Welcome Back MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 10/9 Topic: No HW - Unit 1 Test Friday	0 1 2	
Tuesday Date: 10/10 Topic: 26A, 26B	0 1 2	I love vectors!
Wednesday Date: ______ Topic: ____________	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

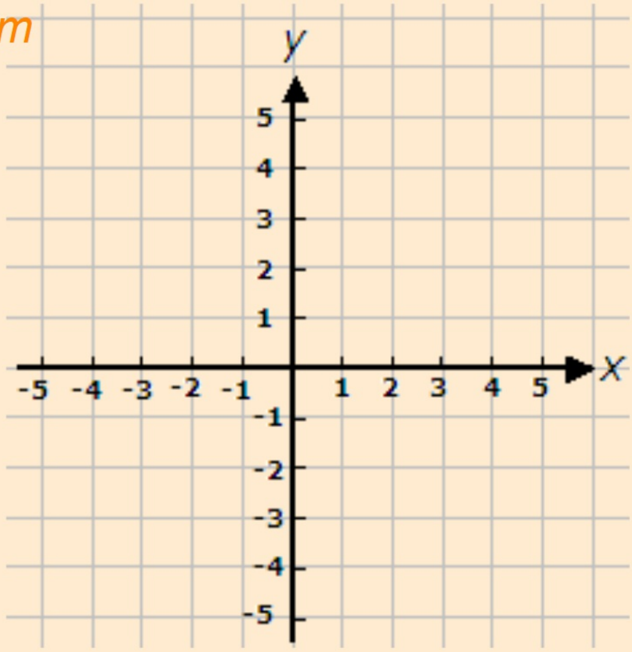
1. Warm-up
2. 26C Equal Vectors
26D Vector Addition
3. Joke!
4. Real-life example
5. Practice

Warm-up:

Find the vector from (2,3) to (5,5).

Find the vector from (1,-1) to (4,1).

Write the vectors in *component form*



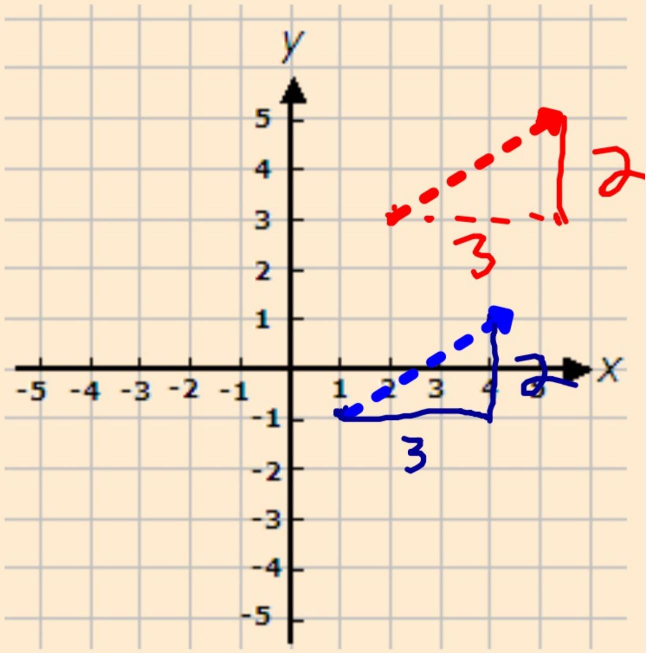
Warm-up:

Find the vector from (2,3) to (5,5).

Find the vector from (1,-1) to (4,1).

$$\begin{pmatrix} 5-2 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 4-1 \\ 1--1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



C EQUAL VECTORS

Two vectors are called equal if they have the same magnitude and direction.

Alternatively...

Two vectors are **equal** if they have the same x and y-components.

If $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c \\ d \end{pmatrix}$, then $a = c$ and $b = d$.

Example: Is the vector from $\left(\frac{1}{2}, 3\right)$ to $(3, 5)$ equal to the vector from $\left(\frac{17}{8}, 0\right)$ to $\left(\frac{35}{8}, 2\right)$?

$$\begin{pmatrix} 2\frac{1}{2} \\ 2 \end{pmatrix} \text{ vs. } \begin{pmatrix} \frac{18}{8} = 2\frac{1}{4} \\ 2 \end{pmatrix}$$

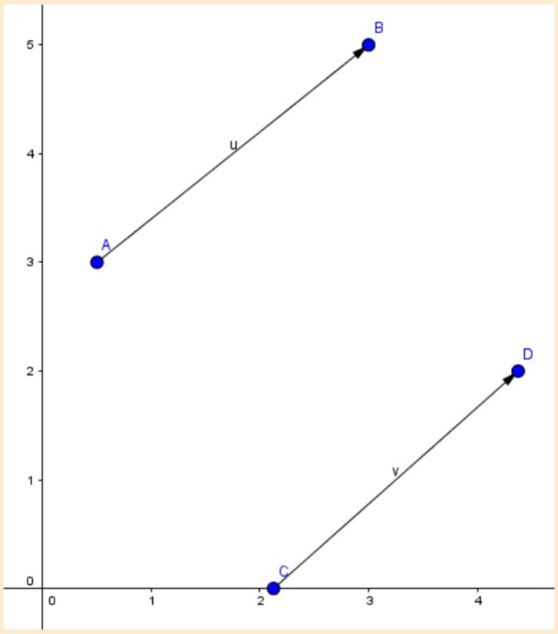
$$(3, 5) - \left(\frac{1}{2}, 3\right) = \left(\frac{5}{2}, 2\right)$$

$$\left(\frac{35}{8}, 2\right) - \left(\frac{17}{8}, 0\right) = \left(\frac{9}{4}, 2\right)$$

Not Equivalent

$$\left|\vec{u}\right| = \sqrt{\left(3-\tfrac{1}{2}\right)^2 + (5-3)^2}$$
$$= \sqrt{\frac{41}{4}}$$

$$\left|\vec{v}\right| = \sqrt{\left(\tfrac{35}{8}-\tfrac{17}{8}\right)^2 + (2-0)^2}$$
$$= \sqrt{\frac{145}{16}} = \frac{\sqrt{145}}{4}$$



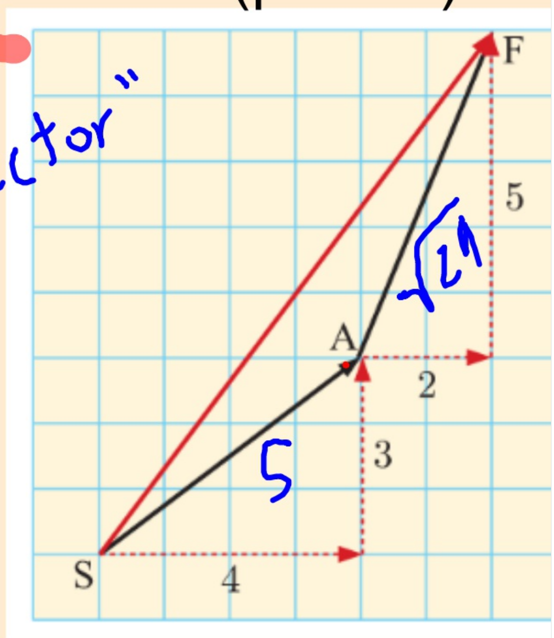
Different magnitudes, so not equal.

D VECTOR ADDITION

Suppose Victor walks from Southwest (point S) to Agri Culture (point A) for a healthy meal. Then they walk to their friend's house (point F).

1) Write the ~~displacement vector~~ "vector" from S to A, and from A to F.

$SA = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $AF = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$



2) Think like a Physicist, what is the total distance that Victor walked?

$5 + \sqrt{29}$

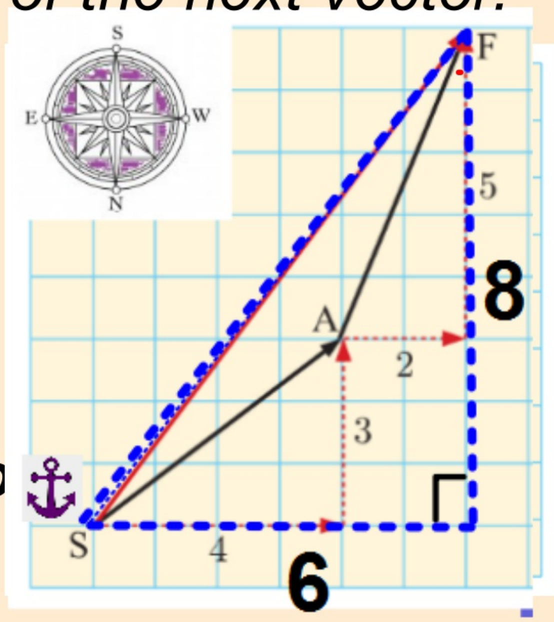
D Real-life ⇔ Vocab — VECTOR ADDITION

1) *"Total Distance Traveled'* **(14 blocks)**

Shows the Head-to-tail addition.

Adding arrowhead to the tail of the next vector.

$$\begin{pmatrix}4\\3\end{pmatrix}+\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}6\\8\end{pmatrix}$$



2) Distance & direction from SWHS to house?

(10 blocks SW)

The ***Magnitude*** and ***directio***

Found by $6^2 + 8^2 = 10^2$.

Trigonometry is used when given angles.

D Algebraically

VECTOR ADDITION

If $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ then $\mathbf{a} + \mathbf{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$.

Example 4 Complete the addition at your tables:

If $\mathbf{a} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$, find:

a $\mathbf{a} + \mathbf{b}$ **b** $\mathbf{b} + \mathbf{c}$ **c** $2\mathbf{a} + 3\mathbf{c}$

a+b $= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

b+c $= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

2a+3c $= \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

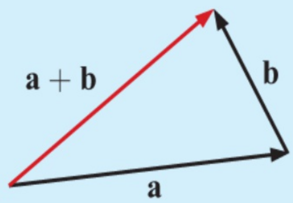
D Geometrically — VECTOR ADDITION

To find $\mathbf{a} + \mathbf{b}$ geometrically:

Step 1: Draw vector **a**.

Step 2: At the arrow end of **a**, draw vector **b**.

Step 3: Draw a vector from the start of **a** to the end of **b**. The resultant vector is $\mathbf{a} + \mathbf{b}$.

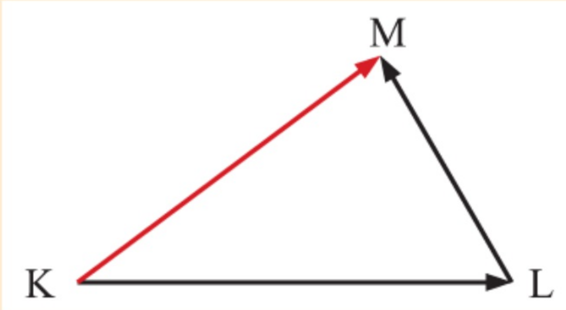


DEMO

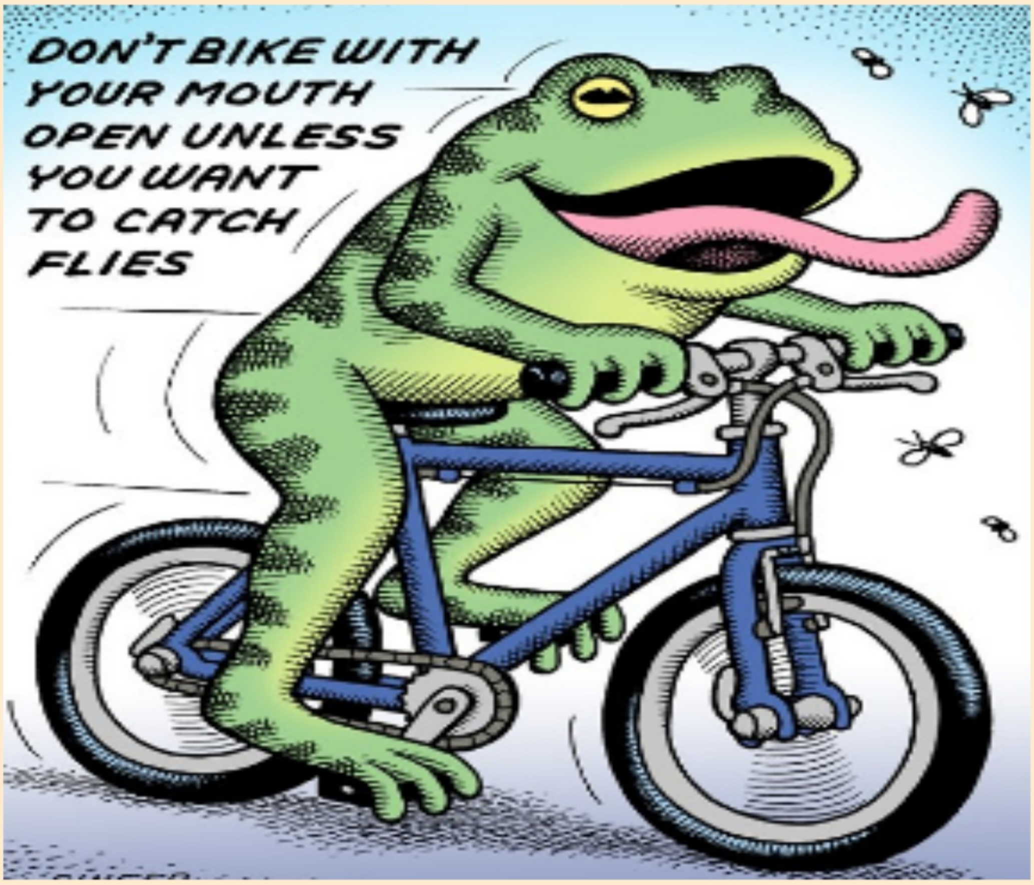
Example:

What is the resultant vector shown in ΔKLM? Write the vector equation.

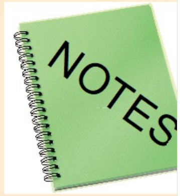
$$\overrightarrow{KL} + \overrightarrow{LM} = \overrightarrow{KM}$$



D Joke break! VECTOR ADDITION



D **Real-life practice** VECTOR ADDITION



You ride your bike 0.5 km north and then turn left, traveling west for 1.1 km.

a) What is the total distance you rode your bike? Displacement?

b) Draw a detailed diagram to represent your ride.

c) How can we determine the angle at which you are traveling from the north?

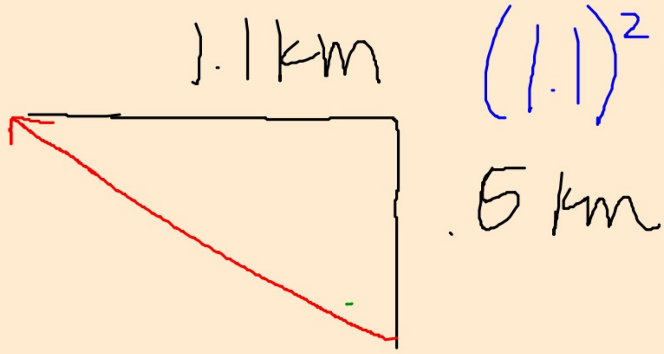
Example: You ride your bike 0.5 km north and then turn left, traveling west for 1.1 km

*Draw a detailed diagram

What is the total distance you rode your bike?

What was your displacement?

$1.1 + .5 = 1.6$ km



$(1.1)^2 + (.5)^2 = d^2$

$1.21 + .25 = d^2$

$\sqrt{1.46} = \sqrt{d^2}$

$1.2\text{ km} \approx d$

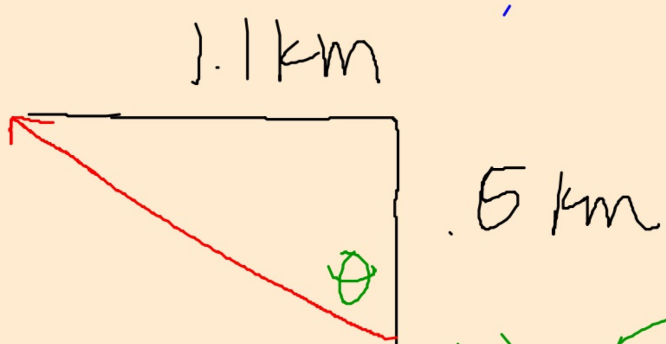
Extension: How can we determine the angle at which you are traveling from the north?

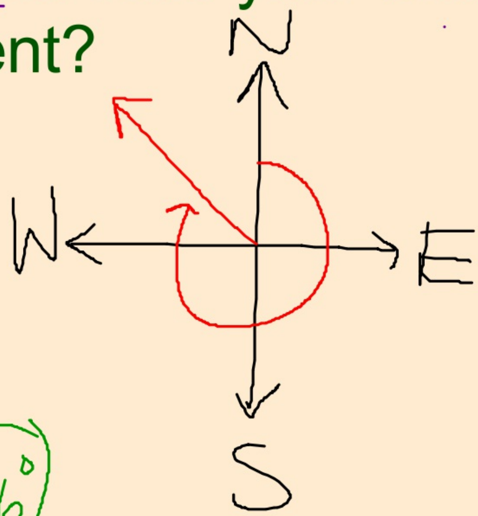
Example: You ride your bike 0.5 km north and then turn left, traveling west for 1.1 km

*Draw a detailed diagram

What is the total distance you rode your bike?

What was your displacement?





$\tan^{-1}\left(\frac{1.1}{.5}\right) \approx 65.6^\circ$

Extension: How can we determine the angle at which you are traveling from the north?

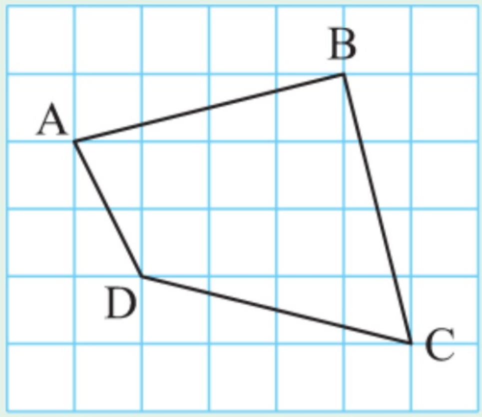
D Additional examples — VECTOR ADDITION

a Find in component form:

i $\overrightarrow{BC}$ **ii** $\overrightarrow{BD}$

b Simplify $\overrightarrow{AD} + \overrightarrow{DC}$.

c Find $|\overrightarrow{AC}|$.



8 **a** **i** $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ **ii** $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$ **b** $\overrightarrow{AC}$ **c** $\sqrt{34}$ units

D Additional examples

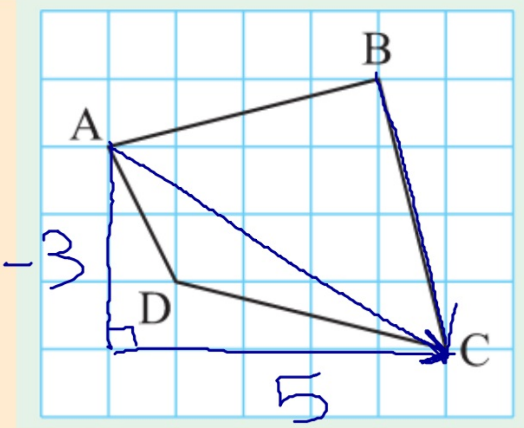
VECTOR ADDITION

a Find in component form:

i $\overrightarrow{BC}$ **ii** $\overrightarrow{BD}$

b Simplify $\overrightarrow{AD} + \overrightarrow{DC}$.

c Find $|\overrightarrow{AC}|$.



$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$ -3

$\overrightarrow{BC} = \begin{pmatrix}1\\-4\end{pmatrix}$

$(-3)^2 + (5)^2 = \overrightarrow{AC}^2$

$\sqrt{34} = \overrightarrow{AC}$

8 **a** **i** $\begin{pmatrix}1\\-4\end{pmatrix}$ **ii** $\begin{pmatrix}-3\\-3\end{pmatrix}$ **b** $\overrightarrow{AC}$ **c** $\sqrt{34}$ units

Exercises...

26C Evens, 26D Odds

- **If you need more practice... you can always do the entire set!**

Assigned Practice

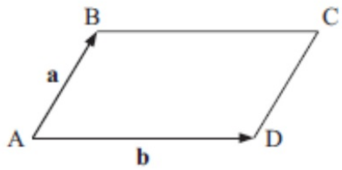
EXERCISE 26C

1 Explain why $\begin{pmatrix} 5 \\ 2 \end{pmatrix} \neq \begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

2 What can be deduced if $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$?

3 Find a and b such that $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^2 \\ -b \end{pmatrix}$.

4



ABCD is a parallelogram with $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$. State, with reasons, vector expressions for $\overrightarrow{DC}$ and $\overrightarrow{BC}$.

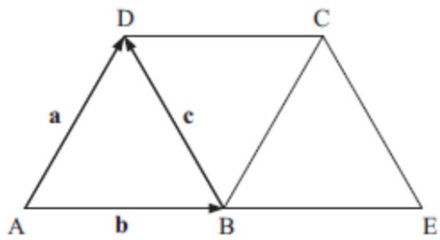
5 Triangles ABD, BEC, and BCD are equilateral.
Suppose $\overrightarrow{AD} = \mathbf{a}$, $\overrightarrow{AB} = \mathbf{b}$, and $\overrightarrow{BD} = \mathbf{c}$.

a Find, in terms of vectors **a**, **b**, and **c**, vectors representing:

i $\overrightarrow{BC}$ **ii** $\overrightarrow{BE}$ **iii** $\overrightarrow{DC}$ **iv** $\overrightarrow{EC}$

b Explain why $\mathbf{a} \neq \mathbf{b}$.

c Does $|\mathbf{a}| = |\mathbf{b}|$?



Assigned Practice

26D: ODD#'s

1 Find:

a $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ **c** $\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

d $\begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ **e** $\begin{pmatrix} -7 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ **f** $\begin{pmatrix} -6 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

g $\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **h** $\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

2 If $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$, find:

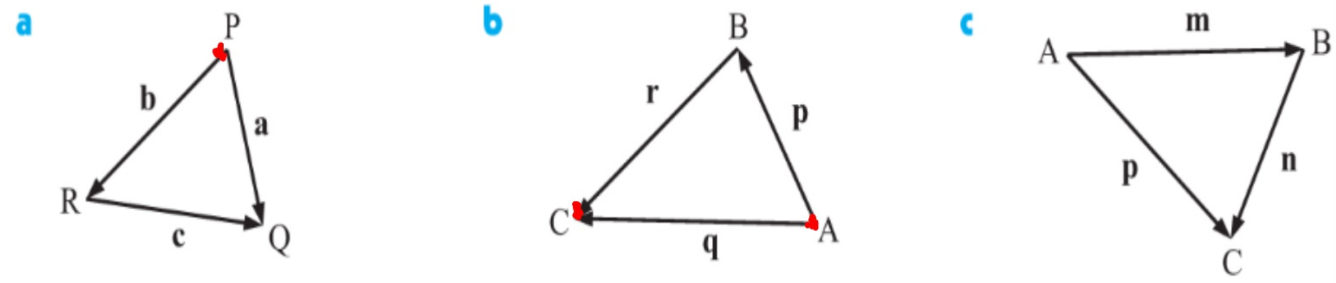
a $\mathbf{a} + \mathbf{b}$ **b** $\mathbf{b} + \mathbf{a}$ **c** $\mathbf{b} + \mathbf{c}$ **d** $\mathbf{c} + \mathbf{a}$

e $\mathbf{a} + \mathbf{a}$ **f** $\mathbf{b} + \mathbf{b}$ **g** $\mathbf{c} + \mathbf{c} + \mathbf{c}$ **h** $\mathbf{a} + \mathbf{b} + \mathbf{c}$

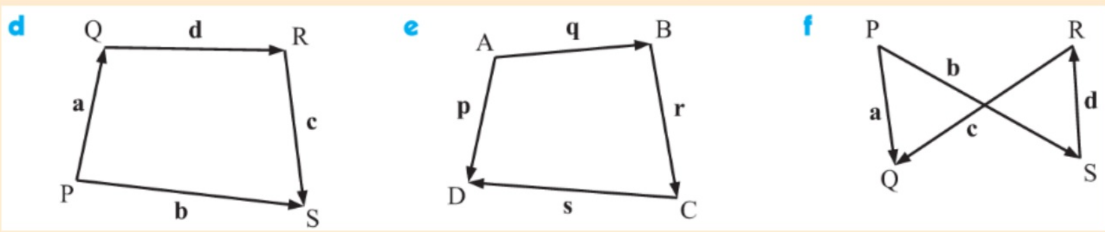
Assigned Practice

26D: ODD#'s

Write a vector equation to connect the vectors in:



$\vec{b} + \vec{c} = \vec{a}$ $\vec{p} + \vec{r} = \vec{q}$

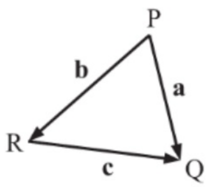


Assigned Practice

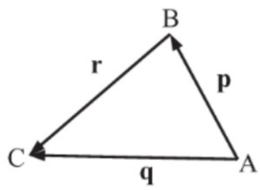
26D: ODD#'s

3 Write a vector equation to connect the vectors in:

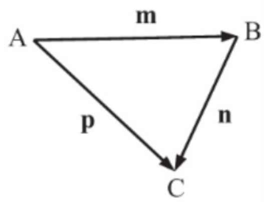
a



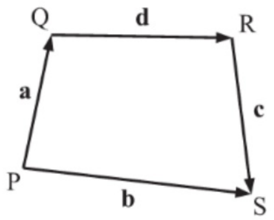
b



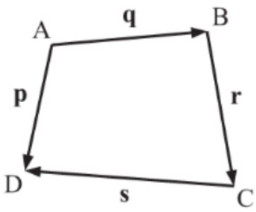
c



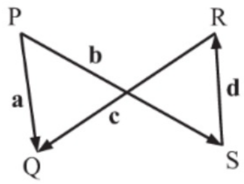
d



e



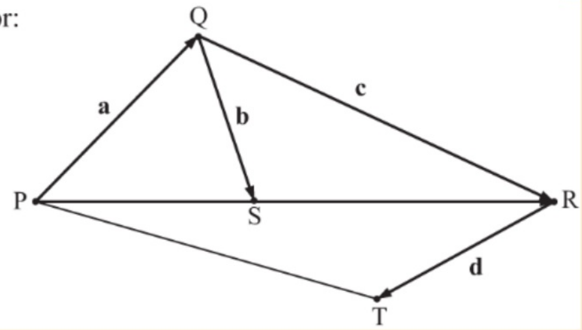
f



4 Write an expression in terms of **a**, **b**, **c**, and **d**, for:

a $\overrightarrow{PS}$ **b** $\overrightarrow{PR}$

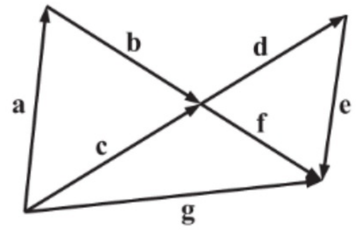
c $\overrightarrow{QT}$ **d** $\overrightarrow{PT}$



Assigned Practice

26D: ODD#'s

5

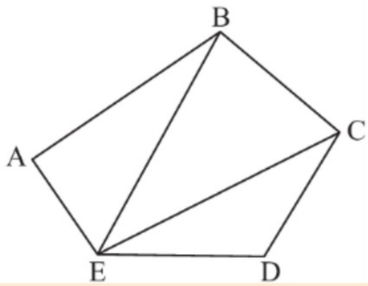


Simplify:

- **a** $\mathbf{a} + \mathbf{b}$
- **b** $\mathbf{d} + \mathbf{e}$
- **c** $\mathbf{c} + \mathbf{f}$
- **d** $\mathbf{a} + \mathbf{b} + \mathbf{f}$
- **e** $\mathbf{c} + \mathbf{d} + \mathbf{e}$

6 Simplify:

- **a** $\overrightarrow{AB} + \overrightarrow{BE}$
- **b** $\overrightarrow{BC} + \overrightarrow{CE}$
- **c** $\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$
- **d** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$



Assigned Practice

26D: ODD#'s

7 Simplify:

a $\overrightarrow{AP} + \overrightarrow{PB}$

b $\overrightarrow{PX} + \overrightarrow{XY} + \overrightarrow{YQ}$

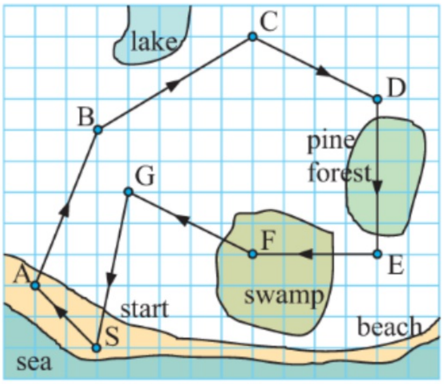
c $\overrightarrow{LM} + \overrightarrow{MN} + \overrightarrow{ND}$

d $\overrightarrow{SP} + \overrightarrow{PQ} + \overrightarrow{QN}$

e $\overrightarrow{EF} + \overrightarrow{FD} + \overrightarrow{DQ} + \overrightarrow{QR}$

f $\overrightarrow{CX} + \overrightarrow{XN} + \overrightarrow{ND} + \overrightarrow{DP}$

9



The diagram alongside shows an orienteering course run by Kahu.

a Write a column vector to describe each leg of the course.

b Find the sum of all of the vectors.

c What does the sum in **b** tell us?

Assigned Practice

26D: ODD#'s

8 Alongside is a hole at Hackers Golf Club.

a Jack tees off from T and his ball finishes at A. Write a vector to describe the displacement of the ball from T to A.

b Jack plays his second stroke from A to B. Write a vector to describe the displacement of the ball from this shot.

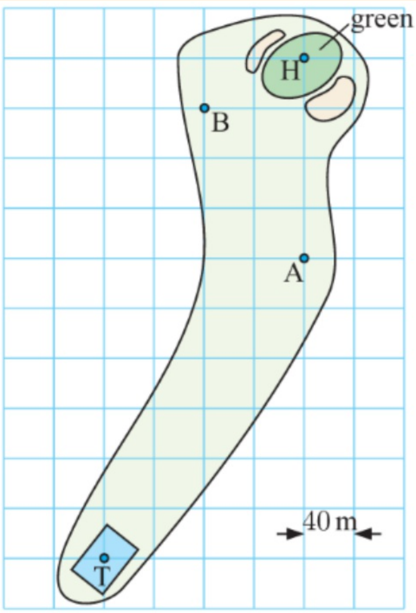
c By great luck, Jack's next shot finishes in the hole H. Write a vector which describes this shot.

d Find:

i $\overrightarrow{TA} + \overrightarrow{AB} + \overrightarrow{BH}$ **ii** $\overrightarrow{TH}$

Comment on your answers.

e Find the straight line distance between the tee T and the hole H.



Assigned Practice

26D: ODD#'s;

8 Alongside is a hole at Hackers Golf Club.

a Jack tees off from T and his ball finishes at A. Write a vector to describe the displacement of the ball from T to A.

b Jack plays his second stroke from A to B. Write a vector to describe the displacement of the ball from this shot.

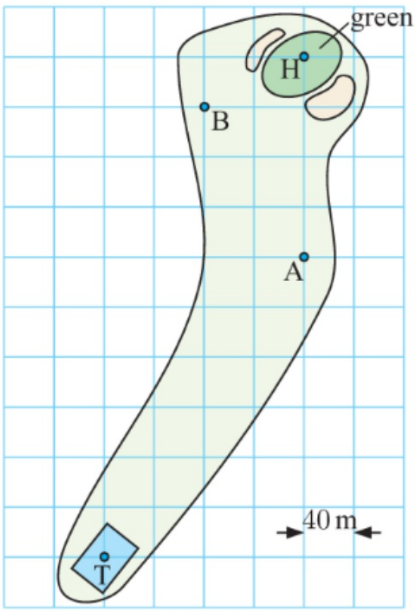
c By great luck, Jack's next shot finishes in the hole H. Write a vector which describes this shot.

d Find:

i $\overrightarrow{TA} + \overrightarrow{AB} + \overrightarrow{BH}$ **ii** $\overrightarrow{TH}$

Comment on your answers.

e Find the straight line distance between the tee T and the hole H.



Assigned Practice SOLUTIONS

26D: ODD#'s

EXERCISE 26D

1 a $\begin{pmatrix}6\\10\end{pmatrix}$ b $\begin{pmatrix}11\\11\end{pmatrix}$ c $\begin{pmatrix}8\\2\end{pmatrix}$ d $\begin{pmatrix}4\\-3\end{pmatrix}$
e $\begin{pmatrix}-3\\-3\end{pmatrix}$ f $\begin{pmatrix}-8\\-2\end{pmatrix}$ g $\begin{pmatrix}6\\7\end{pmatrix}$ h $\begin{pmatrix}5\\-2\end{pmatrix}$

2 a $\begin{pmatrix}7\\-1\end{pmatrix}$ b $\begin{pmatrix}7\\-1\end{pmatrix}$ c $\begin{pmatrix}-2\\-2\end{pmatrix}$ d $\begin{pmatrix}1\\3\end{pmatrix}$
e $\begin{pmatrix}10\\4\end{pmatrix}$ f $\begin{pmatrix}4\\-6\end{pmatrix}$ g $\begin{pmatrix}-12\\3\end{pmatrix}$ h $\begin{pmatrix}3\\0\end{pmatrix}$

3 a $\mathbf{a} = \mathbf{b} + \mathbf{c}$ b $\mathbf{q} = \mathbf{p} + \mathbf{r}$ c $\mathbf{p} = \mathbf{m} + \mathbf{n}$
d $\mathbf{b} = \mathbf{a} + \mathbf{d} + \mathbf{c}$ e $\mathbf{p} = \mathbf{q} + \mathbf{r} + \mathbf{s}$ f $\mathbf{a} = \mathbf{b} + \mathbf{d} + \mathbf{c}$

4 a $\mathbf{a} + \mathbf{b}$ b $\mathbf{a} + \mathbf{c}$ c $\mathbf{c} + \mathbf{d}$ d $\mathbf{a} + \mathbf{c} + \mathbf{d}$

5 a $\mathbf{c}$ b $\mathbf{f}$ c $\mathbf{g}$ d $\mathbf{g}$ e $\mathbf{g}$

6 a $\overrightarrow{AE}$ b $\overrightarrow{BE}$ c $\overrightarrow{BE}$ d $\overrightarrow{AE}$

7 a $\overrightarrow{AB}$ b $\overrightarrow{PQ}$ c $\overrightarrow{LD}$ d $\overrightarrow{SN}$ e $\overrightarrow{ER}$ f $\overrightarrow{CP}$

8 a $\begin{pmatrix}160\\240\end{pmatrix}$ b $\begin{pmatrix}-80\\120\end{pmatrix}$ c $\begin{pmatrix}80\\40\end{pmatrix}$
d i $\begin{pmatrix}160\\400\end{pmatrix}$ ii $\begin{pmatrix}160\\400\end{pmatrix}$ e ≈ 431 m
$\overrightarrow{TA} + \overrightarrow{AB} + \overrightarrow{BH} = \overrightarrow{TH}$

9 a $\overrightarrow{SA} = \begin{pmatrix}-2\\2\end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix}2\\5\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}5\\3\end{pmatrix}$,
$\overrightarrow{CD} = \begin{pmatrix}4\\-2\end{pmatrix}$, $\overrightarrow{DE} = \begin{pmatrix}0\\-5\end{pmatrix}$, $\overrightarrow{EF} = \begin{pmatrix}-4\\0\end{pmatrix}$,
$\overrightarrow{FG} = \begin{pmatrix}-4\\2\end{pmatrix}$, $\overrightarrow{GS} = \begin{pmatrix}-1\\-5\end{pmatrix}$

3-d challenge!

Read:

MYP 9 Math Extended

(**3D Vector Practice**)

Note: Vectors are noted in ordered triples using brackets.
For example: **v = <x, y, z>**

EXAMPLE 5 **Using Vectors to Determine Collinear Points**

Determine whether the following points lie on the same line.

$P(2, -1, 4)$, $Q(5, 4, 6)$, and $R(-4, -11, 0)$

Solution

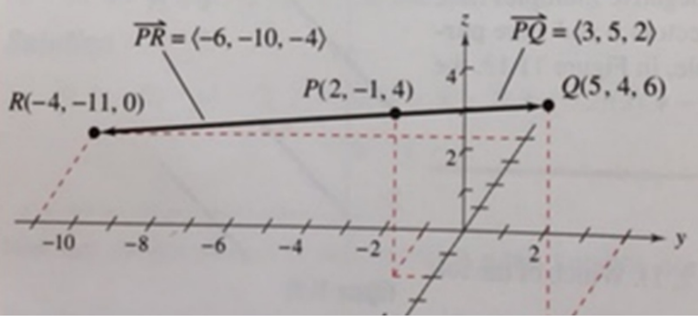
The component forms of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ are

$$\overrightarrow{PQ} = \langle 5 - 2, 4 - (-1), 6 - 4 \rangle = \langle 3, 5, 2 \rangle$$

and

$$\overrightarrow{PR} = \langle -4 - 2, -11 - (-1), 0 - 4 \rangle = \langle -6, -10, -4 \rangle.$$

Because $\overrightarrow{PR} = -2\overrightarrow{PQ}$, you can conclude that they are parallel. Therefore, the points P, Q, and R lie on the same line, as shown in Figure 11.14.



3-d challenge!

EXERCISES: (Refer to example 5 above)

1) Determine whether the following are collinear points.

a) P(5, 4, 1), Q(7, 3, -1), R(4, 5, 3)

b) A(1, 3, 2), B(-1, 2, 5), C(3, 4, -1)

3-d challenge!

Read:

EXAMPLE 6 Finding the Terminal Point of a Vector

The initial point of the vector $\mathbf{v} = \langle 4, 2, -1 \rangle$ is $P(3, -1, 6)$. What is the terminal point of this vector?

Solution

Using the component form of the vector whose initial point is P and whose terminal point is Q, you can write

$$\begin{aligned} \overrightarrow{PQ} &= \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle \\ &= \langle q_1 - 3, q_2 + 1, q_3 - 6 \rangle \\ &= \langle 4, 2, -1 \rangle. \end{aligned}$$

This implies that $q_1 - 3 = 4$, $q_2 + 1 = 2$, and $q_3 - 6 = -1$. The solutions of these three equations are

$$q_1 = 7, \qquad q_2 = 1, \qquad \text{and} \qquad q_3 = 5.$$

So, the terminal point is $Q(7, 1, 5)$.

3-d challenge!

Solutions!

2) (Refer to example 6) Using the initial point of **v** and the terminal point of **v:**

1) Write the component form of the vector **v.**

2) Find the length of **v.**

a) Initial point: (-1, -2, 1) Terminal point: (3, 2, 5)

b) Initial: (-4, 5, 5) Terminal: (4, 0, 0)

a) $\langle 5-1, 2--2, 3--1 \rangle$

$\vec{v} = \langle 4, 4, 4 \rangle$

$|\vec{v}| = \sqrt{16+16+16}$

$|\vec{v}| = \sqrt{48} = 4\sqrt{3}$

b) $\langle 0--4, 0-5, 0-5 \rangle$

$\vec{v} = \langle 4, -5, -5 \rangle$

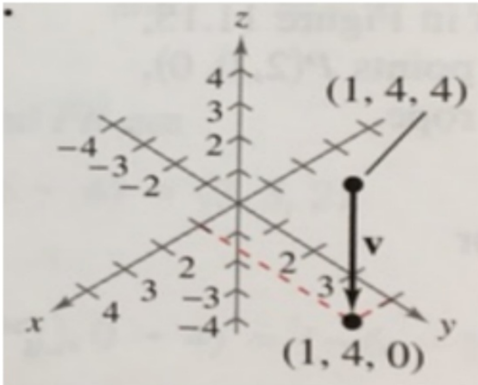
$|\vec{v}| = \sqrt{16+25+25}$

$|\vec{v}| = \sqrt{66}$

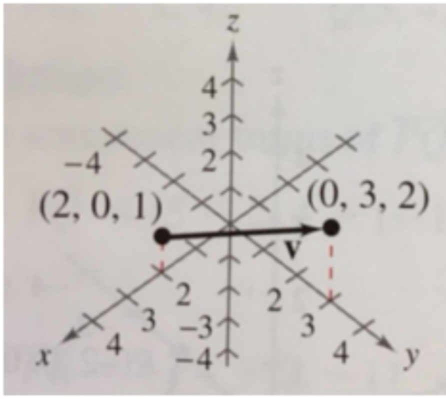
3-d challenge!

3) Find component form of vector **v**. Sketch the vector with its initial point at the origin.

a)



b)

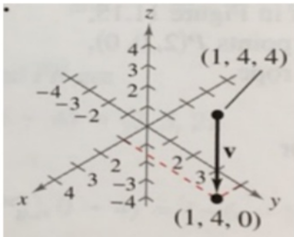


3-d challenge!

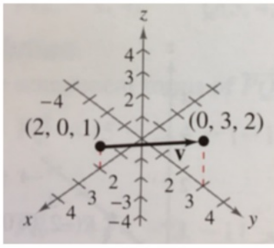
Solutions!

3) Find component form of vector **v**. Sketch the vector with its initial point at the origin.

a)



b)



$\vec{v} = \langle 0-2, 3-0, 2-1 \rangle$

$\vec{v} = \langle 0, 0, -4 \rangle$