Friday! Please reflect....Don't turn in.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 7 Topic: Hidden Figures	0 1 2	
Tuesday Date: 5 - 8 Topic: Experimental vs. Theoretical Probability	0 1 2	
Wednesday Date: 5 - 9 Topic: Tree Diagrams - **Independent Events**	0 1 2	
Thursday Date: 5 - 10 Topic: Tree Diagrams - **Dependent Events**	0 1 2	
Friday Date: 5 - 11 Topic: Sampling Populations	0 1 2	

Warm-up:

How many possible outcomes are there for rolling a six-sided die and flipping a coin? 12

List the outcomes of the sample space.

1H 2H 3H 4H 5H 6H
1T 2T 3T 4T 5T 6T

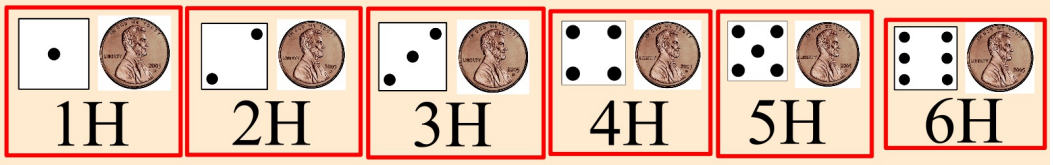
$P(6H) = \frac{1}{12}$

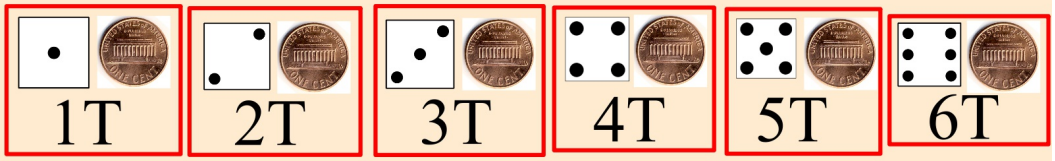
P(Rolling 6 and Flipping Head) = $\frac{1}{12}$

Sample Space - Possible Outcomes

6 outcomes from the die
2 outcomes from the coin

$6 \times 2 = 12$





$S = \{1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T\}$

$$P(6H) = \frac{1}{12}$$

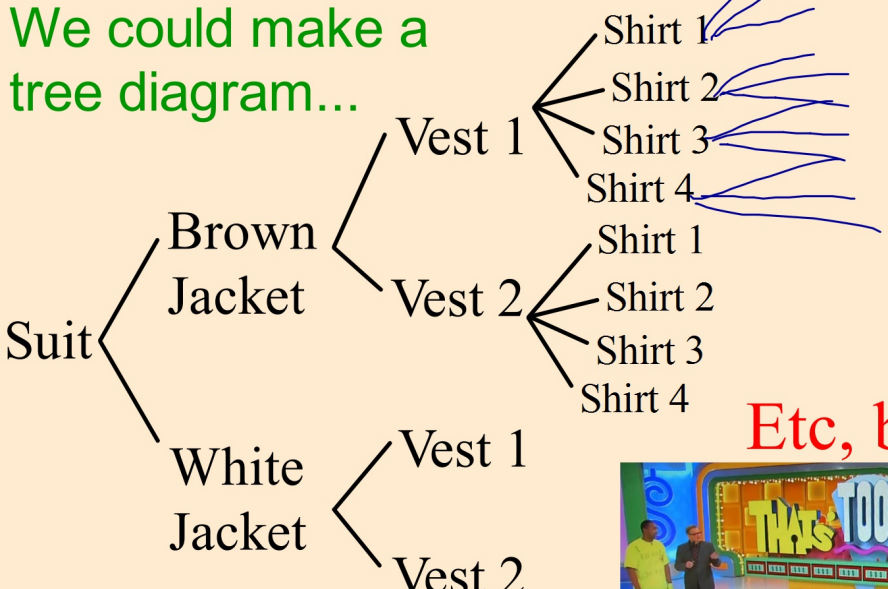
1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

How could we display these many options?

W B
V1 V1
S1 S1
SH1 SH1
B1 B1

1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

We could make a tree diagram...



Etc, but...

But The Counting Principle is Easier!

(2 Jackets)·(2 Vests)·(4 Shirts)·
(3 Shoes)·(5 Belts)

Example: Mr. Sinclair is renting a suit.

1) Mr. Sinclair is renting a suit. The store has the following: a white or brown jacket, 2 vest options, 4 shirt options, 3 shoe options, and 5 types of belts.

How many different suit outfits can Mr. Sinclair choose from?

$$\underset{\text{Jackets}}{2} \times \underset{\text{Vests}}{2} \times \underset{\text{Shirts}}{4} \times \underset{\text{Shoes}}{3} \times \underset{\text{Belts}}{5}$$

240 suit options


NOW SHOWING
Counting Techniques
Starring:
The Multiplaction Principle

The Multiplication Principle
(a.k.a. The Fundamental Counting Principle)

$$n_1 \times n_2 \times n_3 \times \ldots$$

Jackets Vests Shirts . . . .

If there are:

- n_1 outcomes from event E_1,
- n_2 outcomes from event E_2,
- n_3 outcomes from event E_3,
- ...etc...

How many MN license plates could be made?

2) i. Currently, Minnesota license plates have 3 numbers followed by 3 letters.

How many license plates could be created with these options?

$10^3 \cdot 26^3$

10 · 10 · 10 · 26 · 26 · 26

\# \# \# A-Z A-Z A-Z

17,576,000 Plates

How many MN license plates could be made?

2) i. Currently, Minnesota license plates have 3 numbers followed by 3 letters.

How many license plates could be created with these options?

2) i. Currently, Minnesota license plates have 3 numbers followed by 3 letters.

How many license plates could be created with these options? 10 digits (0-9)
26 letters (A-Z)

$\frac{10}{\text{Number}} \times \frac{10}{\text{Number}} \times \frac{10}{\text{Number}} \times \frac{26}{\text{letter}} \times \frac{26}{\text{letter}} \times \frac{26}{\text{letter}} = 17{,}576{,}000$ plates!

ii. Suppose numbers and letters could be used, as in old MN plates

How many MN license plates could be made?

ii. Suppose numbers and letters could be used, as in old MN plates.

How many license plates could be created with these options?

2,176,782,336 plates

36 · 36 · 36 ___ ___ ___

AAAAAA
AAAAZA
A

$= 36^6$

Example: License - **Number or Letters**

What if the numbers and letters could be swapped as well, as in old MN plates?

How many possible license plates could Minnesota issue?

$36 \times 36 \times 36 \times 36 \times 36 \times 36$

36^6

2,176,782,336 plates

How many MN license plates could be made?

iii. Suppose that numbers and letters cannot be repeated and there are exactly 3 numbers and 3 letters used in the license plate.

How many license plates could be created with these options?

11,232,000

$$10 \times 9 \times 8 \times \underset{A\text{-}Z}{26} \times \underset{A\text{-}Z}{25} \times \underset{A\text{-}Z}{24}$$

Example: License Plates - **No Repeats!**

Let's go back to the current issue of license plates. Suppose that numbers and letters cannot be repeated.

How many possible license plates could Minnesota issue?

$10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24$

$11{,}232{,}068$ plates

Permuations

A ***permutation*** of *n* objects is an ordered arrangement of the *n* objects.

Example:

How many ways could 5 friends arrange themselves in 5 seats at a movie theater?

Example:

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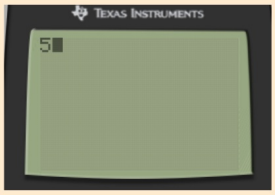
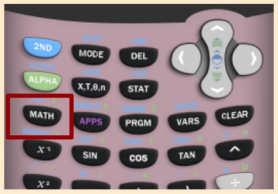
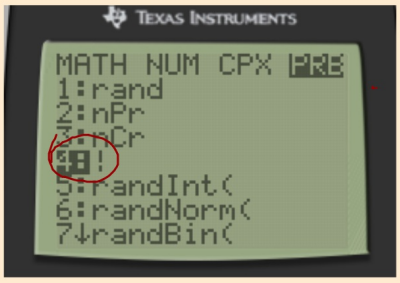
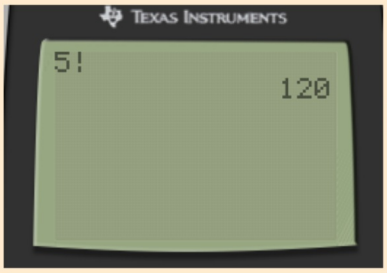
Example: $5 \times 4 \times 3 \times 2 \times 1$

How many ways could 5 friends arrange themselves in 5 seats at a movie theater?









5!

Factorials

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$$

The ***factorial*** of a non-negative integer *n*, denoted *n*!, is the product of all positive integers less than or equal to *n*.

Example: 1969 Military Draft

The draft lottery of 1969 for military service ranked all 366 days (Jan 1, Jan 2, ..., Feb 29, ..., Dec 31) of the year. The men who were eligible for service whose birthday was selected first were the first to be drafted. Those whose birthday was selected second were the second to be drafted. And so on. How many possible ways can the 366 days be ranked?

1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
2	159	144	029	271	298	228	350	045	161	125	034	328
3	251	297	267	083	040	301	115	261	049	244	348	157
4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	021	071	220	282	041
11	329	150	136	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	042	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	226	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	0
31	211	---	030	---	313	---	193	011	---	079	---	1

$$366 \times 365 \times 364 \times \ldots \times 3 \times 2 \times 1 = \ ?$$

Example: 1969 Military Draft

1970 RANDOM SELECTION SEQUENCE, BY MONTH AND DAY

	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	305	086	108	032	330	249	093	111	225	359	019	129
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4	215	210	275	081	276	020	279	145	232	202	266	165
5	101	214	293	269	364	028	188	054	082	024	310	056
6	224	347	139	253	155	110	327	114	006	087	076	010
7	306	091	122	147	035	085	050	168	008	234	051	012
8	199	181	213	312	321	366	013	048	184	283	097	105
9	194	338	317	219	197	335	277	106	263	342	080	043
10	325	216	323	218	065	206	284	024	071	220	282	041
11	329	150	136	014	037	134	248	324	158	237	046	039
12	221	068	300	346	133	272	015	142	242	072	066	314
13	318	152	259	124	295	069	060	307	175	138	126	163
14	238	004	354	231	178	356	331	198	001	294	127	026
15	017	089	169	273	130	180	322	102	113	171	131	320
16	121	212	166	148	055	274	120	044	207	254	107	096
17	235	189	033	260	112	073	098	154	255	288	143	304
18	140	292	332	090	278	341	190	141	246	005	146	128
19	058	025	200	336	075	104	227	311	177	241	203	240
20	280	302	239	345	183	360	187	344	063	192	185	135
21	186	363	334	062	250	060	027	291	204	243	156	070
22	337	290	265	316	326	247	153	339	160	117	009	053
23	118	057	256	252	319	109	172	116	119	201	182	162
24	059	236	258	002	031	358	023	036	195	196	230	095
25	052	179	343	351	361	137	067	286	149	176	132	084
26	092	365	170	340	357	022	303	245	018	007	309	173
27	355	205	268	074	296	064	289	352	233	264	047	078
28	077	299	223	262	308	222	088	167	257	094	281	123
29	349	285	362	191	026	353	270	061	151	229	099	016
30	164	---	217	208	103	209	287	333	315	038	174	003
31	211	---	030	---	313	---	193	011	---	079	---	100

Ms. Paulson's father, 216th

Mr. Nelson's father, 2nd

Mic Nelson helps run after school help T & W. Former 9th grade math teacher :)

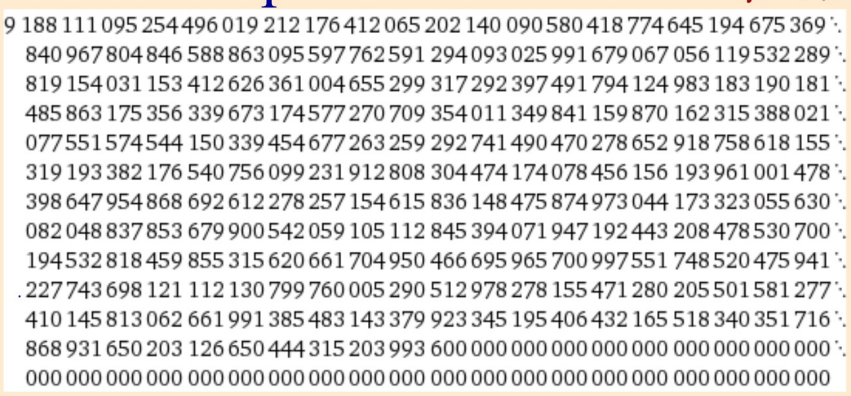
1st, 9-14

Factorials and Permuations

The number of different arrangements (permuations) of *n* objects is given by *n*!

In our draft example there would be 60X13

$366! =$



781 digits

different arrangements.

Exercises...

Practice Counting Techniques (Backside of handout)

Work Together!

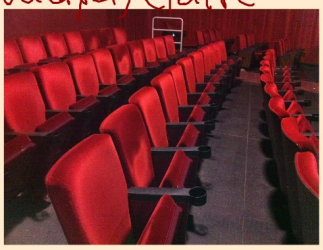
Done?
Raise hand -
Challenge Problems!!!

Exercises...

123, 132, 213, 231, 312, 321

1. **List and count** the many ways could 3 friends arrange themselves in 3 seats at a movie theater.

Abdinasir, Nadira, Claire



ANC | NAC
ACN | NCA
CAN
CNA

$3 \times 2 \times 1$

$= 6$

Exercises…

2. In how many ways can 9 softball players be listed for batting order?

Exercises…

3. If you have 12 shirts and 8 pairs of shorts, how many outfits could be created to wear to a picnic?

Exercises…

4. You need to choose a password to access your grades. The password must have two letters, two numbers, and one symbol. [Symbols include: # % $]

How many different passwords could you create?

Exercises…

5. You are working at Bruegger's Bagels and must build a bagel sandwich with cream cheese and meat. Bruegger's has 24 different bagels, 12 flavors of cream cheese, and roast beef, ham, turkey, and roasted chicken. How many different sandwiches could you build?

1. **List and count** the many ways could 3 friends arrange themselves in 3 seats at a movie theater.

Friends: A, B, C

ABC BAC CAB
ACB BCA CBA } 6 ways!

$\underset{\text{1st seat}}{3} \times \underset{\text{2nd seat}}{2} \times \underset{\text{3rd seat}}{1} = 6$

2. In how many ways can 9 softball players be listed for batting order?

$\underset{\text{1st}}{9} \times \underset{\text{2nd}}{8} \times \underset{\text{3rd}}{7} \times \underset{\text{4th}}{6} \times 5 \times 4 \times 3 \times 2 \times 1$

9 Players

(362,880 batting orders)

3. If you have 12 shirts and 8 pairs of shorts, how many outfits could be created to wear to a picnic?

$\frac{12}{\text{shirts}} \times \frac{8}{\text{shorts}} = 96$ options for picnic!

4. You need to choose a password to access your grades. The password must have two letters, two numbers, and one symbol. [Symbols include: # % $] How many different passwords could you create?

$\frac{26}{\text{letters}} \times \frac{26}{\text{letters}} \times \frac{10}{\text{numbers}} \times \frac{10}{\text{numbers}} \times \frac{3}{\text{symbol}} = 202{,}800$ passwords

5. You are working at Bruegger's Bagels and must build a bagel sandwich with cream cheese and meat. Bruegger's has 24 different bagels, 12 flavors of cream cheese, and roast beef, ham, turkey, and roasted chicken. How many different sandwiches could you build?

$\frac{12}{\text{Cream Cheese}} \times \frac{24}{\text{Babels}} \times \frac{4}{\text{Meat}} = 1152$ sandwiches