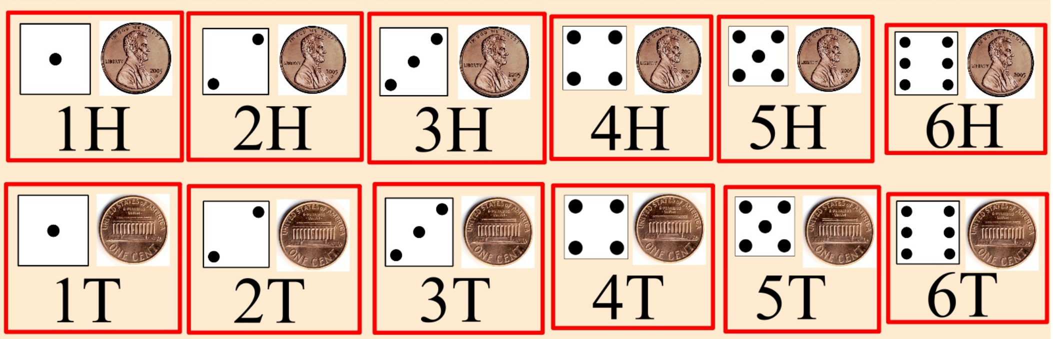
$\approx .083 \approx 8.3\%$

P(Rolling 6 and Flipping Head) = $\frac{1}{12}$

Sample Space - Possible Outcomes

6 outcomes from the die
2 outcomes from the coin

$$6 \times 2 = \boxed{12}$$



$$S = \{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$$

$$P(6H) = \frac{1}{12}$$

1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

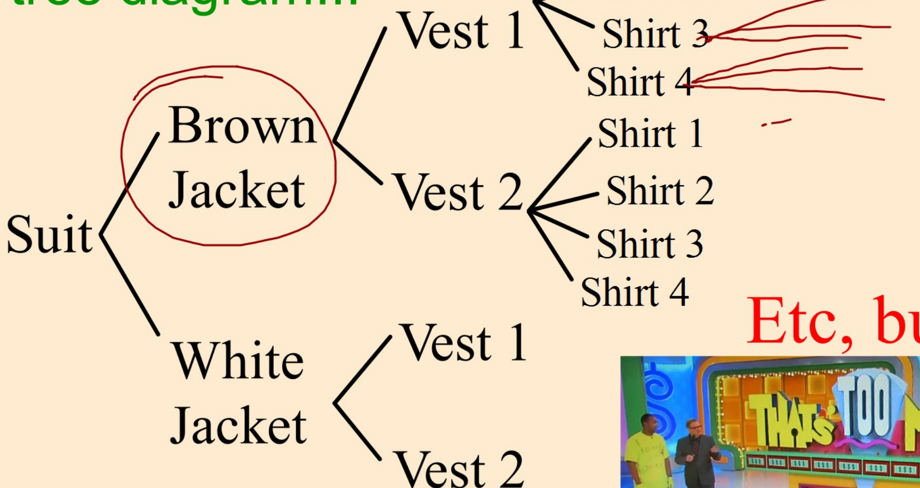
How could we display these many options?

B, VO, SH1, Shoe1, Belt1

B, VO, SH1, Shoe1, Belt2

1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

We could make a tree diagram...



Etc, but...

But The Counting Principle is Easier!

(2 Jackets)·(2 Vests)·(4 Shirts)·
(3 Shoes)·(5 Belts)

Example: Mr. Sinclair is renting a suit.

1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

How many different suit outfits can Mr. Sinclair choose from?

$$\underset{\text{Jackets}}{\underline{2}} \times \underset{\text{Shirts}}{\underline{4}} \times \underset{\text{Vests}}{\underline{2}} \times \underset{\text{Shoes}}{\underline{3}} \times \underset{\text{Belts}}{\underline{5}}$$

240 options

NOW SHOWING

Counting Techniques

Starring:

The Multiplaction Principle

The Multiplication Principle
(a.k.a. The Fundamental Counting Principle)

$$n_1 \times n_2 \times n_3 \times \ldots$$

Jackets Vests Shirts . . . . —

If there are:

- n_1 outcomes from event E_1,
- n_2 outcomes from event E_2,
- n_3 outcomes from event E_3,
- ...etc...

Example: License Plates - **Numbers 1st**

Currently issued Minnesota license plates have 3 numbers followed by 3 letters.

How many possible license plates could Minnesota issue?

17,576,000 plates

$$\underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{A-Z}{\underline{26}}\ \underset{A-Z}{\underline{26}}\ \underset{A-Z}{\underline{26}}$$

$$10^3 \cdot 26^3$$

Example: License Plates - **Numbers 1st**

Currently issued Minnesota license plates have 3 numbers followed by 3 letters.

How many possible license plates could Minnesota issue?

$$\underline{10} \times \underline{10} \times \underline{10} \times \underline{26} \times \underline{26} \times \underline{26}$$

$\boxed{17{,}576{,}000 \text{ plates}}$

Example: License - **Number or Letters**

What if the numbers and letters could be swapped as well, as in old MN plates?

How many possible license plates could Minnesota issue?

$\underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{\quad}\ \underline{\quad}$

$36^6 = 2,176,782,336$ plates

Example: License - **Number or Letters**

What if the numbers and letters could be swapped as well, as in old MN plates?

How many possible license plates could Minnesota issue?

$\underline{36} \times \underline{36} \times \underline{36} \times \underline{36} \times \underline{36} \times \underline{36}$

36^6

$\boxed{2{,}176{,}782{,}336 \text{ plates}}$

Example: License Plates - **No Repeats!**

Let's go back to the current issue of license plates. Suppose that numbers and letters cannot be repeated.

How many possible license plates could Minnesota issue?

$\underline{10}$ $\underline{9}$ $\underline{8}$ $\underline{26}$ $\underline{25}$ $\underline{24}$

\# \# \# A-Z A-Z A-Z

11,232,000

Example: License Plates - **No Repeats!**

Let's go back to the current issue of license plates. Suppose that numbers and letters cannot be repeated.

How many possible license plates could Minnesota issue?

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{26} \cdot \underline{25} \cdot \underline{24}$

$\boxed{11{,}232{,}000 \text{ plates}}$

Permuations

A ***permutation*** of *n* objects is an <u>ordered</u> arrangement of the *n* objects.

<u>Example</u>:

How many ways could 5 friends arrange themselves in 5 seats at a movie theater?

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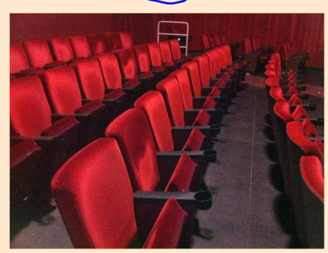


$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

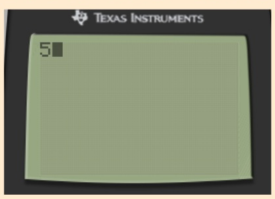
120

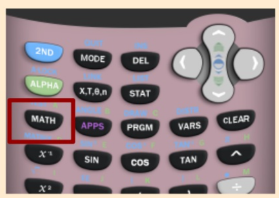
ABCDE
ABCED
ABDCE

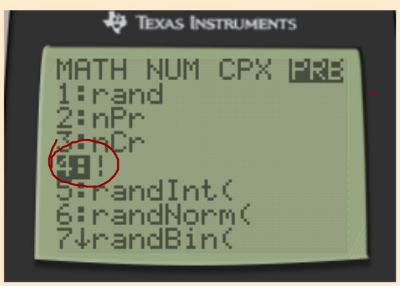
Example:

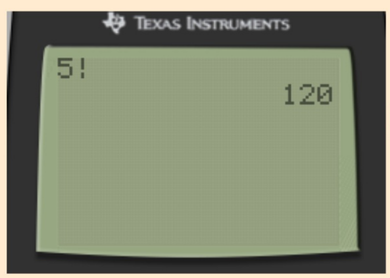
$\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

How many ways could 5 friends arrange themselves in 5 seats at a movie theater?









5!

Factorials

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

The ***factorial*** of a non-negative integer *n*, denoted *n*!, is the product of all positive integers less than or equal to *n*.

$51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44$

Example: 1969 Military Draft

The draft lottery of 1969 for military service ranked all 366 days (Jan 1, Jan 2, ..., Feb 29, ..., Dec 31) of the year. The men who were eligible for service whose birthday was selected first were the first to be drafted. Those whose birthday was selected second were the second to be drafted. And so on. How many possible ways can the 366 days be ranked?

1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
2	159	144	029	271	298	228	350	045	161	125	034	328
3	251	297	267	083	040	301	115	261	049	244	348	157
4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	021	071	220	282	041
11	329	150	136	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	042	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	226	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	003
31	211	---	030	---	313	---	193	011	---	079	---	100

$$366 \times 365 \times 364 \times \ldots \times 3 \times 2 \times 1 = \ ?$$

Example: 1969 Military Draft

1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
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4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	021	071	220	282	041
11	329	150	136	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	042	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	226	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	003
31	211	---	030	---	313	---	193	011	---	079	---	100

Ms. Paulson's father, 216th

Mr. Nelson's father, 2nd

Mic Nelson helps run after school help T & W. Former 9th grade math teacher :)

1st, 9-14

Factorials and Permuations

The number of different arrangements (permuations) of *n* objects is given by *n*!

In our draft example there would be

366! =
9 188 111 095 254 496 019 212 176 412 065 202 140 090 580 418 774 645 194 675 369
840 967 804 846 588 863 095 597 762 591 294 093 025 991 679 067 056 119 532 289
819 154 031 153 412 626 361 004 655 299 317 292 397 491 794 124 983 183 190 181
485 863 175 356 339 673 174 577 270 709 354 011 349 841 159 870 162 315 388 021
077 551 574 544 150 339 454 677 263 259 292 741 490 470 278 652 918 758 618 155
319 193 382 176 540 756 099 231 912 808 304 474 174 078 456 156 193 961 001 478
398 647 954 868 692 612 278 257 154 615 836 148 475 874 973 044 173 323 055 630
082 048 837 853 679 900 542 059 105 112 845 394 071 947 192 443 208 478 530 700
194 532 818 459 855 315 620 661 704 950 466 695 965 700 997 551 748 520 475 941
227 743 698 121 112 130 799 760 005 290 512 978 278 155 471 280 205 501 581 277
410 145 813 062 661 991 385 483 143 379 923 345 195 406 432 165 518 340 351 716
868 931 650 203 126 650 444 315 203 993 600 000 000 000 000 000 000 000 000 000
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

different arrangements.

60 columns
13 rows
+1 = 781
digits!!!!!!

Exercises...#5 & #7 Challenge

Counting Technique Exercises
(Lots of passwords & lock combos!)

Counting Technique Exercises Name:________________________

3. A single dial combination lock has a passcode involving three different numbers that need to be entered between spinning the face. There are digits 0 to 39 to choose from on the face (repeats are allowed). How many possible passcodes are possible?

4. Consider making a four-digit I.D. number using the digits 3, 5, 8, and 0.

a. How many I.D. numbers can be formed using each digit once?

b. How many can be formed using each digit once and not using 0 first?

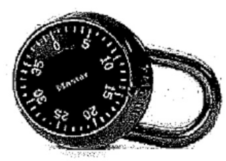
c. How many can be formed if repetition is allowed and any digit can be first?

d. How many can be formed if repetition is allowed but 0 is not used first?

Solutions...

3. A single dial combination lock has a passcode involving three different numbers that need to be entered between spinning the face. There are digits 0 to 39 to choose from on the face (repeats are allowed). How many possible passcodes are possible?

$\underline{40} \times \underline{40} \times \underline{40} = 64{,}000$ combinations



4. Consider making a four-digit I.D. number using the digits 3, 5, 8, and 0.

a. How many I.D. numbers can be formed using each digit once? $\underset{\text{choices}}{\underline{4}} \times \underline{3} \times \underline{2} \times \underset{\text{choice}}{\underline{1}} = 24$ I.D.'s

b. How many can be formed using each digit once and not using 0 first? $\underline{3} \times \underline{3} \times \underline{2} \times \underline{1} = 18$ I.D.'s

c. How many can be formed if repetition is allowed and any digit can be first? $\underset{\text{choices}}{\underline{4}}\ \underset{\text{choices}}{\underline{4}}\ \underline{4}\ \underline{4} =$ [redacted] $= 256$

[redacted] $4^4 = 256$

d. How many can be formed if repetition is allowed but 0 is not used first?

D.) $\underset{\text{choices}}{\underline{3}} \times \underset{\text{choices}}{\underline{4}} \times \underset{\text{choices}}{\underline{4}} \times \underset{\text{choices}}{\underline{4}} = 192$ I.D.'s

Exercises...

5. APPLICATION A combination lock has four dials. On each dial are the digits 0 to 9.

a. Suppose you forget the correct combination to open the lock. How many combinations do you have to try? If it takes 10 s to enter each combination, how long will it take you to try every possibility?

b. Suppose you replace your lock with one that has five dials, each with the digits 0 to 9. How many combinations are possible? If it still takes 10 s to enter each combination, how long will it take to try every possibility?

c. How many combinations are possible on a 5 dial lock when you are not allowed to repeat any digits?

6. You need to create a 6 character password (repeats allowed) for a website: _ _ _ _ _ _

a) How many different passwords can you make if you only use lower case letters?
b) How many different passwords are possible using only lower & upper case letters?
c) How many are possible using only lower & upper case letters and numerical digits?
d) How many are possible with upper & lower case letters, digits, and 33 special symbols?
e) How many characters long will a password that only uses lower case letters need to be to make it more secure than your answer to part (d)?

Solutions...

5. APPLICATION A combination lock has four dials. On each dial are the digits 0 to 9.

a. Suppose you forget the correct combination to open the lock. How many combinations do you have to try? If it takes 10 s to enter each combination, how long will it take you to try every possibility?

b. Suppose you replace your lock with one that has five dials, each with the digits 0 to 9. How many combinations are possible? If it still takes 10 s to enter each combination, how long will it take to try every possibility?

c. How many combinations are possible on a 5 dial lock when you are not allowed to repeat any digits?

A.) $\underset{\text{options}}{\underline{10}} \times \underset{\text{options}}{\underline{10}} \times \underset{\text{options}}{\underline{10}} \times \underset{\text{options}}{\underline{10}} = 10^4 = 10{,}000$ combinations

$10^5 = 100{,}000$ seconds

$= 1{,}666.67$ minutes

$= 27.778$ hours

$= 1.157407$ days

B.) $\underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 10^5 = 100{,}000$ combinations

$10^5 \cdot 10 = 10^6 = 1{,}000{,}000$ seconds or $16{,}666.67$ minutes

or 277.78 hours

or 11.5740741 days

C.) $\underset{\text{choices}}{\underline{10}} \times \underset{\text{choices}}{\underline{9}} \times \underset{\text{choices}}{\underline{8}} \times \underset{\text{choices}}{\underline{7}} \times \underset{\text{choices remain}}{\underline{6}} = 30{,}240$ combinations

Solutions...

6\. a) $\underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{26} = 26^6 = 308{,}915{,}776$ passwords

$\approx 3.09 \times 10^8$

b.) $52^6 = 19{,}770{,}609{,}664$ passwords $\approx 1.98 \times 10^{10}$

c.) $\left.\begin{array}{l}\text{26 lower case}\\ \text{26 upper case}\\ \text{10 numerical digits}\end{array}\right\}$ $62^6 = 56{,}800{,}235{,}584$ passwords

$\approx 5.68 \times 10^{10}$ passwords

d.) $62 + 33 = 95$ choices

so $95^6 = 735{,}091{,}890{,}625$ passwords $\approx 7.35 \times 10^{11}$

Solutions...

e.) $26^7 = 8,031,810,176 \rightarrow$ not as secure

$26^8 = 208,827,064,576 \rightarrow$ not as secure because

$208,827,064,576 < 735,091,890,625$

$26\cdot26\cdot26\cdot26\cdot26\cdot26\cdot26\cdot26 = 26^8 =$

$26^9 = 5,429,503,678,976 \approx 5.43\times10^{12} \rightarrow$ more secure

9 letters needed to make it more secure.

A password (using only lower case letters) with a length of 9 characters/letters is more secure than a password of length 6 characters that utilizes 52 letters, 10 digits, and 33 symbols.

Exercises...

7. An eight-volume set of reference books is kept on a shelf. The books are used frequently and put back in random order.

a. How many ways can the eight books be arranged on the shelf?

b. How many ways can the books be arranged so that Volume 5 will be the rightmost book?

c. Use the answers from 9a and b to find the probability that Volume 5 will be the rightmost book if the books are arranged at random.

d. Explain how to compute the probability in 9c using another method.

e. If the books are arranged randomly, what is the probability that the last book on the right is an *even*-numbered volume? Explain how you determined this probability.

f. How many ways can the books be arranged so that they are in the correct order, with volume numbers increasing from left to right?

g. How many ways can the books be arranged so that they are out of order?

h. What is the probability that the books happen to be in the correct order?

Solutions...

7. a) $8! = \boxed{40{,}320 \text{ ways}}$

b) Set 5 as the rightmost book.

$\_\ \_\ \_\ \_\ \_\ \_\ \_\ \underset{\text{Book 5}}{(1)}$

Arrange other 7 however: $7! = \boxed{5{,}040 \text{ ways}}$

c) $P(5 \text{ Rightmost}) = \dfrac{\#\text{ Ways 5 Rightmost}}{\text{Total way to Order}} = \dfrac{7!}{8!}$

$= \dfrac{5{,}040}{40{,}320} = \boxed{\dfrac{1}{8}}$

d) There are 8 possible positions for Book 5. Being rightmost is one of the 8 equally likely positions. Therefore the probability is $\frac{1}{8}$.

Solutions...

e. $\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \; \underset{\text{Even}}{\underline{(4)}} = \boxed{20{,}160}$

There 4 books that are even, so there are 4 options for the rightmost book.
There are 7! ways to arrange the other 7 books.

$P(\text{even on right}) = \frac{20{,}160}{40{,}320} = \frac{1}{2}$ or 50%

f. Only 1 way.

g. $8! - 1 = 40{,}320 - 1 = \boxed{40{,}319}$

h. $\frac{1}{8!} = \boxed{\frac{1}{40{,}320}}$