Welcome Back 9th Grade!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 5 - 14 Topic: Counting Techniques	0 1 2	
Tuesday Date: _____ Topic: _____	0 1 2	
Wednesday Date: _____ Topic: _____	0 1 2	
Thursday Date: _____ Topic: _____	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Warm-up:

How can this be simplified?

$$\frac{51!}{48!}$$

$$= \frac{51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdots 1}{48 \cdot 47 \cdot 46 \cdots 1}$$

$$= 124{,}950$$

Largest factorial 69.000005?

Class Plan:

1. Warm-up
2. Counting Techniques Continued
   - Icecream: Cones & Bowls

   - When is the order of the outcomes important?

3) If time....
Practice via....

Do:Investigate Permutations&Combinations

(Develop concepts & formulas - we will occassionally come back together!)

Question: What is the difference between arranging scoops of ice cream in **cone** and selecting scoops for a **bowl**?




How can we count the arrangements and selections *efficiently*?

Investigation: Permutations & Combinations

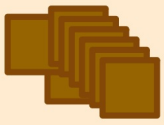
Ice Cream Cones vs. Bowls of Ice Cream **Flavors:** {Chocolate, Vanilla, and Strawberry}

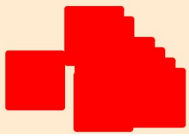
CONES: If you have 3 different flavors of ice-cream, how many different 2-scoop **cones** could you arrange? List **ALL** arrangements.

C
V
S

BOWLS: If you have 3 different flavors of ice-cream, how many different 2-scoop **bowls** could you choose? List **ALL** choices.

CONTRAST CONES & BOWLS: What is the difference between the number of ways we can ***arrange*** cones and ***choose*** bowls using the same flavors of ice-cream?





Investigation: Permutations & Combinations

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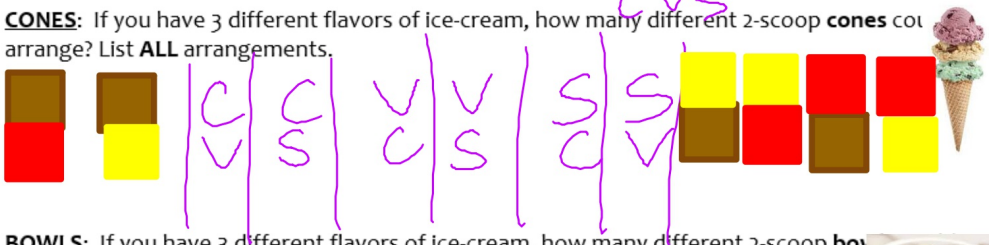
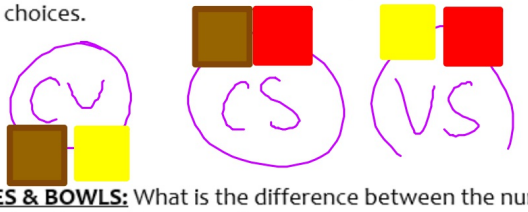


BOWLS: If you have 3 different flavors of ice-cream, how many different 2-scoop **bo**
choose? List **ALL** choices.



CONTRAST CONES & BOWLS: What is the difference between the number of ways we can ***arrange*** cones and ***choose*** bowls using the same flavors of ice-cream?

Cones: order of scoops matters (vanilla then strawberry is different from strawberry then vanilla.)
Bowls: A bowl of chocolate and vanilla is the same as a bowl of vanilla and chocolate. Order is **NOT IMPORTANT**.

SOLUTION - INVESTIGATION

Counting: Arranging and Choosing Name_____

Ice Cream Cones vs. Bowls of Ice Cream **Flavors:** {Chocolate, Vanilla, and Strawberry}

CONES: If you have 3 different flavors of ice-cream, how many different 2-scoop **cones** could you arrange? List **ALL** arrangements.

CV CS VS
VC SC SV } 6 options

BOWLS: If you have 3 different flavors of ice-cream, how many different 2-scoop **bowls** could you choose? List **ALL** choices.

CV, CS, VS } 3 options

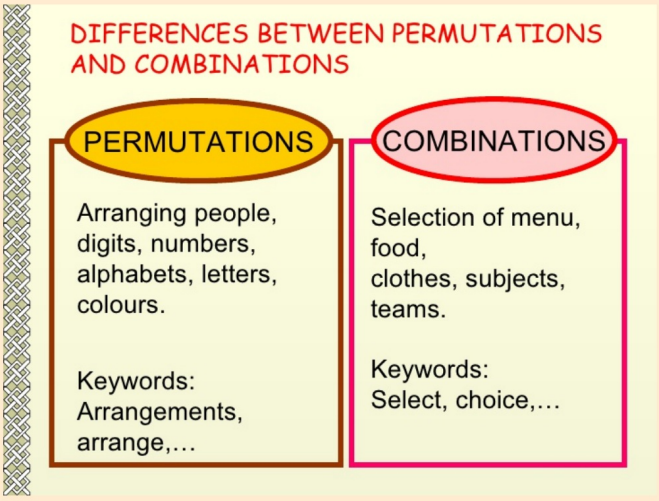
CONTRAST CONES & BOWLS: What is the difference between the number of ways we can ***arrange*** cones and ***choose*** bowls using the same flavors of ice-cream?

Cones need order - one on top of the other scoop...
Bowls can be mixed-up - Chocolate & Vanilla same Van. as Choc.

Counting Principles

Permutation: An ordered arrangement (ones)

Combination: Order doesn't matter (Bowls)



Investigation: Permutations & Combinations

DEVELOPING THE FORMULAS: There are 51 flavors of Ben & Jerry's ice cream.

Scenario 1 - ORDER MATTERS: Arrange 3 of the flavors on a cone. How many ways can you do this?

$51 \cdot 50 \cdot 49 = 124{,}950$ cones

(1st scoop · 2nd scoop · 3rd scoop)

Rewrite the expression above as a ratio of two factorials.

$$\frac{51!}{48!}$$

SOLUTION - INVESTIGATION

Rewrite the expression above as a ratio of two factorials.

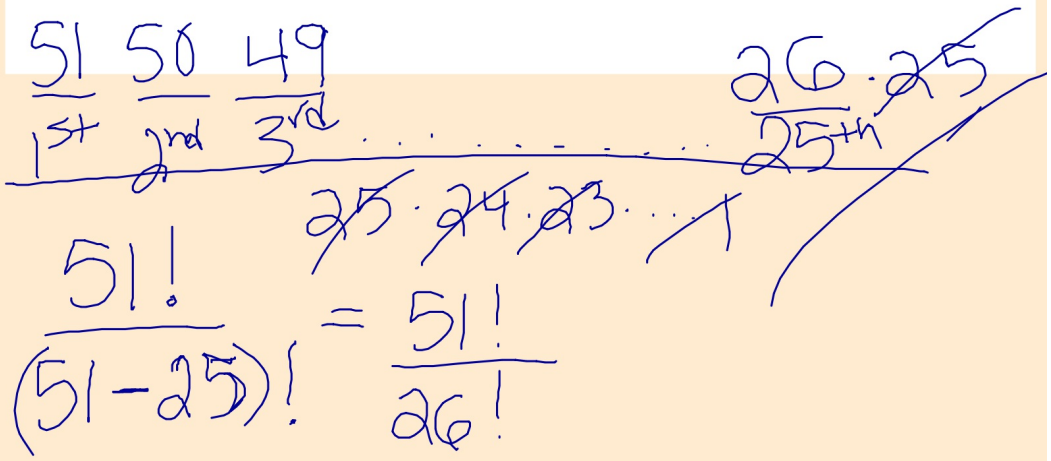
$$\frac{51!}{48!}$$

$$= \frac{51 \cdot 50 \cdot 49 \cdot 48!}{48!}$$

(WARM-UP)

Investigation: Permutations & Combinations

Let's say you wanted to put 25 different flavors on an ice cream cone (World Record?!), how could you solve for this without writing out (51)(50)(49)…?



SOLUTION - INVESTIGATION

Let's say you wanted to put 25 different flavors on an ice cream cone (World Record?!), how could you solve for this without writing out (51)(50)(49)…?

Choosing 25 flavors from 51.

$\underline{51}\ \underline{50}\ \underline{49}\ \underline{48}\ \underline{47}\ \underline{46} \cdots \underline{26}$

$1^{st}\ 2^{nd}\ 3^{rd}\ 4^{th}\ 5^{th}\ 6^{th} \cdots 25^{th}$ flavor!

$= 3.8 \times 10^{39}$ different cones

{40 digits after the 3! Soooo many!}

Investigation: Permutations & Combinations

Permutation Formula: Let's write this in general terms. **Ordering *r* objects from *n* choices.** *(How many different ways could you order r objects from n choices)*

"Choosing 3 flavors out of 51 choices"

Investigation: Permutations & Combinations

Permutation Formula: Let's write this in general terms. **Ordering *r* objects from *n* choices.** *(How many different ways could you order r objects from n choices)*

"Choosing 3 flavors out of 51 choices"

$$\frac{51 \cdot 50 \cdot 49 \cdot \ldots \cdot 1}{48 \cdot 47 \cdot \ldots \cdot 1} = \frac{51!}{48!} = \frac{51!}{(51-3)!} = 124{,}950 \text{ cones}$$

$$_{51}P_3 = {}_nP_r = \frac{n!}{(n-r)!}$$

$$124{,}950 = \frac{51!}{48!} = 51 \cdot 50 \cdot 49 = \frac{51!}{(51-3)!} = {}_{51}P_3$$

SOLUTION - INVESTIGATION
"Choosing 3 flavors out of 51 choices"

Permutation Formula: Let's write this in general terms. **Ordering r objects from n choices.** *(How many different ways could you order r objects from n choices)* 3 51

$$_nP_r = \frac{n!}{(n-r)!} \qquad \frac{51!}{(51-3)!}$$

ARRANGING 3 (DIFFERENT FLAVORED) SCOOPS OF ICECREAM FROM 51 FLAVORS

Investigation: Permutations & Combinations

Scenario 2 - ORDER DOESN'T MATTER: You are choosing 3 of the flavors for a bowl of ice cream.

Hmmm.... let's look at scoops of vanilla, chocolate, and strawberry.

List all the outcomes that represent the same bowl of ice cream with these 3 flavors.

CVS SCV VCS } diff
CSV SVC VSC } cones

6 of same bowl

SOLUTION - INVESTIGATION

Scenario 2 - ORDER DOESN'T MATTER: You are choosing 3 of the flavors for a bowl of ice cream.

Hmmm.... let's look at scoops of vanilla, chocolate, and strawberry.

List all the outcomes that represent the same bowl of ice cream with these 3 flavors.

VCS CVS SCV
VSC CSV SVC

V, C, S are 3 OF THE 51 FLAVORS, ABOVE ARE 6 DIFF CONES BUT THE SAME BOWL.

SOLUTION - INVESTIGATION

How can we **remove the outcomes that represent the same bowl of ice cream,** as order does not matter here?

Cones repeats

$\frac{124,950}{6} = 20,825$ bowls

SOLUTION - INVESTIGATION

How can we **remove the outcomes that represent the same bowl of ice cream,** as order does not matter here?

SCV
SVC
CVS
CSV
SCV
SVC

} 6 of the same bowl

$$\frac{124,950}{6} = 20,825 \text{ Bowls}$$

Investigation: Permutations & Combinations

How can we **remove the outcomes that represent the same bowl of ice cream,** as order does not matter here?

$$\text{Number of bowls} = \frac{\text{total \# cones}}{\text{\# of repeats}}$$

$$\frac{124{,}950}{6} = \frac{51!}{48!\,6} = \frac{51 \cdot 50 \cdot 49}{6} = \frac{51!}{(51-3)!\,3!}$$

Remember: $3! = 3 \cdot 2 \cdot 1 = 6$, 6 of the Same bowl.

Investigation: Permutations & Combinations

Combination Formula: Let's write this in general terms. **Choosing r objects from n choices.** *(How many different ways could you choose r objects from n choices)*

$3! = 3 \cdot 2 \cdot 1 = 6$

Number of bowls = $\frac{\text{total \# cones}}{\text{\# of repeats}}$

$$\frac{51!}{48!3!} = \frac{124{,}950}{6} = 20{,}825 \text{ bowls}$$

$$= \frac{51!}{(51-3)!\cdot 3!} = \frac{n!}{(n-r)!\,r!}$$

divide out extra count

SOLUTION - INVESTIGATION

Combination Formula: Let's write this in general terms. **Choosing *r* objects from *n* choices.** *(How many different ways could you choose r objects from n choices)*

$$_{51}C_{3} = \frac{n!}{(n-r)!\,r!}$$

← To Remove "Double Counting"

$3! = 6 = 3 \cdot 2 \cdot 1$

$$\frac{51!}{48!\,3!}$$

Example:

8) A team of 15 soccer players needs to choose two players to refill the water cooler.

1st, 3rd

$$\frac{15!}{(15-2)!\cdot 2!} = \frac{15\cdot 14}{2}$$

105 arrangements

Combination

Example:

14) A group of 35 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

Permutation

$$\frac{35!}{(35-3)!} = \frac{35 \cdot 34 \cdot 33 \cdot 32 \cdots 1}{32 \cdot 31 \cdots 1}$$

$$\underset{1st}{35} \cdot \underset{2nd}{34} \cdot \underset{3rd}{33} = 39{,}270$$

1st + 2nd + 3rd place arrange.

Exercises...

Permutations and Combinations Worksheet

State if each scenario involves a permutation or a combination.

1) There are 35 applicants for two jobs: computer programmer and software tester.

2) A group of 45 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

State if each scenario involves a permutation or a combination.

3) 4 out of 8 students will ride in a car instead of a van

4) There are 40 applicants for four Software Tester positions.

List all possible permutations.

5) A, B, C, taken two at a time

List all possible permutations.

6) ☺, ☼, ♥, ▲, taken two at a time

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

7) You are setting the combination on a five-digit lock. You want to use the numbers 12345 but don't care what order they are in.

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

8) A team of 15 soccer players needs to choose two players to refill the water cooler.

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

9) There are 60 people at a meeting. They each give a Valentine's Day card to everyone else. How many cards were given?

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

10) A group of 16 people need to take an elevator to the top floor. They will go in groups of eight. They are deciding who will take the elevator on its second trip.

10

1-

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

11) A group of 20 people are going to run a race. The top 12 finishers advance to the finals.

2

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

12) There are 100 athletes at a meeting. They each shake hands with everyone else. How many handshakes were there?

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

13) Bill and Huong are planning trips to ten countries this year. There are 13 countries they would like to visit. They are deciding which countries to skip.

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

14) A group of 35 people are going to run a race. The top three runners earn gold, silver, and bronze medals.

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

15) A team of 16 soccer players needs to choose a captain and co-captain.

State if each scenario involves a permutation or a combination. Then find the number of possibilities.

16) Alberto has homework assignments in four subjects. He only has time to do one of them.

1) Permutation

2) Permutation

3) Combination

4) Combination

5) AB BA CA
AC BC CB

6) ☺☼ ☼☺ ♥☺ ▲☺
☺♥ ☼♥ ♥☼ ▲☼
☺▲ ☼▲ ♥▲ ▲♥

7) Permutation; 120

8) Combination; 105

9) Permutation; 3,540

10) Combination; 12,870

11) Combination; 125,970

12) Combination; 4,950

13) Combination; 286

14) Permutation; 39,270

15) Permutation; 240

16) Combination; 4