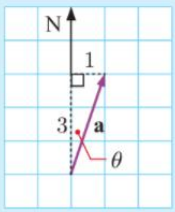
Vector Extension: Problem Solving Name_____

Example 10 Self Tutor

Find the angle that the given vector makes with true north:

a $\mathbf{a} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **b** $\mathbf{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

a



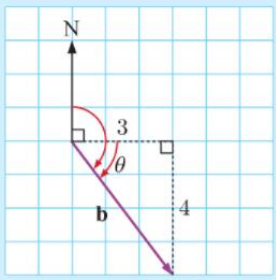
$\tan\theta = \frac{1}{3}$

$\therefore\ \theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\approx 18.4°$

$\therefore$ **a** has the bearing $018.4°$.

b



$\tan\theta = \frac{4}{3}$

$\therefore\ \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\approx 53.1°$

and $\theta + 90° \approx 143.1°$

$\therefore$ **b** has the bearing $143.1°$.

1 Quang runs 10 km to the north, and then 5 km to the west.

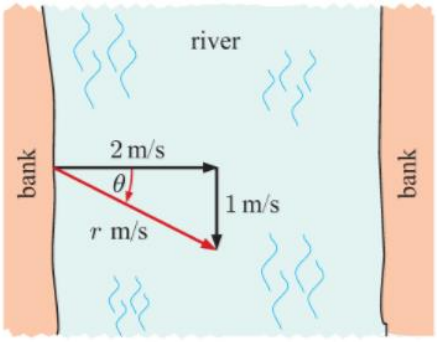
- **a** Write each part of this run in component vector form.
- **b** Find Quang's displacement vector from his starting point.
- **c** Find Quang's distance from his starting point.
- **d** Find Quang's bearing from his starting point.

2 Aleksandra drives with displacement vector $\begin{pmatrix} 14 \\ 2 \end{pmatrix}$. She then changes direction and drives with displacement vector $\begin{pmatrix} 3 \\ -11 \end{pmatrix}$. Units are in kilometres.

- **a** Illustrate Aleksandra's movement on grid paper.
- **b** Find Aleksandra's displacement vector from her starting point.
- **c** How far is Aleksandra from her starting point?
- **d** What is Aleksandra's bearing from her starting point?

3 The diagram shows a river running from north to south at 1 m/s. A swimmer attempts to swim directly out from the bank at 2 m/s.

- **a** What will the actual speed of the swimmer be?
- **b** Find θ.
- **c** On what bearing will the swimmer actually be heading?



4 The Interislander ferry is steaming due east across Cook Strait at a speed of 20 km/h. Johanna is a passenger on the ferry. She walks from the front of the ferry towards the back at a speed of 5 km/h.

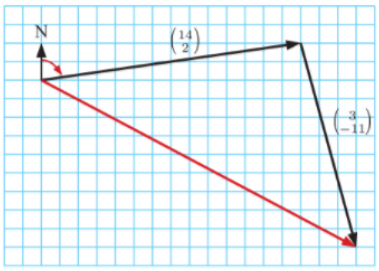
- **a** Find Johanna's resultant speed.
- **b** Is it possible for Johanna to move faster than the ferry? Explain your answer.

Answers:

1 **a** $\begin{pmatrix} 0 \\ 10 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \end{pmatrix}$ **b** $\begin{pmatrix} -5 \\ 10 \end{pmatrix}$ **c** $\sqrt{125}$ km

d $\approx 333.4°$

2 **a**



b $\begin{pmatrix} 17 \\ -9 \end{pmatrix}$ **c** $\sqrt{370} \approx 19.2$ km **d** $\approx 117.9°$

3 **a** $\sqrt{5} \approx 2.24$ m/s **b** $\theta \approx 26.6°$ **c** $\approx 116.6°$

4 **a** 15 km/h

b Yes, if she walks from the back forwards to the front, her relative speed is greater than the ferry's speed.