Welcome Back MYP Math 9!

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 10/23 Topic: Create your own problem!	0 1 2	I'm almost done!
Tuesday Date: 10/24 Topic: Finished create your own problem	0 1 2	
Wednesday Date: ______ Topic: ____________	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

1. Finish self-asses, turn in
2. Warm-up
3. Parallel and Orthogonal Vectors
4. Dot Product
5. Practice

Do: Self-assess and turn in! (4 min)

Reflect: Why did you earn that score?

What evidence supports your score?

Your Level		*****Draw check marks in the appropriate boxes to determine your score.** **Student Reflection:** *(Why did you earn this score?)*

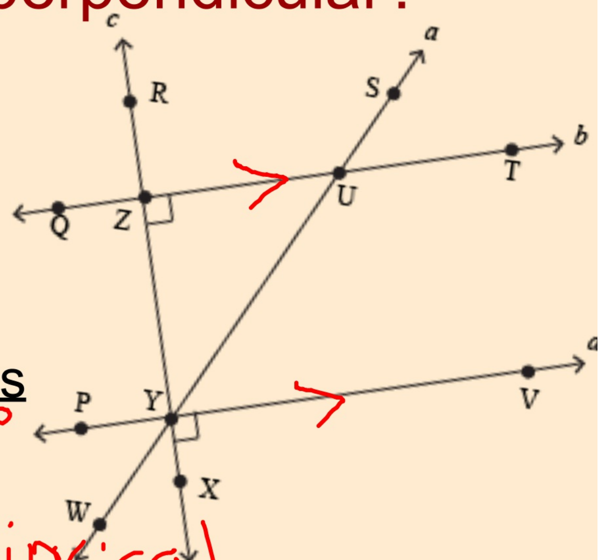
Deadline: Monday all Q1 assessments

End of quarter: Wednesday 11/1

Opener: How do we know if lines are parallel or perpendicular?

Parallel lines

Same slope



Perpendicular lines

Right angle 90°

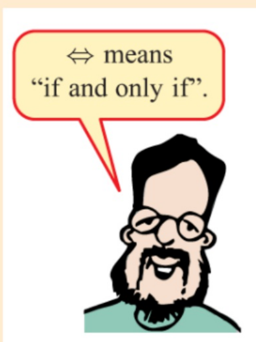
opposite reciprical

Recall from previous learning...

D Chapter 8 PARALLEL AND PERPENDICULAR LINES

For lines which are not horizontal or vertical:

- the lines are **parallel** ⇔ they have **equal gradient**
- the lines are **perpendicular** ⇔ their gradients are **negative reciprocals**.



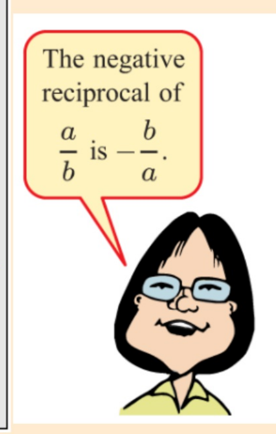
Two non-vertical lines with slopes m_1 and m_2 are:

Parallel
if the lines have the same slope,
$m_1 = m_2$.

Perpendicular
if the slopes are negative reciprocals,

$$m_2 = -\frac{1}{m_1}$$

or equivalently, if $m_1 \cdot m_2 = -1$.



Warm-Up

Sketch the following vectors.

$$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \vec{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}, \text{ and } \vec{d} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

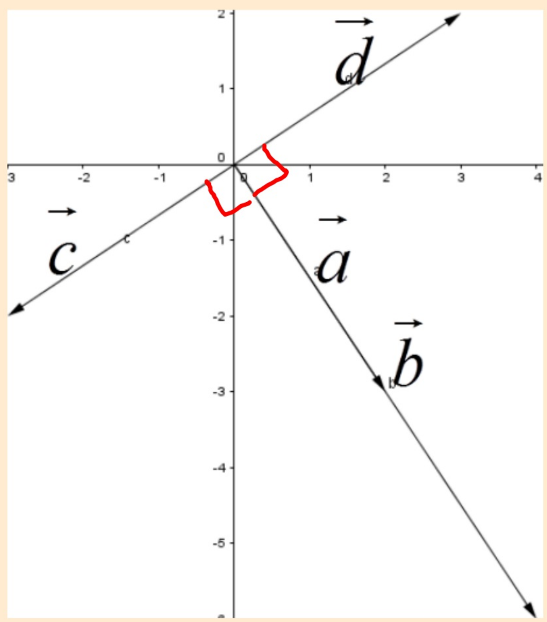
How could we defend whether any of these ***vectors*** are "parallel" or "perpendicular"?

When vectors are sketched, we can see the relationships!

$\vec{b} = 2\vec{a}$

$\vec{b} \parallel \vec{a}$

$\vec{a} \perp \vec{c}$



Parallel

$\vec{c} + \vec{d}$

$\vec{a} + \vec{b}$

Perpendicular

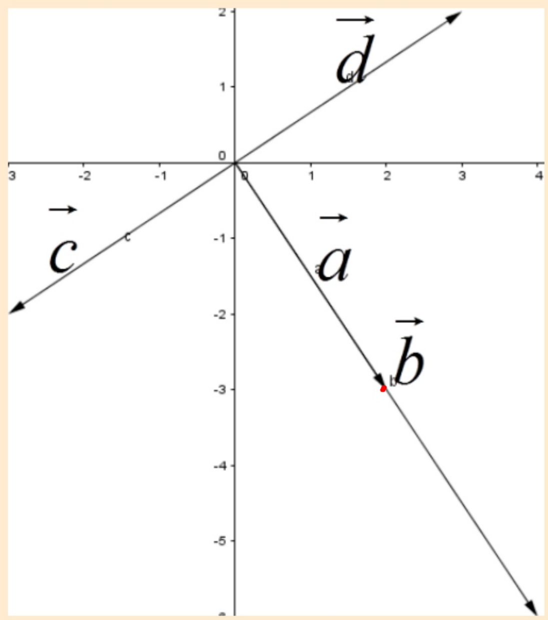
$\vec{c} + \vec{a}$

$\vec{d} + \vec{a}$

$\vec{c} + \vec{b}$

$d + \vec{b}$

When vectors are sketched, we can see the relationships!



Parallel Vectors

Two vectors are ***parallel*** if they are scalar multiples of one another.

How are $\vec{a}$ and $\vec{b}$ parallel?

$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

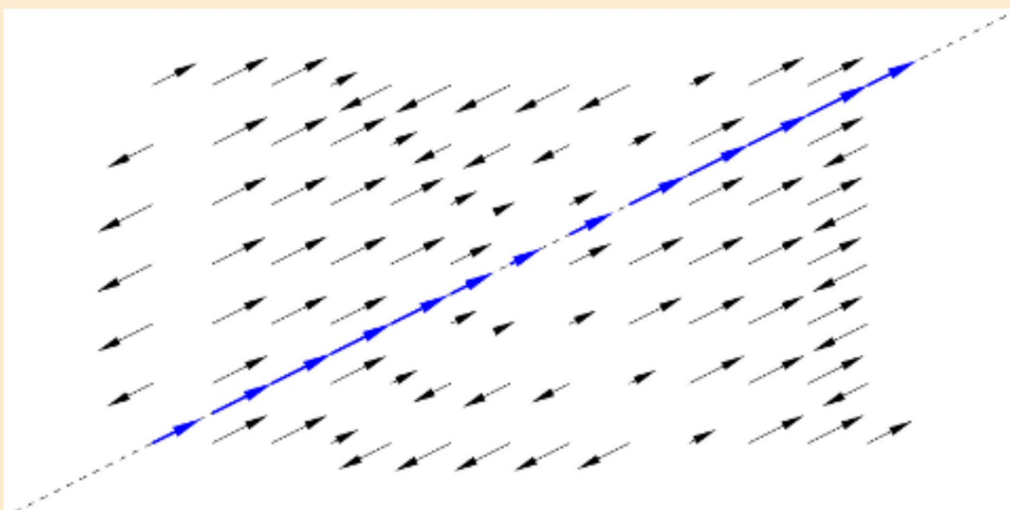
$2\vec{a} = \vec{b}$

How are $\vec{d}$ and $\vec{c}$ parallel?

$\vec{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$, and $\vec{d} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\vec{c} = -\vec{d}$

A bunch of parallel vectors:



(Like parallel lines, they have the same "slopes".)

Orthogonal Vectors

(a.k.a. "perpendicular" or "normal" vectors)

Investigate: How do the orthogonal vectors relate to each other?

$\vec{b} \perp \vec{c}$

$\vec{b} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \vec{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

$4(-3) + -6(-2)$

$-12 + 12 = 0$

$\vec{a} \perp \vec{d}$

$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$2(3) + -3(2)$

$6 - 6 = 0$

Can we figure out a rule for algebraically finding if vectors are orthogonal?

$\vec{b} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}, \vec{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ $-\frac{1}{2}\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Consider slopes!

$\frac{-6}{4} \rightarrow -\frac{4}{6}$ $\frac{-2}{-3} = \frac{2}{3}$

$\vec{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\rightarrow \begin{pmatrix} +3 \\ 2 \end{pmatrix}$

Can we verify if two vectors are orthogonal quickly?

Two vectors are orthogonal if their *dot product* is equal to zero.

The Dot Product: *aka* ***scalar product***

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = ac + bd$$

To find the dot product of two vectors we multiply their *x*-components and the *y*-components, then take the sum.

If ac + bd = 0, then the vectors are orthogonal!

Examples: Find the dot product

$\overrightarrow{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\overrightarrow{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. Find $\overrightarrow{a} \cdot \overrightarrow{b}$.

$\vec{a} \cdot \vec{b} = 2(1) + -3(4)$

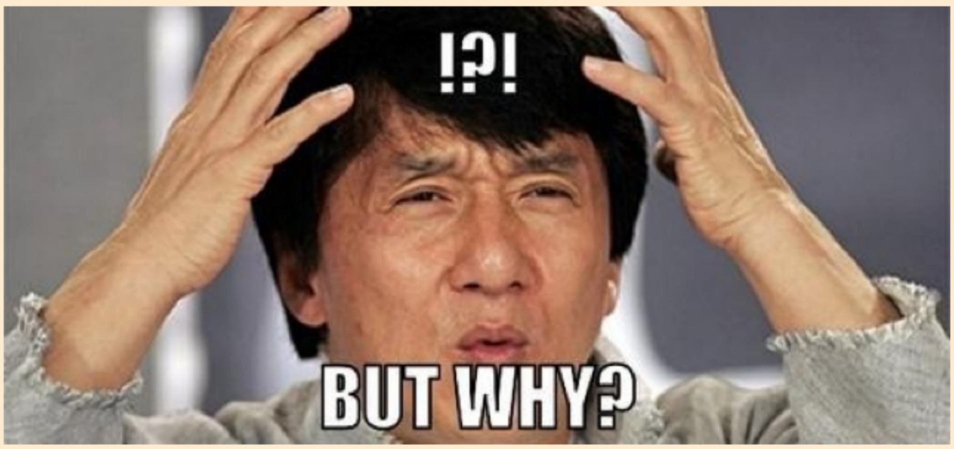
$\vec{a} \cdot \vec{b} = 2 - 12 = -10 \neq 0$

NOT $\perp$

$\overrightarrow{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\overrightarrow{v} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$. Find $\overrightarrow{u} \cdot \overrightarrow{v}$.

$\vec{u} \cdot \vec{v} = 2(6) + -3(4) = 0$

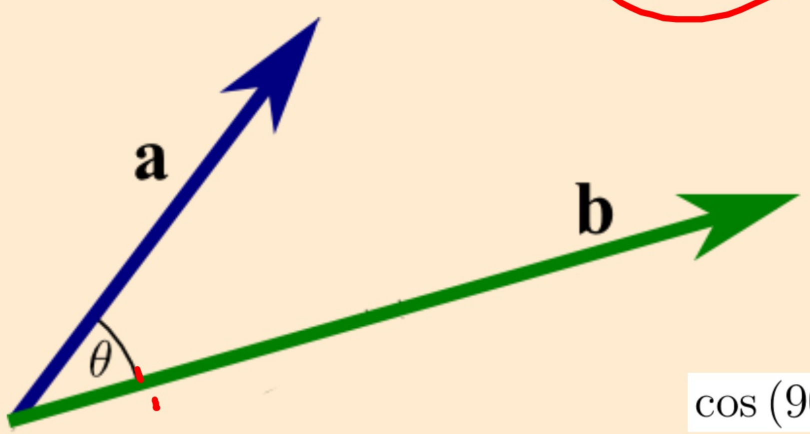
$\vec{u} \perp \vec{v}$



Why does the dot product tell us that two vectors are orthogonal?

The dot product is actually related to the angle between the vectors.

$$\overrightarrow{a} \cdot \overrightarrow{b} = \left|\overrightarrow{a}\right| \left|\overrightarrow{b}\right| \cos(\theta)$$



$$\cos(90^\circ) = 0$$

Proof: From MIT Math

The easiest way to prove this is by using the concepts of vector and dot product.

We represent a point A in the plane by a pair of coordinates, x(A) and y(A) and can define a vector associated with a line segment AB to consist of the pair (x(B)-x(A), y(B)-y(A)).

A vector consists of a pair of numbers, (a,b); the dot product of two vectors (a,b) and (c,d) is the quantity ac + bd.

The Pythagorean Theorem tells us that the square of the length of a line segment is the dot product of its vector with itself.

In general the dot product of two vectors is the product of the lengths of their line segments times the cosine of the angle between them.

Moreover, if ABC is a triangle, the vector $\overrightarrow{AB}$ obeys

$$\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$$

Taking the dot product of $\overrightarrow{AB}$ with itself, we get the desired conclusion.

$$\begin{aligned} AB^2 &= \overrightarrow{AB}.\overrightarrow{AB} = (\overrightarrow{AC} - \overrightarrow{BC}).(\overrightarrow{AC} - \overrightarrow{BC}) \\ &= \overrightarrow{AC}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BC} - 2\overrightarrow{AC}\overrightarrow{BC} \\ &= AC^2 + BC^2 - 2(AC)(BC)\cos C. \end{aligned}$$

http://www-math.mit.edu/~djk/18_01/chapter05/proof05.html

Tonight's Practice:

Vector Dot Product: $\begin{pmatrix} a \\ b \end{pmatrix} \bullet \begin{pmatrix} c \\ d \end{pmatrix}$ = a•c + b•d = scalar number

Parallel Vectors $\vec{m}$ and $\vec{n}$ when k is a scalar: $\vec{m} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\vec{n} = \begin{pmatrix} ka \\ kb \end{pmatrix}$

1) State whether each pair of vectors are orthogonal, parallel, or neither. Show work.

A. $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ 12 \end{pmatrix}$

B. $\overset{\vec{f}}{\begin{pmatrix} 40 \\ 56 \end{pmatrix}}$ and $\overset{\vec{v}}{\begin{pmatrix} 5 \\ 7 \end{pmatrix}}$ $\quad \begin{pmatrix} 5(8) \\ 7(8) \end{pmatrix} = \begin{pmatrix} 40 \\ 56 \end{pmatrix} \quad 8\vec{v} = \vec{f}$

C. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 18 \\ -6 \end{pmatrix}$

D. $\begin{pmatrix} -72 \\ 48 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$

Tonight's Practice:

2) Give a vector parallel to each given vector. Defend that the vectors are parallel.

A. $\begin{pmatrix} 12 \\ 4 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ C. $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$ D. $\begin{pmatrix} -8 \\ -6 \end{pmatrix}$

3) Give a vector orthogonal to each given vector. Use the dot product to defend that your vectors are perpendicular. Show your work.

A. $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ B. $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ C. $\begin{pmatrix} -11 \\ 7 \end{pmatrix}$ D. $\begin{pmatrix} -9 \\ -8 \end{pmatrix}$

Homework Solutions:

1) State whether each pair of vectors are perpendicular, parallel, or neither. Show work

A. $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ 12 \end{pmatrix}$

Parallel?

$3(4)=12$

$4(4)\neq -9$

No

Orthagonal?

$4(-9)+3(12)=?$

$-36+36=?$

$0=?$ YES

B. $\begin{pmatrix} 40 \\ 56 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Parallel

$5(8)=40$

$7(8)=56$

YES

Orthagonal?

$40(5)+56(7)=?$

$200+392=?$

$592=?$

NO

C. $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 18 \\ -6 \end{pmatrix}$

$\parallel$

$3(6)=18$

$-2(6)=-12$

No

$\perp$

$3(18)+(-2)(-6)=?$

$54+12=?$

$66=?$ NO

D. $\begin{pmatrix} -72 \\ 48 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$

$\parallel$

$12(4)=48$

$7(4)=28$

No

$\perp$

$(-72)(7)+48(12)=?$

$-504+576=?$

$72=?$

No

Homework Solutions:

2) Give a vector parallel to each given vector. Defend that the vectors are parallel.

A. $\begin{pmatrix} 12 \\ 4 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ C. $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$ D. $\begin{pmatrix} -8 \\ -6 \end{pmatrix}$

A. $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$4\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} -3 \\ 15 \end{pmatrix}$

$(-3)\begin{pmatrix} 1 \\ -5 \end{pmatrix} = \begin{pmatrix} -3 \\ 15 \end{pmatrix}$

C. $\begin{pmatrix} 1.5 \\ 5 \end{pmatrix}$

$\frac{1}{2}\begin{pmatrix} 3 \\ 10 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 5 \end{pmatrix}$

D. $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$-\frac{1}{2}\begin{pmatrix} -8 \\ -6 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Homework Solutions:

3) Give a vector perpendicular to each given vector. Use the dot product to defend that your vectors are perpendicular. Show your work.

A. $\begin{pmatrix} -4 \\ 0 \end{pmatrix} \perp \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$-4(0) + 0(2) = ?$

$0 + 0 = ?$

0

B. $\begin{pmatrix} 5 \\ 2 \end{pmatrix} \perp \begin{pmatrix} 4 \\ -10 \end{pmatrix}$

$5(4) + 2(-10) = ?$

$20 - 20 = ?$

0

C. $\begin{pmatrix} -11 \\ 7 \end{pmatrix} \perp \begin{pmatrix} 11 \\ 7 \end{pmatrix}$

$-11(11) + 7(7) = ?$

$-77 + 77 = ?$

0

D. $\begin{pmatrix} -9 \\ -8 \end{pmatrix} \perp \begin{pmatrix} -8 \\ 9 \end{pmatrix}$

$-9(-8) + (-8)(9) = ?$

$72 - 72 = ?$

0