

Assignment Self-Monitoring Sheet

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: <u>9/18</u> Topic: <u>Movie Ticket Analysis</u>	0 1 2	
Tuesday Date: <u>9/19</u> Topic: <u>HowOld.net Analysis</u>	0 1 2	
Wednesday Date: <u>9/20</u> Topic: <u>No homework - yesterday was a quiz.</u>	0 1 2	
Thursday Date: <u>9/21</u> Topic: <u>19A Graphing Systems of Equations</u>	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Warm-up:

How can we solve this? How many methods?

Example 4

Solve simultaneously by elimination:
$$\begin{cases} 4x + 3y = 2 \\ x - 3y = 8 \end{cases}$$

Warm-up:

How can we solve this? How many methods?

Example 4

Solve simultaneously by elimination:
$$\begin{cases} 4x + 3y = 2 \\ x - 3y = 8 \end{cases}$$

- 1) Elimination!
- 2) Substitution! (write 2nd equation in terms of x and substitute in $4x + 3y = 2$)
- 3) Graph using intercepts, or rewrite in intercept form to graph.

Class Plan

1. Warm-up
2. 19B Substitution & 19C Elimination
3. Joke Break :)
4. Practice

B**SOLUTION BY SUBSTITUTION**

We will now consider some **algebraic** methods for solving simultaneous equations.

The method of **solution by substitution** is used when at least one equation is given with either x or y as the **subject** of the formula. We substitute an expression for this variable into the other equation.

When at least 1 eqn. is given with x/y as the subject

Example 3**Self Tutor**

Solve simultaneously by substitution:
$$\begin{cases} y = x + 5 \\ 3x - y = 1 \end{cases}$$

B**SOLUTION BY SUBSTITUTION****Example 3****Self Tutor**

Solve simultaneously by substitution:
$$\begin{cases} y = x + 5 \\ 3x - \underline{y} = 1 \end{cases}$$

$$3x - (x + 5) = 1$$

$$3x - x - 5 = 1$$

$$2x - 5 = 1$$

$$2x = 6$$

$$\boxed{x = 3}$$

$$y = 3 + 5$$

$$\boxed{y = 8}$$

Solution (3, 8)

Example 3**Solution**

Solve simultaneously by substitution:
$$\begin{cases} y = x + 5 \\ 3x - y = 1 \end{cases}$$

$$\begin{aligned} y &= x + 5 && \dots (1) \\ 3x - y &= 1 && \dots (2) \end{aligned}$$

$$\begin{aligned} \text{Substituting (1) into (2) gives } & 3x - (x + 5) = 1 \\ & \therefore 3x - x - 5 = 1 \\ & \therefore 2x = 6 \\ & \therefore x = 3 \end{aligned}$$

$$\begin{aligned} \text{Substituting } x = 3 \text{ into (1) gives } & y = 3 + 5 \\ & \therefore y = 8 \end{aligned}$$

The solution is $x = 3, y = 8$.

$$\begin{aligned} \text{Check: In (1): } & 8 = 3 + 5 \quad \checkmark \\ \text{In (2): } & 3(3) - 8 = 9 - 8 = 1 \quad \checkmark \end{aligned}$$

We substitute $(x + 5)$ for y in equation (2).

**C****SOLUTION BY ELIMINATION**

If both equations are presented in the general form $Ax + By = C$, then solution by substitution is tedious. We instead use the method of **elimination**.

In this method, we make the coefficients of x (or y) the **same size** but **opposite in sign**. We then add the equations, which has the effect of **eliminating** one of the variables.

When both equations are in General, sub. is tedious

***We make coefficient of x or y same size, opp. sign.
We add the equations. (Why can we do this?)**

Example 4

Solve simultaneously by elimination:
$$\begin{cases} 4x + 3y = 2 \\ x - 3y = 8 \end{cases}$$

C**SOLUTION BY ELIMINATION****Example 4**

Solve simultaneously by elimination: $\begin{cases} 4x + 3y = 2 \\ x - 3y = 8 \end{cases}$

$$\begin{array}{r} 4x + 3y = 2 \\ x - 3y = 8 \\ \hline 5x = 10 \end{array}$$

$$\boxed{x = 2}$$

$$\begin{array}{r} 2 - 3y = 8 \\ -3y = 6 \\ \hline \boxed{y = -2} \end{array}$$

Solution $(2, -2)$

Example 4**Solution**

Solve simultaneously by elimination: $\begin{cases} 4x + 3y = 2 \\ x - 3y = 8 \end{cases}$

The coefficients of y are the same size but opposite in sign.

We **add** the LHSs and the RHSs to get an equation which contains x only.

$$4x + 3y = 2 \quad \dots (1)$$

$$x - 3y = 8 \quad \dots (2)$$

$$\text{Adding, } \begin{array}{r} 4x + 3y = 2 \\ x - 3y = 8 \\ \hline 5x = 10 \\ \therefore x = 2 \end{array}$$

$$\begin{array}{l} \text{Substituting } x = 2 \text{ into (1) gives } 4(2) + 3y = 2 \\ \therefore 8 + 3y = 2 \\ \therefore 3y = -6 \\ \therefore y = -2 \end{array}$$

The solution is $x = 2, y = -2$.

Check: In (2): $(2) - 3(-2) = 2 + 6 = 8 \quad \checkmark$

By adding the equations,
we **eliminate** y .



Joke Break :)

Parallel lines
have so much in
common...

it's a shame that
they'll never
meet.

PLAN
(P+L)(A+N)
PA+PN+LA+LN

Your plan has been
foiled

C

SOLUTION BY ELIMINATION

Example 5

Solve simultaneously by elimination: $\begin{cases} 3x + 2y = 7 & (-2) \\ 2x - 5y = 11 & (3) \end{cases}$

$$\begin{array}{r} -6x - 4y = -14 \\ 6x - 15y = 33 \\ \hline -19y = 19 \\ \boxed{y = -1} \end{array}$$

$$\begin{array}{r} 2x - 5(-1) = 11 \\ 2x + 5 = 11 \\ 2x = 6 \\ \boxed{x = 3} \end{array}$$

Solution (3, -1)

Find smallest integer interval to produce all 3 equations!

C
H
A
L
L
E
N
G
E :)

Linear Equation with one solution:

$$\frac{\square}{\square} X + \square = \square X + \frac{\square}{\square}$$

Linear Equation with no solutions:

$$\frac{\square}{\square} X + \square = \square X + \frac{\square}{\square}$$

Linear Equation with infinite solutions:

$$\frac{\square}{\square} X + \square = \square X + \frac{\square}{\square}$$

What is the smallest integer interval that could produce all three equations?

Answers can vary, an example is:

1 Solution: $(-3/-1)X + 0 = 1X + (-4/-2)$

No Solution: $(-2/-1)X + 1 = 2X + 0$

Infinite Solutions: $(-2/-1)X + 3 = 2X + (-9/-3)$

Interval of: 11

<http://www.openmiddle.com/one-solution-no-solutions-infinite-solutions/>

B

Chapter 19

SOLUTION BY SUBSTITUTION**#1 (choose 2), #2 (choose 2),
#3 (choose 2), #4, #5****EXERCISE 19B****1** Solve simultaneously by substitution:

a
$$\begin{cases} y = x - 3 \\ 2x + 3y = 26 \end{cases}$$

b
$$\begin{cases} y = 3x - 2 \\ 5x - 2y = 5 \end{cases}$$

c
$$\begin{cases} 7x + 4y = -7 \\ y = 8 - 5x \end{cases}$$

d
$$\begin{cases} y = 2x - 12 \\ y = 13 - 3x \end{cases}$$

e
$$\begin{cases} y = 3x + 4 \\ 5x + 3y = 5 \end{cases}$$

f
$$\begin{cases} 5x - 6y = 2 \\ y = 3 - 7x \end{cases}$$

2 Solve simultaneously by substitution:

a
$$\begin{cases} x = y + 9 \\ 5x + 2y = 10 \end{cases}$$

b
$$\begin{cases} 7x - 5y = 1 \\ x = 2y + 4 \end{cases}$$

c
$$\begin{cases} x = 2y - 7 \\ x = 13 - 6y \end{cases}$$

d
$$\begin{cases} y = 2x - 2 \\ x = 5y + 7 \end{cases}$$

e
$$\begin{cases} 2x + 4y = 1 \\ x = 10y - 7 \end{cases}$$

f
$$\begin{cases} x = -4y - 3 \\ 9x + 5y = 2 \end{cases}$$

B

Chapter 19

SOLUTION BY SUBSTITUTION**Exercises: 19B****3** Solve simultaneously by substitution:

a
$$\begin{cases} y = \frac{1}{2}x - 2 \\ 3x - 8y = 11 \end{cases}$$

b
$$\begin{cases} x = -\frac{2}{3}y \\ 6x + 7y = 6 \end{cases}$$

c
$$\begin{cases} 5x - 12y = 1 \\ y = \frac{1}{4}x - 1 \end{cases}$$

d
$$\begin{cases} y = -\frac{1}{2}x - 1 \\ 4x + 3y = -2 \end{cases}$$

e
$$\begin{cases} 2x - 3y = 3 \\ x = \frac{11}{5}y + 2 \end{cases}$$

f
$$\begin{cases} 14x + 15y = 1 \\ y = \frac{2}{5}x - 3 \end{cases}$$

4 a Try to solve by substitution:
$$\begin{cases} y = 3x + 1 \\ 9x - 3y = 5 \end{cases}$$
b What is the simultaneous solution for the equations in **a**?**5 a** Try to solve by substitution:
$$\begin{cases} y = 3x + 1 \\ 6x - 2y = -2 \end{cases}$$
b How many simultaneous solutions do the equations in **a** have?

C

Chapter 19

SOLUTION BY ELIMINATION

Section 19C: #1, #2 (choose 2), #4 or #5 (choose 2), #6

EXERCISE 19C

1 What equation results when the following are added vertically?

a $\begin{cases} 3x + 4y = 6 \\ 8x - 4y = 5 \end{cases}$

b $\begin{cases} 2x - y = 7 \\ -2x + 5y = 5 \end{cases}$

c $\begin{cases} 7x - 3y = 2 \\ 2x + 3y = 7 \end{cases}$

d $\begin{cases} 6x - 11y = 12 \\ 3x + 11y = -6 \end{cases}$

e $\begin{cases} -7x + 2y = 5 \\ 7x - 3y = 6 \end{cases}$

f $\begin{cases} 2x - 3y = -7 \\ -2x - 8y = -4 \end{cases}$

2 Solve simultaneously by elimination:

a $\begin{cases} 5x - y = 4 \\ 2x + y = 10 \end{cases}$

b $\begin{cases} 3x - 2y = 7 \\ 3x + 2y = -1 \end{cases}$

c $\begin{cases} -5x - 3y = 14 \\ 5x + 8y = -29 \end{cases}$

d $\begin{cases} 4x + 3y = -11 \\ -4x - 2y = 6 \end{cases}$

e $\begin{cases} 2x - 5y = 14 \\ 4x + 5y = -2 \end{cases}$

f $\begin{cases} -6x - y = 23 \\ 6x + 5y = -13 \end{cases}$

C

Chapter 19

SOLUTION BY ELIMINATION

4 Solve simultaneously by elimination:

a $\begin{cases} 2x + y = 8 \\ x - 3y = 11 \end{cases}$

b $\begin{cases} 3x + 2y = 7 \\ x + 3y = 7 \end{cases}$

c $\begin{cases} 5x - 2y = 17 \\ 3x - y = 9 \end{cases}$

d $\begin{cases} 2x + 5y = -14 \\ -6x + 2y = -9 \end{cases}$

e $\begin{cases} 7x + 3y = 5 \\ 5x - 6y = 9 \end{cases}$

f $\begin{cases} 4x + 9y = 24 \\ 12x - 7y = -30 \end{cases}$

5 Solve simultaneously by elimination:

a $\begin{cases} 2x + 3y = 13 \\ 3x + 2y = 17 \end{cases}$

b $\begin{cases} 4x - 3y = 1 \\ 2x + 5y = 7 \end{cases}$

c $\begin{cases} 2x + 5y = 14 \\ 5x - 3y + 27 = 0 \end{cases}$

d $\begin{cases} 7x + 2y = 20 \\ 13x + 3y = 34 \end{cases}$

e $\begin{cases} 3x + 7y = 5 \\ 5x + 11y = 10 \end{cases}$

f $\begin{cases} 5x - 7y - 9 = 0 \\ 4x - 5y - 5 = 0 \end{cases}$

6 Try to solve by elimination:

a $\begin{cases} 2x - y = 3 \\ 4x - 2y = 6 \end{cases}$

b $\begin{cases} 3x + 4y = 6 \\ 6x + 8y = 7 \end{cases}$

Comment on your results.

B

Chapter 19

SOLUTION BY SUBSTITUTION**Solutions**4 **b** no solutions5 **b** infinitely many solutions**EXERCISE 19B**

1 **a** $x = 7, y = 4$

c $x = 3, y = -7$

e $x = -\frac{1}{2}, y = \frac{5}{2}$

2 **a** $x = 4, y = -5$

c $x = -2, y = \frac{5}{2}$

e $x = -\frac{3}{4}, y = \frac{5}{8}$

3 **a** $x = 5, y = \frac{1}{2}$

c $x = 3, y = -\frac{1}{4}$

e $x = \frac{3}{7}, y = -\frac{5}{7}$

b $x = -1, y = -5$

d $x = 5, y = -2$

f $x = \frac{20}{47}, y = \frac{1}{47}$

b $x = -2, y = -3$

d $x = \frac{1}{3}, y = -\frac{4}{3}$

f $x = \frac{23}{31}, y = -\frac{29}{31}$

b $x = -\frac{4}{3}, y = 2$

d $x = \frac{2}{5}, y = -\frac{6}{5}$

f $x = \frac{7}{2}, y = -\frac{8}{5}$

C

Chapter 19

SOLUTION BY ELIMINATION**Solutions****EXERCISE 19C**

1 **a** $11x = 11$ **b** $4y = 12$ **c** $9x = 9$ **d** $9x = 6$

e $-y = 11$ **f** $-11y = -11$

2 **a** $x = 2, y = 6$

c $x = -1, y = -3$

e $x = 2, y = -2$

3 **a** $10x + 25y = 5$

c $3x - 21y = 24$

e $-18x - 12y = 12$

b $x = 1, y = -2$

d $x = 1, y = -5$

f $x = -\frac{17}{4}, y = \frac{5}{2}$

b $-3x + y = -4$

d $-10x - 8y = -18$

f $-16x + 8y = -12$

4 **a** $x = 5, y = -2$

c $x = 1, y = -6$

e $x = 1, y = -\frac{2}{3}$

5 **a** $x = 5, y = 1$

c $x = -3, y = 4$

e $x = \frac{15}{2}, y = -\frac{5}{2}$

b $x = 1, y = 2$

d $x = \frac{1}{2}, y = -3$

f $x = -\frac{3}{4}, y = 3$

b $x = 1, y = 1$

d $x = \frac{8}{5}, y = \frac{22}{5}$

f $x = -\frac{10}{3}, y = -\frac{11}{3}$

6 **a** infinitely many solutions, the lines are coincident**b** no solutions, the lines are parallel