

Welcome Back MYP Math 9!

Self-assess:

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: <u>9/25</u> Topic: <u>System Applications</u>	0 1 2	I tried the 3 variable system!
Tuesday Date: <u>9/26</u> Topic: <u>Matrices!</u>	0 1 2	I tried the 3 variable system!
Wednesday Date: _____ Topic: _____	0 1 2	
Thursday Date: _____ Topic: _____	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Exercises: Systems Matrices Worksheet (Do your best on #5!)

Solve each system using matrices and row operations.

$$1) \begin{cases} x - y = -1 \\ -3x - y = 15 \end{cases} \quad \text{Yesterday's homework!} \quad 2) \begin{cases} 2x - 7y = -24 \\ 2x + 3y = -4 \end{cases}$$

$$3) \begin{cases} 5x - 8y = 16 \\ -9x + 2y = -4 \end{cases} \quad 4) \begin{cases} 4x - 8y = 17 \\ 2x - 4y = 10 \end{cases}$$

$$5) \begin{cases} -2x - 4y - 2z = -2 \\ 6x + y + 2z = 11 \\ -x + y - 4z = 14 \end{cases}$$

Solutions

$$1) (-4, -3) \quad 2) (-5, 2) \quad 3) (0, -2) \quad 4) \text{No solution}$$

$$5) (3, 1, -4)$$

$$1) \begin{cases} x - y = -1 \\ -3x - y = 15 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ -3 & -1 & 15 \end{array} \right] \begin{array}{l} (3)R_1 + R_2 \\ \rightarrow R_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & -4 & 12 \end{array} \right] \begin{array}{l} -\frac{1}{4}R_2 \rightarrow R_2 \\ \rightarrow R_2 \end{array}$$

$$R_1 + R_2 \rightarrow R_1 \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & -3 \end{array} \right] \begin{array}{l} x = -4 \\ y = -3 \end{array}$$

$$\boxed{(-4, -3)}$$

$$1) \begin{cases} x - y = -1 \\ -3x - y = 15 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -1 & -1 \\ -3 & -1 & 15 \end{array} \right] \begin{array}{l} -R_2 \cdot R_1 \\ \rightarrow R_2 \end{array}$$

$$\left[\begin{array}{cc|c} 4 & 0 & -16 \\ -3 & -1 & 15 \end{array} \right] \begin{array}{l} \frac{1}{4}R_1 \\ \rightarrow R_1 \end{array} \left[\begin{array}{cc|c} 1 & 0 & -4 \\ -3 & -1 & 15 \end{array} \right]$$

$$2) \begin{cases} 2x - 7y = -24 \\ 2x + 3y = -4 \end{cases} \begin{bmatrix} 2 & -7 & -24 \\ 2 & 3 & -4 \end{bmatrix} (-1)R_2 + R_1 \rightarrow R_2$$

$$\begin{bmatrix} 2 & -7 & -24 \\ 0 & -10 & -20 \end{bmatrix} \begin{matrix} -\frac{1}{10}R_2 \rightarrow R_2 \\ \end{matrix} \begin{bmatrix} 2 & -7 & -24 \\ 0 & 1 & 2 \end{bmatrix}$$

$$7R_2 + R_1 \rightarrow R_1 \begin{bmatrix} 2 & 0 & -10 \\ 0 & 1 & 2 \end{bmatrix} \frac{1}{2}R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} x = -5 \\ y = 2 \end{matrix} \quad \boxed{(-5, 2)}$$

$$3) \begin{cases} 5x - 8y = 16 \\ -9x + 2y = -4 \end{cases} \begin{bmatrix} 5 & -8 & 16 \\ -9 & 2 & -4 \end{bmatrix} 4R_2 + R_1 \rightarrow R_2$$

$$\begin{bmatrix} 5 & -8 & 16 \\ -40 & 0 & 0 \end{bmatrix} \begin{matrix} -\frac{1}{40}R_2 \rightarrow R_2 \\ \end{matrix} \begin{bmatrix} 5 & -8 & 16 \\ 1 & 0 & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & 0 \\ 5 & -8 & 16 \end{bmatrix} (-5)R_1 + R_2 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -8 & 16 \end{bmatrix} \begin{matrix} -\frac{1}{8}R_2 \rightarrow R_2 \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix} \begin{matrix} x = 0 \\ y = -2 \end{matrix}$$

$$\boxed{(0, -2)}$$

$$4) \begin{cases} 4x - 8y = 17 \\ 2x - 4y = 10 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -8 & 17 \\ 2 & -4 & 10 \end{array} \right] \xrightarrow{-2R_2 + R_1 \rightarrow R_2}$$

$$\left[\begin{array}{cc|c} 4 & -8 & 17 \\ 0 & 0 & -3 \end{array} \right]$$

← This shows there is no y-value solution. Parallel lines

$$\begin{aligned} &\rightarrow 4x - 8y = 17 \\ &\rightarrow 0x + 0y = -3 \text{ (Impossible!)} \\ &\quad 0x + 0y = -3 \end{aligned}$$

$$5) \begin{cases} -2x - 4y - 2z = -2 \\ 6x + y + 2z = 11 \\ -x + y - 4z = 14 \end{cases}$$

$$\left[\begin{array}{ccc|c} -2 & -4 & -2 & -2 \\ 6 & 1 & 2 & 11 \\ -1 & 1 & -4 & 14 \end{array} \right] \xrightarrow{-\frac{1}{2}R_1 \rightarrow R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 6 & 1 & 2 & 11 \\ -1 & 1 & -4 & 14 \end{array} \right] \xrightarrow{\begin{array}{l} -6R_1 + R_2 \rightarrow R_2 \\ \rightarrow R_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -11 & -4 & 5 \\ -1 & 1 & -4 & 14 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 + R_3 \\ \rightarrow R_3 \end{array}}$$

$$\xrightarrow{\begin{array}{l} \frac{1}{3}R_3 \\ \rightarrow R_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -11 & -4 & 5 \\ 0 & 1 & -1 & 5 \end{array} \right] \xrightarrow{\begin{array}{l} R_2 \leftrightarrow R_3 \\ \rightarrow R_1 \end{array}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -9 \\ 0 & 1 & -1 & 5 \\ 0 & -11 & -4 & 5 \end{array} \right] \xrightarrow{\begin{array}{l} 11R_2 + R_3 \\ \rightarrow R_3 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & -9 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & -15 & 60 \end{array} \right] \xrightarrow{-\frac{1}{15}R_3 \rightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & -9 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 1 & -4 \end{array} \right] \xrightarrow{\begin{array}{l} R_3 + R_2 \\ \rightarrow R_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & 3 & -9 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \end{array} \right] \xrightarrow{\begin{array}{l} -3R_3 + R_1 \\ \rightarrow R_1 \end{array}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -4 \end{array} \right] \quad x=3 \quad y=1 \quad z=-4$$

Warm-up:

Solve this system using matrices:

$$\begin{array}{l} x = 6 \\ x + 3y = 3 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 1 & 3 & 3 \end{array} \right] \begin{array}{l} -R_1 + R_2 \\ \rightarrow R_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 3 & -3 \end{array} \right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & -1 \end{array} \right] \begin{array}{l} \\ (6, -1) \end{array}$$

What is our goal?

Warm-up:

Solve this system using matrices:

$$\begin{array}{l} x = 6 \\ x + 3y = 3 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 1 & 3 & 3 \end{array} \right] \begin{array}{l} -R_1 + R_2 \\ \rightarrow R_2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 3 & -3 \end{array} \right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & -1 \end{array} \right] \begin{array}{l} x=6 \\ y=-1 \\ (6, -1) \end{array}$$

What is our goal?
$$\left[\begin{array}{cc|c} x & y & c \\ 1 & 0 & a \\ 0 & 1 & b \end{array} \right]$$

Class Plan

1. Warm-up

2. Day 2: Matrix
(One additional example)

3. Exercises, practice



$$\left[\begin{array}{cc|c} 1 & 0 & h \\ 0 & 1 & k \end{array} \right] \quad \text{or} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \end{array} \right]$$

Tuesday: calculator solution!

One additional example:

$$4) \begin{cases} -5y + 21 + 7x = 0 \\ -2x = -12 - 4y \end{cases}$$

$$7x - 5y = -21$$

$$2x - 4y = 12$$

$$\left[\begin{array}{cc|c} 7 & -5 & -21 \\ 2 & -4 & 12 \end{array} \right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 7 & -5 & -21 \\ 1 & -2 & 6 \end{array} \right]$$

$$R_1 \leftrightarrow R_2 \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 7 & -5 & -21 \end{array} \right] \xrightarrow{\begin{array}{l} -7R_1+R_2 \\ \rightarrow R_2 \end{array}} \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 9 & -63 \end{array} \right]$$

$$\xrightarrow{\frac{1}{9}R_2} \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 1 & -7 \end{array} \right] \xrightarrow{\begin{array}{l} 2R_2+R_1 \\ \rightarrow R_1 \end{array}} \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -7 \end{array} \right]$$

$$\boxed{(-8, -7)}$$

One additional example:

$$4) \begin{cases} -5y + 21 + 7x = 0 \\ -2x = -12 - 4y \end{cases} \quad \begin{cases} 7x - 5y = -21 \\ -2x + 4y = -12 \end{cases}$$

$$\left[\begin{array}{cc|c} 7 & -5 & -21 \\ -2 & 4 & -12 \end{array} \right] \xrightarrow[\rightarrow R_1]{3R_2 + R_1} \left[\begin{array}{cc|c} 1 & 7 & -57 \\ -2 & 4 & -12 \end{array} \right]$$

$$\xrightarrow[\rightarrow R_2]{2R_1 + R_2} \left[\begin{array}{cc|c} 1 & 7 & -57 \\ 0 & 18 & -126 \end{array} \right] \xrightarrow[\rightarrow R_2]{\frac{1}{18}R_2} \left[\begin{array}{cc|c} 1 & 7 & -57 \\ 0 & 1 & -7 \end{array} \right]$$

$$\xrightarrow[\rightarrow R_1]{-7R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -7 \end{array} \right] \quad (-8, -7)$$

One additional example:

$$4) \begin{cases} -5y + 21 + 7x = 0 \\ -2x = -12 - 4y \end{cases} \quad \begin{cases} 7x - 5y = -21 \\ -2x + 4y = -12 \end{cases}$$

$$\left[\begin{array}{cc|c} 7 & -5 & -21 \\ -2 & 4 & -12 \end{array} \right] \xrightarrow[\rightarrow R_2]{-\frac{1}{2}R_2} \left[\begin{array}{cc|c} 7 & -5 & -21 \\ 1 & -2 & 6 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 7 & -5 & -21 \end{array} \right]$$

$$\xrightarrow[\rightarrow R_2]{-7R_1 + R_2} \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 9 & -63 \end{array} \right] \xrightarrow[\rightarrow R_2]{\frac{1}{9}R_2} \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 1 & -7 \end{array} \right] \xrightarrow[\rightarrow R_1]{2R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -7 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & -7 \end{array} \right] \quad \begin{cases} x = -8 \\ y = -7 \end{cases} \quad (-8, -7)$$

Additional example using calculator:

4) $-5y + 21 + 7x = 0$
 $-2x = -12 - 4y$

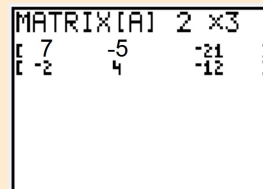
$7x - 5y = -21$
 $-2x + 4y = -12$

Enter the matrix!

1) 2nd, x^{-1} , EDIT, ENTER



2) 2 X 3, Enter equation values



Additional example using calculator:

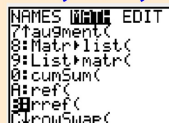
4) $-5y + 21 + 7x = 0$
 $-2x = -12 - 4y$

$7x - 5y = -21$
 $-2x + 4y = -12$

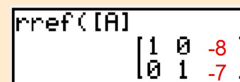
Enter the matrix!

3) 2nd, MODE (go to main screen)

4) 2nd, x^{-1} , MATH, "rref("



5) 2nd, x^{-1} , ENTER



**Joke
Break :)**



Exercises: Systems Matrices Worksheet

1) $6x - y = 14$
 $-6x + 2y = -10$

2) $-6x + 8y = 2$
 $3x + 7y = 10$

3) $-2x + y = -4$
 $12x - 3y = 30$

4) $-5y + 21 + 7x = 0$
 $-2x = -12 - 4y$

5) $-x - 2y + z = 2$
 $4x + y - z = -4$
 $-x - 2y + 4z = 20$

6) $-4x - 5z = -3$
 $-5x - y - z = 9$
 $-2x - 6y + z = -9$

Answers to 9-26 Extended Level Homework

1) (3, 4)
5) (0, 2, 6)

2) (1, 1)
6) (-3, 3, 3)

3) (3, 2)

4) (-8, -7)

$$1) \begin{cases} 6x - y = 14 \\ -6x + 2y = -10 \end{cases}$$

$$\begin{bmatrix} 6 & -1 & | & 14 \\ -6 & 2 & | & -10 \end{bmatrix} \begin{array}{l} R_1 + R_2 \\ \rightarrow R_2 \end{array} \begin{bmatrix} 6 & -1 & | & 14 \\ 0 & 1 & | & 4 \end{bmatrix} \begin{array}{l} R_2 + R_1 \\ \rightarrow R_1 \end{array}$$

$$\begin{bmatrix} 6 & 0 & | & 18 \\ 0 & 1 & | & 4 \end{bmatrix} \begin{array}{l} \frac{1}{6} R_1 \\ \rightarrow R_1 \end{array} \begin{bmatrix} 1 & 0 & | & 3 \\ 0 & 1 & | & 4 \end{bmatrix}$$

$$(3, 4)$$

$$2) \begin{cases} -6x + 8y = 2 \\ 3x + 7y = 10 \end{cases}$$

$$\begin{bmatrix} -6 & 8 & | & 2 \\ 3 & 7 & | & 10 \end{bmatrix} \begin{array}{l} -\frac{1}{2} R_1 \\ \rightarrow R_1 \end{array} \begin{bmatrix} 3 & -4 & | & -1 \\ 3 & 7 & | & 10 \end{bmatrix} \begin{array}{l} -R_1 + R_2 \\ \rightarrow R_2 \end{array}$$

$$\begin{bmatrix} 3 & -4 & | & -1 \\ 0 & 11 & | & 11 \end{bmatrix} \begin{array}{l} \frac{1}{11} R_2 \\ \rightarrow R_2 \end{array} \begin{bmatrix} 3 & -4 & | & -1 \\ 0 & 1 & | & 1 \end{bmatrix} \begin{array}{l} 4R_2 + R_1 \\ \rightarrow R_1 \end{array}$$

$$\begin{bmatrix} 3 & 0 & | & 3 \\ 0 & 1 & | & 1 \end{bmatrix} \begin{array}{l} \frac{1}{3} R_1 \\ \rightarrow R_1 \end{array} \begin{bmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 1 \end{bmatrix} (1, 1)$$

3) $-2x + y = -4$
 $12x - 3y = 30$

$$\begin{bmatrix} -2 & 1 & -4 \\ 12 & -3 & 30 \end{bmatrix} \xrightarrow[6R_1 + R_2]{\rightarrow R_2} \begin{bmatrix} -2 & 1 & -4 \\ 0 & 3 & 6 \end{bmatrix} \xrightarrow[\frac{1}{3}R_2]{\rightarrow R_2}$$

$$\begin{bmatrix} -2 & 1 & -4 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[-R_2 + R_1]{\rightarrow R_1} \begin{bmatrix} -2 & 0 & -6 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow[\frac{-1}{2}R_1]{\rightarrow R_1}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \quad \boxed{(3, 2)}$$

4) $-5y + 21 + 7x = 0$
 $-2x = -12 - 4y$

Let's get it
matrix-ready

$$7x - 5y = -21$$

$$-2x + 4y = -12$$

$$\begin{bmatrix} 7 & -5 & -21 \\ -2 & 4 & -12 \end{bmatrix} \xrightarrow[-\frac{1}{2}R_2]{\rightarrow R_2} \begin{bmatrix} 7 & -5 & -21 \\ 1 & -2 & 6 \end{bmatrix} \xrightarrow[R_1 \leftrightarrow R_2]{} \begin{bmatrix} 1 & -2 & 6 \\ 7 & -5 & -21 \end{bmatrix}$$

$$\xrightarrow[-7R_1 + R_2]{\rightarrow R_2} \begin{bmatrix} 1 & -2 & 6 \\ 0 & 9 & -63 \end{bmatrix} \xrightarrow[\frac{1}{9}R_2]{\rightarrow R_2} \begin{bmatrix} 1 & -2 & 6 \\ 0 & 1 & -7 \end{bmatrix} \xrightarrow[2R_2 + R_1]{\rightarrow R_1}$$

$$\begin{bmatrix} 1 & 0 & -8 \\ 0 & 1 & -7 \end{bmatrix} \quad \begin{matrix} x = -8 \\ y = -7 \end{matrix} \quad (-8, -7)$$

$$\begin{aligned} 5) \quad & -x - 2y + z = 2 \\ & 4x + y - z = -4 \\ & -x - 2y + 4z = 20 \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} -1 & -2 & 1 & | & 2 \\ 4 & 1 & -1 & | & -4 \\ -1 & -2 & 4 & | & 20 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 4 & 1 & -1 & | & -4 \\ -1 & -2 & 4 & | & 20 \end{bmatrix} \xrightarrow{-4R_1 + R_2} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & -7 & 3 & | & 4 \\ -1 & -2 & 4 & | & 20 \end{bmatrix} \xrightarrow{-R_1 + R_3} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & -7 & 3 & | & 4 \\ 0 & -4 & 5 & | & 22 \end{bmatrix} \xrightarrow{-4R_2 + 7R_3} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & -7 & 3 & | & 4 \\ 0 & 0 & 23 & | & 138 \end{bmatrix} \xrightarrow{\frac{1}{23}R_3} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & -7 & 3 & | & 4 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-3R_3 + R_2} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & -7 & 0 & | & -14 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-\frac{1}{7}R_2} \begin{bmatrix} 1 & 2 & -1 & | & -2 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-R_2 + R_1} \begin{bmatrix} 1 & 1 & -1 & | & -4 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 1 & 0 & -1 & | & -6 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 & | & -8 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-R_1 + R_2} \begin{bmatrix} 0 & 0 & 0 & | & -10 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 6 \end{bmatrix} \quad \boxed{(0, 2, 6)} \end{aligned}$$

$$\begin{aligned} 6) \quad & -4x - 5z = -3 \\ & -5x - y - z = 9 \\ & -2x - 6y + z = -9 \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} -4 & 0 & -5 & | & -3 \\ -5 & -1 & -1 & | & 9 \\ -2 & -6 & 1 & | & -9 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ -5 & -1 & -1 & | & 9 \\ -4 & 0 & -5 & | & -3 \end{bmatrix} \xrightarrow{-2R_1 + R_3} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ -5 & -1 & -1 & | & 9 \\ 0 & 12 & -7 & | & 15 \end{bmatrix} \xrightarrow{-5R_1 + 2R_2} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 28 & -7 & | & 63 \\ 0 & 12 & -7 & | & 15 \end{bmatrix} \xrightarrow{\frac{1}{7}R_2} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 4 & -1 & | & 9 \\ 0 & 12 & -7 & | & 15 \end{bmatrix} \xrightarrow{-3R_2 + R_3} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 4 & -1 & | & 9 \\ 0 & 0 & -4 & | & -12 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_3} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 4 & -1 & | & 9 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 4 & 0 & | & 12 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{\frac{1}{4}R_2} \begin{bmatrix} -2 & -6 & 1 & | & -9 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-R_3 + R_1} \begin{bmatrix} -2 & -6 & 0 & | & -12 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 2 & 6 & 0 & | & 12 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-\frac{1}{2}R_1} \begin{bmatrix} 1 & 3 & 0 & | & 6 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-3R_2 + R_1} \begin{bmatrix} 1 & 0 & 0 & | & 3 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{-R_1} \begin{bmatrix} 0 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & 3 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \quad \boxed{(-3, 3, 3)} \end{aligned}$$