## Welcome Back MYP Math 9!

Self-assess:

| | Assignment Effort Grade (Circle One) | Comments (What was interesting or challenging?) |
|---|---|---|
| Monday Date: 9/25 Topic: System Applications | 0 1 2 | I tried the 3 variable system! |
| Tuesday Date: _____ Topic: ___________ | 0 1 2 | |
| Wednesday Date: _____ Topic: ___________ | 0 1 2 | |
| Thursday Date: _____ Topic: ___________ | 0 1 2 | |
| Friday Date: _____ Topic: ___________ | 0 1 2 | |

## Warm-up: **How can we visualize the solution?**

**1)** Solve this system using elimination:

$$3x - 2y = 14$$
$$x + 3y = 1$$

**DONE? 2)** Solve this system using elimination:

$$2x + 3y - z = 5$$
$$-x - 3y + 4z = 2$$
$$x + 6y - 5z = 7$$

... a little tedious!!!

How can we avoid this mess?

## Warm-up:

**1)** Solve this system using elimination:

$$3x - 2y = 14$$
$$x + 3y = 1$$

$(x + 3y = 1)(-3)$

$3x - 2y = 14$

$-3x - 9y = -3$

$-11y = 11$

$y = -1$

$x + 3(-1) = 1$

$x - 3 = 1$

$x = 4$

## Warm-up:

**1)** Solve this system using elimination:

$$2x + 3y - z = 5$$
$$-x - 3y + 4z = 2$$
$$x + 6y - 5z = 7$$

$-2x - 6y + 8z = 4$

$-x + 3z = 11$

$x + 3z = 7$

$-(-x + 3z = 11)$

$6z = 18$

$z = 3$

... a little tedious!!!

How can we avoid this mess?

## Class Plan

1. Warm-up
2. Mathematician Mondays!
3. What is a matrix? 


... and why would we use them?

4. Exercises, practice

$$\left[\begin{array}{cc:c} 1 & 0 & h \\ 0 & 1 & k \end{array}\right] \quad \text{or} \quad \left[\begin{array}{ccc:c} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \end{array}\right]$$

## Dr. Isabel Alicia Hubard Escalera

**Pronounced as: "**

As a child, Isabel Alicia Hubbard wanted to be a bullfighter. She has said of her family, "My mother is an engineer and my father an accountant. My brother wanted to become a mathematician and my sister a physicist. I never thought that I would like math. I simply found it easy and fun, but nothing more. However, my mathematics teacher in junior high and high school, Óscar Chávez, inspired me."

She was the organiser of the 2015 Mexico City Mathematics Olympiad of the Federal District, an organization that played a prominent role in recent national competitions, achieving the second place medal of the 2015 Olimpiada Nacional de Matemáticas para Alumnos de Primaria y Secundaria competition.[6] She is also a delegate for Mexico City in the Mexican Mathematics Olympiad of the Mexican Mathematics Society

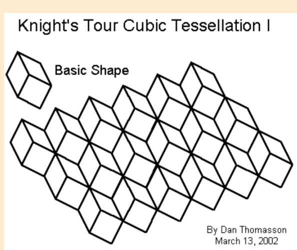
# Dr. Isabel Alicia Hubard Escalera

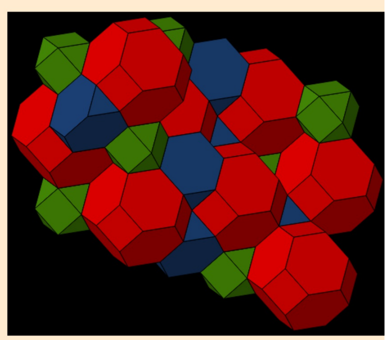
Hubard Escalera began her studies in the Faculty of Sciences of the UNAM, where in 2001, she graduated in Mathematics with a baccalaureate thesis titled *Polyhedra colored with cyclic orders*

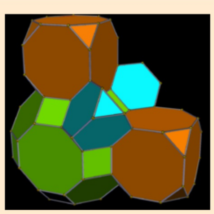
(Top Univeristy of Mexico)

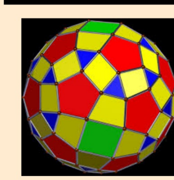
In 2008, she earned a Ph.D. from York University of Canada, with a dissertation titled *From geometry to groups and back: the study of highly symmetric polytopes*

In 2012 she was the first Mexican mathematician to receive the L´Oréal-UNESCO-AMC Fellowship in the area of Exact Sciences for her work, titled *Algebra, combinatorics and geometry of abstract two-orbit polytopes.*[2][5] The Fellowship is awarded to **"promote the participation of women in science for advanced scientific studies in universities or other recognized Mexican institutions in the areas of exact sciences, natural sciences and engineering and technology."**







# Which Method is Best?

(To solve the system of equations)

① 
$$y = 2x - 8$$
$$y = 5x - 14$$

2 in Grad-Int Form

Substution

② 
$$y = 2x - 5$$
$$-x - 4y = 2$$

1 in Grad-Int Form
1 in Standard Form

Subst....

③ 
$$4x + 7y = -2$$
$$x + 6y = 8$$

2 in Standard Form

Elimination

④ Matrices!

$$-5x + 3y + 6z = 4$$
$$-3x + y + 5z = -5$$
$$-4x + 2y + z = 13$$

3 Equations...!

~~Elim~~

# Augmented Matrix:

A matrix with a column for the coefficients of each variable and a final column for the constant terms.

**Record the example in notes**

$$3x - 2y = 14$$
$$x + 3y = 1$$

→

x y = c

$$\left[\begin{array}{cc|c} 3 & -2 & 14 \\ 1 & 3 & 1 \end{array}\right]$$

## Row Reduction Method:

Transforms an augmented matrix into a solution matrix. **Why do we need 1s and 0s?**

Instead of combing equations to get to one variable...

**You add multiples of rows to other rows until you obtain the solution matrix.**

$$\begin{cases} 1x + 0y = a \\ 0x + 1y = b \end{cases}, \text{ or } x = a \text{ and } y = b.$$

**The "end" goal for all 2 x 2 matrices!**

The solution

$$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$$

1's along the diagonal

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -5 \end{array}\right] \quad x = 2, \; y = -5$$

$1x + 0y = a$

$0x + 1y = b$

## Row Operations

An augmented matrix represents a system of equations, so the same rules apply to row operations in a matrix as to equations in a system of equations.

**Row Operations in a Matrix**

- You can multiply (or divide) all numbers in a row by a nonzero number.
- You can add all numbers in a row to corresponding numbers in another row.
- You can add a multiple of the numbers in one row to the corresponding numbers in another row.
- You can exchange two rows.

**Record in your notes:**

**1) Multiply row by a nonzero number.**

**2) Add rows.**

**3) Exchange two rows.**

**Example:** Record system in your notes and write the augmented matrix.

$$3x - 2y = 14$$
$$x + 3y = 1$$

$\left[\begin{array}{cc|c} 3 & -2 & 14 \\ 1 & 3 & 1 \end{array}\right]$

Goal (x y = constant): $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ $x = a$, $y = b$

$R_2 \leftrightarrow R_1$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 3 & -2 & 14 \end{array}\right]$ $-3R_1 + R_2$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -11 & 11 \end{array}\right]$

$-\frac{1}{11}R_2 \rightarrow$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -1 \end{array}\right]$ $-3R_2 + R_1 \rightarrow$ $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$

$y = -1$ $x = 4$

**Example:** WITH ALL ROW OPERATIONS!

$$3x - 2y = 14$$
$$x + 3y = 1$$

$\left[\begin{array}{cc|c} 3 & -2 & 14 \\ 1 & 3 & 1 \end{array}\right]$

Goal (x y = constant): $\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$ $x = a$, $y = b$

$R_2 \leftrightarrow R_1$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 3 & -2 & 14 \end{array}\right]$ $-3R_1$ $\left[\begin{array}{cc|c} -3 & -9 & -3 \\ 3 & -2 & 14 \end{array}\right]$

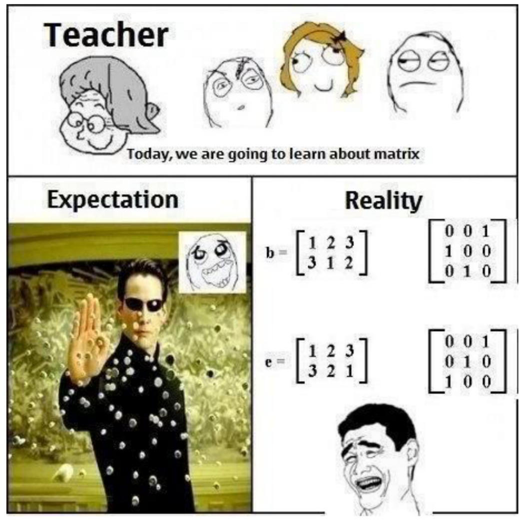
$R_1 + R_2 \rightarrow R_2$ $\left[\begin{array}{cc|c} -3 & -9 & -3 \\ 0 & -11 & 11 \end{array}\right]$ $-\frac{1}{11}R_2 \rightarrow R_2$ $\left[\begin{array}{cc|c} -3 & -9 & -3 \\ 0 & 1 & -1 \end{array}\right]$

$-\frac{1}{3}R_1 \rightarrow R_1$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -1 \end{array}\right]$ $-3R_2 \rightarrow R_2$ $\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -3 & 3 \end{array}\right]$ $R_1 + R_2 \rightarrow R_1$ $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & -3 & 3 \end{array}\right]$

$-\frac{1}{3}R_2 \rightarrow R_2$ $\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$

$X = 4$
$Y = -1$

**Joke Break :)**




Example: Record system in your notes and write the augmented matrix.

$$-2x + y = -3$$
$$x - 4y = -2$$

$\left[\begin{array}{cc|c} -2 & 1 & -3 \\ 1 & -4 & -2 \end{array}\right] \xrightarrow{R_2 \Leftrightarrow R_1} \left[\begin{array}{cc|c} 1 & -4 & -2 \\ -2 & 1 & -3 \end{array}\right]$

$\xrightarrow{2R_1 + R_2} \left[\begin{array}{cc|c} 1 & -4 & -2 \\ 0 & -7 & -7 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{cc|c} 1 & -4 & -2 \\ 0 & 1 & 1 \end{array}\right]$

$\xrightarrow{4R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$ $\quad x = 2, \; y = 1$

$\left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right]$

$(2, 1)$

**Example:** Record system in your notes and write the augmented matrix.

$$3x - 6y = -9$$
$$-2x + -2y = 12$$

$$\left[\begin{array}{cc|c} 3 & -6 & -9 \\ -2 & -2 & 12 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ -2 & -2 & 12 \end{array}\right]$$

$$-\tfrac{1}{2}R_2 \rightarrow \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 1 & 1 & -6 \end{array}\right] \xrightarrow{-1R_1+R_2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 3 & -3 \end{array}\right]$$

$$\tfrac{1}{3}R_2 \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow{2R_2+R_1} \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & -1 \end{array}\right]$$

$y = -1$

$x = -5$

$(-5, -1)$

Another worked out example...

$$-x - 3y = -30$$
$$-6x + y = -9$$

$$\left[\begin{array}{cc|c} -1 & -3 & -30 \\ -6 & 1 & -9 \end{array}\right] \xrightarrow{(3)R_2+R_1} \left[\begin{array}{cc|c} -19 & 0 & -57 \\ -6 & 1 & -9 \end{array}\right]$$

$$\xrightarrow{\frac{-6}{19}R_1+R_2} \left[\begin{array}{cc|c} -19 & 0 & -57 \\ 0 & 1 & -27 \end{array}\right] \xrightarrow{\frac{-1}{19}R_1} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 9 \end{array}\right]$$

## Another worked out example...

**3X - 2Y = 3**
**X + 2Y = 17**

$$\left[\begin{array}{cc|c} 3 & -2 & 3 \\ 1 & 2 & 17 \end{array}\right] \xrightarrow{R_2 \longleftrightarrow R_1} \left[\begin{array}{cc|c} 1 & 2 & 17 \\ 3 & -2 & 3 \end{array}\right]$$

$$\xrightarrow{-3R_1 + R_2} \left[\begin{array}{cc|c} 1 & 2 & 17 \\ 0 & -8 & -48 \end{array}\right] \xrightarrow[\frac{-1}{8}R_2]{\frac{1}{4}R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 6 \end{array}\right]$$

∴ **X = 5**
**Y = 6**

## Exercises: Systems Matrices Worksheet

*(Do your best on #5!)*

**Solve each system using matrices and row operations.**

1) $x - y = -1$
$-3x - y = 15$

2) $2x - 7y = -24$
$2x + 3y = -4$

3) $5x - 8y = 16$
$-9x + 2y = -4$

4) $4x - 8y = 17$
$2x - 4y = 10$

5) $-2x - 4y - 2z = -2$
$6x + y + 2z = 11$
$-x + y - 4z = 14$

afterschool
W125
OR W101

## Solutions

1) $(-4, -3)$ 2) $(-5, 2)$ 3) $(0, -2)$ 4) No solution
5) $(3, 1, -4)$

1) $x - y = -1$
$-3x - y = 15$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ -3 & -1 & 15 \end{array}\right] \quad (3)R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & -4 & 12 \end{array}\right] \quad -\frac{1}{4}R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 1 & -3 \end{array}\right]$$

$$R_1 + R_2 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & -3 \end{array}\right] \quad \begin{array}{l} x = -4 \\ y = -3 \end{array}$$

$$\boxed{(-4, -3)}$$

2) $2x - 7y = -24$
$2x + 3y = -4$

$$\left[\begin{array}{cc|c} 2 & -7 & -24 \\ 2 & 3 & -4 \end{array}\right] \quad (-1)R_2 + R_1 \rightarrow R_2$$

$$\left[\begin{array}{cc|c} 2 & -7 & -24 \\ 0 & -10 & -20 \end{array}\right] \quad -\frac{1}{10}R_2 \rightarrow R_2 \quad \left[\begin{array}{cc|c} 2 & -7 & -24 \\ 0 & 1 & 2 \end{array}\right]$$

$$7R_2 + R_1 \rightarrow R_1 \quad \left[\begin{array}{cc|c} 2 & 0 & -10 \\ 0 & 1 & 2 \end{array}\right] \quad \frac{1}{2}R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} x = -5 \\ y = 2 \end{array} \quad \boxed{(-5, 2)}$$

3) $5x - 8y = 16$
$\quad -9x + 2y = -4$

$$\left[\begin{array}{cc|c} 5 & -8 & 16 \\ -9 & 2 & -4 \end{array}\right] \quad 4R_2 + R_1 \to R_2$$

$$\left[\begin{array}{cc|c} 5 & -8 & 16 \\ -40 & 0 & 0 \end{array}\right] \quad -\frac{1}{40}R_2 \to R_2 \quad \left[\begin{array}{cc|c} 5 & -8 & 16 \\ 1 & 0 & 0 \end{array}\right]$$

$$R_1 \leftrightarrow R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 5 & -8 & 16 \end{array}\right] \quad (-5)R_1 + R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -8 & 16 \end{array}\right] \quad \frac{1}{-8}R_2 \to R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right]$$

$x = 0$
$y = -2$

$(0, -2)$

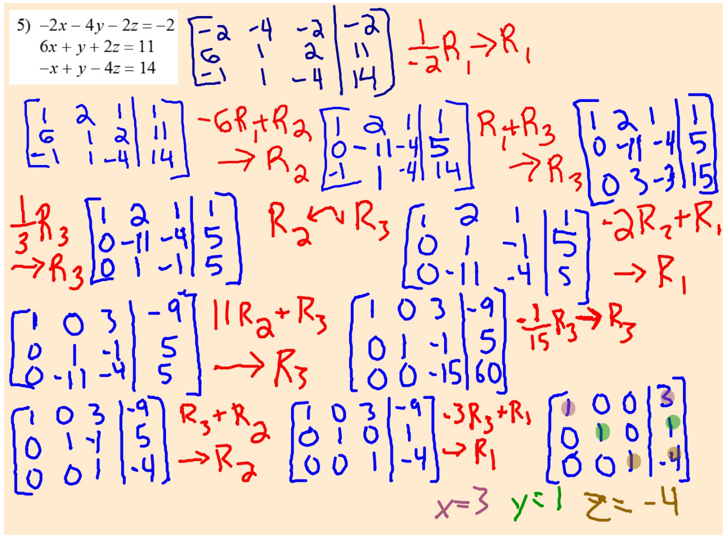
4) $4x - 8y = 17$
$\quad 2x - 4y = 10$

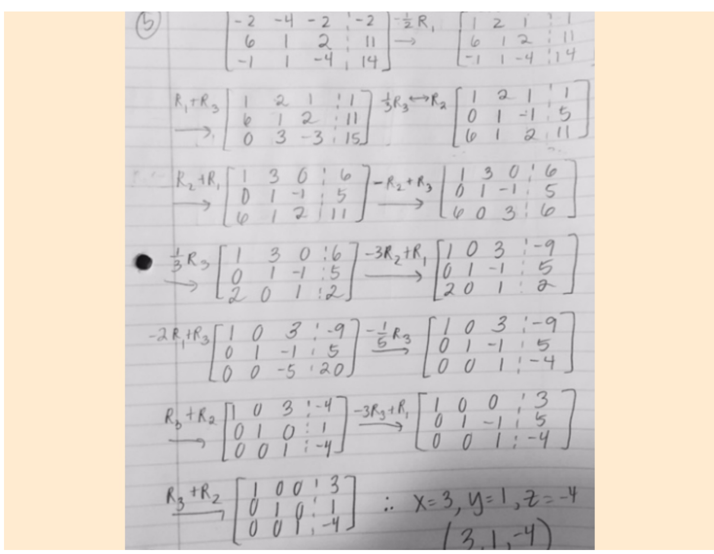
$$\left[\begin{array}{cc|c} 4 & -8 & 17 \\ 2 & -4 & 10 \end{array}\right] \quad -2R_2 + R_1 \to R_2$$

$$\left[\begin{array}{cc|c} 4 & -8 & 17 \\ 0 & 0 & -3 \end{array}\right]$$

This shows there is no y-value solution. Parallel lines

$0y = -3$ (Impossible!)


5) −2x − 4y − 2z = −2
6x + y + 2z = 11
−x + y − 4z = 14
x=3 y=1 z=-4


∴ x=3, y=1, z=-4
(3,1,-4)

**ADDITIONAL....System of Equations with 3 variables:**

**Augmented Matrix:**

$$-5x + 3y + 6z = 4$$
$$-3x + y + 5z = -5$$
$$-4x + 2y + z = 13$$

$$\left[\begin{array}{ccc|c} -5 & 3 & 6 & 4 \\ -3 & 1 & 5 & -5 \\ -4 & 2 & 1 & 13 \end{array}\right]$$

**Augmented Matrix:**

$$\left[\begin{array}{ccc|c} -5 & 3 & 6 & 4 \\ -3 & 1 & 5 & -5 \\ -4 & 2 & 1 & 13 \end{array}\right]$$

---------Row Reduction Steps --------

**Goal / Solution:**

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & r \\ 0 & 1 & 0 & s \\ 0 & 0 & 1 & t \end{array}\right]$$

Read:

X = r
Y = s
Z = t

$$\left[\begin{array}{ccc|c} -5 & 3 & 6 & 4 \\ -3 & 1 & 5 & -5 \\ -4 & 2 & 1 & 13 \end{array}\right] \xrightarrow[-2R_2 + R_3]{-3R_2 + R_1} \left[\begin{array}{ccc|c} 4 & 0 & -9 & 19 \\ -3 & 1 & 5 & -5 \\ 2 & 0 & -9 & 23 \end{array}\right]$$

$$\xrightarrow{-1R_3 + R_1} \left[\begin{array}{ccc|c} 2 & 0 & 0 & -4 \\ -3 & 1 & 5 & -5 \\ 2 & 0 & -9 & 23 \end{array}\right] \xrightarrow[\frac{3}{2}R_1 + R_2]{-1R_1 + R_3} \left[\begin{array}{ccc|c} 2 & 0 & 0 & -4 \\ 0 & 1 & 5 & -11 \\ 0 & 0 & -9 & 27 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -4 \\ 0 & 1 & 5 & -11 \\ 0 & 0 & -9 & 27 \end{array}\right] \xrightarrow[\frac{1}{2}R_1]{\frac{5}{9}R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -9 & 27 \end{array}\right]$$

$$\xrightarrow{\frac{-1}{9}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{array}\right]$$

**therefore,**

**X = -2**

**Y = 4**

**Z = -3**