Welcome Back MYP Math 9!

Reflect on last night's exercises.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 11/27 Topic: 13F Bearings & Trigonometry	0 1 2	
Tuesday Date: 11/28 Topic: Review Set A/B Trigonometry Applications	0 1 2	
Wednesday Date: _____ Topic: _____	0 1 2	
Thursday Date: _____ Topic: _____	0 1 2	
Friday Date: _____ Topic: _____	0 1 2	

Class Plan:

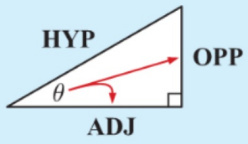
1. Warm-up
2. Investigation: Area & Trig

B Chapter 25 **THE AREA OF A TRIANGLE**

3. Examples
4. Practice

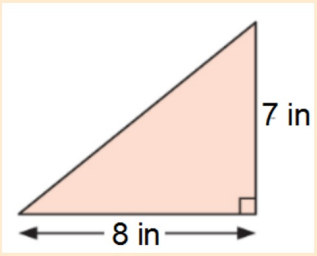
In any right angled triangle with one angle θ, we have:

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}}, \quad \cos\theta = \frac{\text{ADJ}}{\text{HYP}}, \quad \tan\theta = \frac{\text{OPP}}{\text{ADJ}}$$

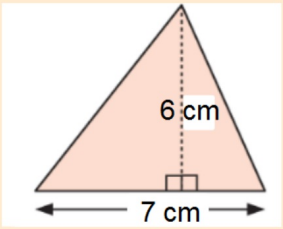


B Chapter 25 **THE AREA OF A TRIANGLE**

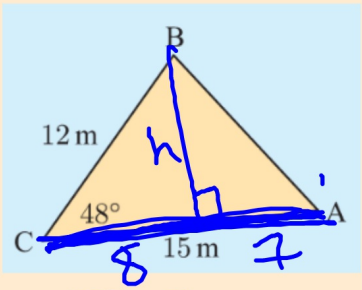
Warm-up: Find the area of each triangle.



28 in²

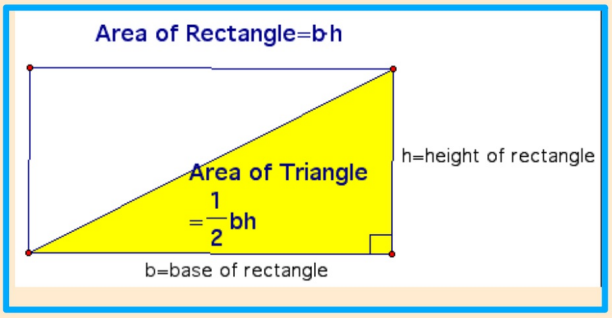


21 cm²

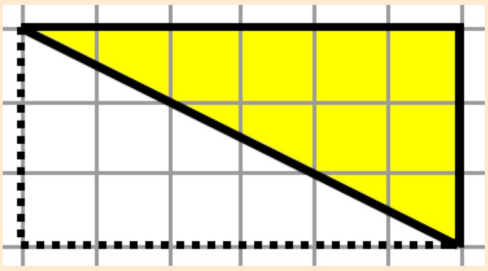


Area of a Triangle

(half a rectangle)



$$A = \frac{1}{2}(6)(3)$$



B Investigation **THE AREA OF A TRIANGLE**

Do: Steps 1 - 4

Whole group:

Go over 1-4,

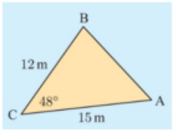
Part 5 (aka "6")

Done?

Keep going! & Help others in steps 1 - 4.

#4: Use exact value of ***h*** to calculate area.

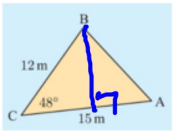
Area of a triangle using trigonometry Name_____



1) Find 48°. How do the side lengths AC and BC relate to the angle 48°?

This angle will be call the _____ angle.

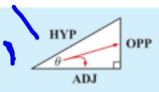
2) Draw segment BD so that BD and AC form a right angle.



3) Use trigonometry to find the length of BD.

In any right angled triangle with one angle θ, we have:

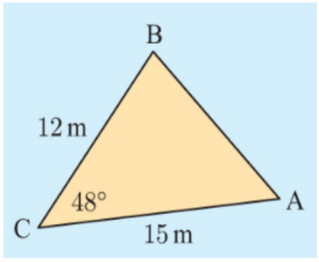
$$\sin\theta = \frac{\text{OPP}}{\text{HYP}}, \quad \cos\theta = \frac{\text{ADJ}}{\text{HYP}}, \quad \tan\theta = \frac{\text{OPP}}{\text{ADJ}}$$



4) Use the length of BD as the height and AC as the base of the triangle. Find the area of triangle ABC.

B Investigation **THE AREA OF A TRIANGLE**

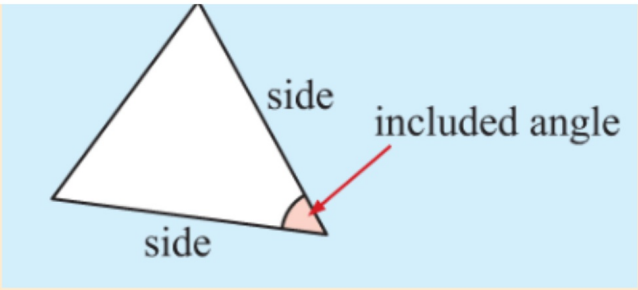
Solution



1) Find 48°. How do the side lengths AC and BC relate to the angle 48°?

Included (in between)

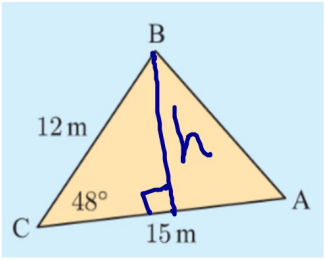
This angle will be call the called **angle.**



B Investigation THE AREA OF A TRIANGLE

Solution

2) Draw segment BD so that BD and AC form a right angle.



3) Use trigonometry to find the length of BD

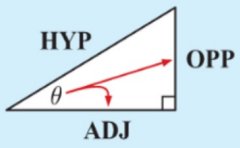
$\sin 48 = \frac{h}{12}$

$h = 12\sin(48°)$

$h \approx 8.92$ m

In any right angled triangle with one angle θ, we have:

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}}, \quad \cos\theta = \frac{\text{ADJ}}{\text{HYP}}, \quad \tan\theta = \frac{\text{OPP}}{\text{ADJ}}$$



B Investigation **THE AREA OF A TRIANGLE**

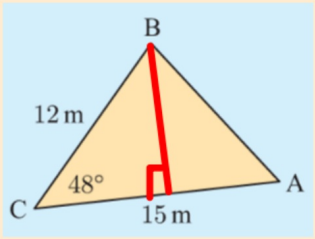
Solution

4) Use the length of BD as the height and AC as the base of the triangle. Find the area of triangle ABC.

$A = \frac{1}{2}(\text{base})(\text{height})$

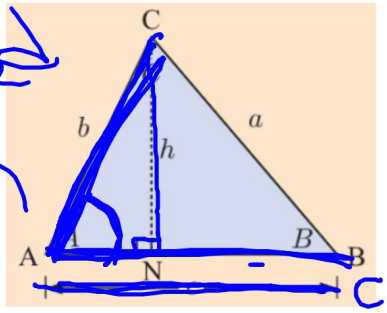
$A = \frac{1}{2}(15)(12\sin 48^\circ)$

$A \approx 66.9 m^2$



B Investigation THE AREA OF A TRIANGLE

5) Use the triangle below to write a formula for area of ΔABC.



Included

a) Use side AB (**side c**) as the ***base*** of the triangle. Find the height (***h***) of ΔABC in terms of ***b*** and the sine of an angle.

$\left(\sin A = \frac{h}{b}\right)b \quad h = b\sin A$

b) Find the area of ΔABC.

$A = \frac{1}{2}(b\sin A)c = \frac{1}{2}b \cdot c\sin A$

B Investigation THE AREA OF A TRIANGLE

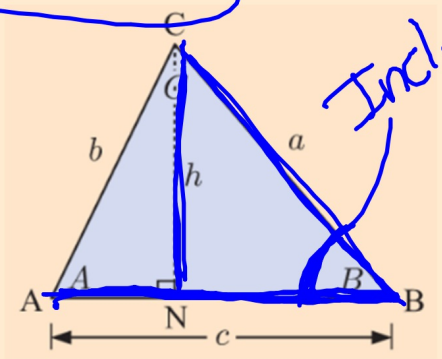
c) Use side AB (**side c**) as the ***base*** of the triangle. Find the height (***h***) of ΔABC in terms of ***a*** and the sine of an angle.

$$\sin B = \frac{opp}{hyp} = \frac{h}{a}$$

d) Find the area of ΔABC.

$$h = a \cdot \sin B$$

$$A = \frac{1}{2} c \cdot a \cdot \sin B$$



B **Investigation** **THE AREA OF A TRIANGLE**

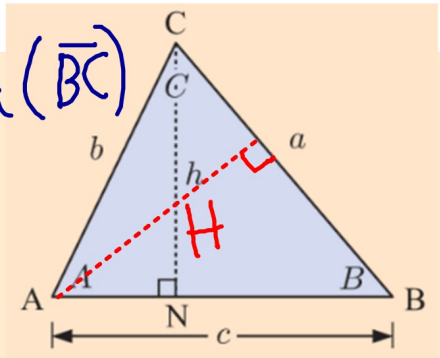
Solution

e) Suppose there is a different base and height drawn in ΔABC. What is the formula of the area of ΔABC?

$\sin C = \frac{H}{b}$ base $= a\,(\overline{BC})$

$H = b(\sin C)$

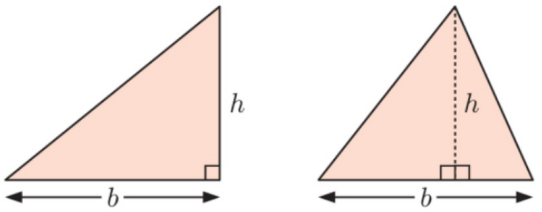
$A = \frac{1}{2}(a)(b)(\sin C)$



B (in textbook) THE AREA OF A TRIANGLE

So far we have only been able to calculate the area of a triangle using the base and the perpendicular height.

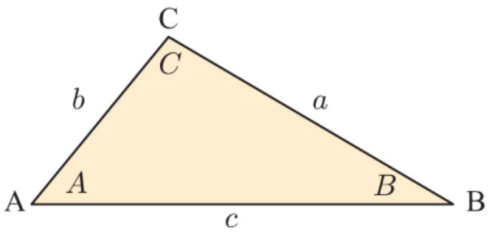
However, we may not know the perpendicular height, especially if the triangle is not right angled.



We can use trigonometry to find the area of a triangle if we are given the lengths of **two sides**, as well as the **included angle** between the sides.

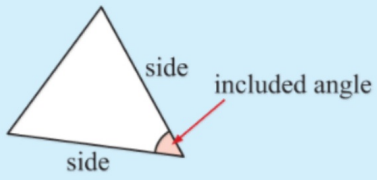
B NOTES **THE AREA OF A TRIANGLE**

$$\text{area} = \frac{1}{2}ab\sin C$$



"one half **a** times **b** times the **sine** of the **angle** between them!"

The **area of a triangle** is a half of the product of two sides and the sine of the included angle.



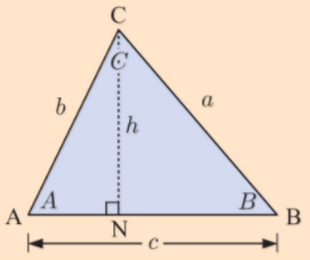
B THE AREA OF A TRIANGLE

Proof (for the case where A is acute):

Consider triangle ABC in which the sides opposite angles A, B, and C are labelled a, b, and c respectively.

Suppose N lies on [AB] such that [CN] is perpendicular to [AB].

The area of $\triangle$ABC$= \frac{1}{2} \times$AB$\times$CN$= \frac{1}{2}ch$ .... (1)



However, in $\triangle$ANC, $\sin A = \dfrac{h}{b}$

$$\therefore \quad h = b \sin A \quad \text{.... (2)}$$

Substituting (2) into (1) gives area $= \frac{1}{2}c(b \sin A) = \frac{1}{2}bc \sin A$

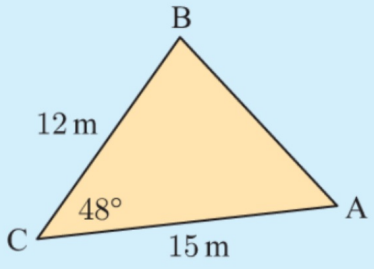
Similarly, if the altitudes from A and B were drawn, it could be shown that area $= \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$.

Break time!

Sign a card for Mr. Ehlke!

Example 3

Find the area of $\triangle ABC$:



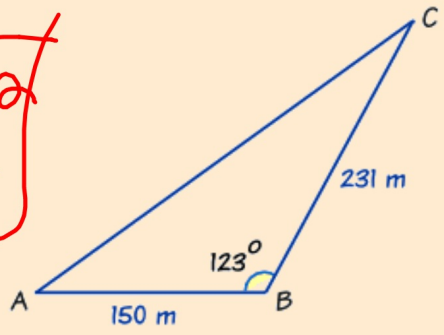
$$\begin{aligned}\text{Area} &= \tfrac{1}{2}ab\sin C \\ &= \tfrac{1}{2} \times 12 \times 15 \times \sin 48^\circ \\ &\approx 66.9 \text{ m}^2\end{aligned}$$

Example: Find the area of triangle ABC.

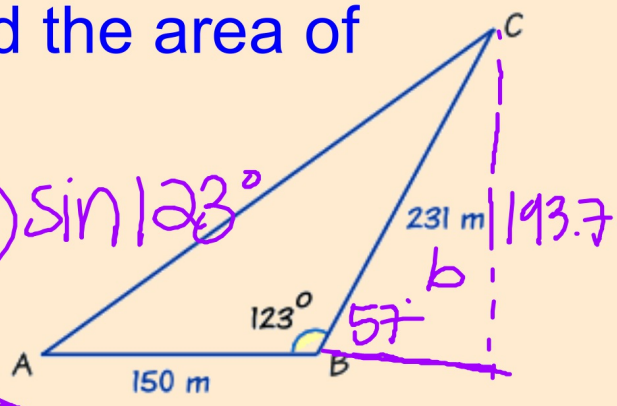
$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(150)(231)\sin 123^\circ$

$A \approx 14530\ m^2$



Example: Find the area of triangle ABC.



$A = \frac{1}{2}(150)(231)\sin 123°$

$A \approx 14{,}529\ m^2$

a

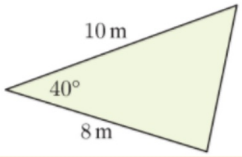
$\sin 57° = \frac{h}{231}$

Exercises...Page 491

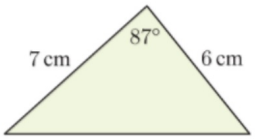
EXERCISE 25B

1 Find the area of:

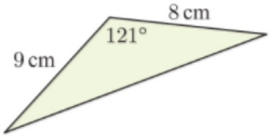
a



b



c



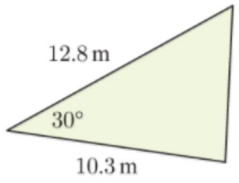
Confident 4-8 ← challenge

Building confidence 1 or 2, 3-5

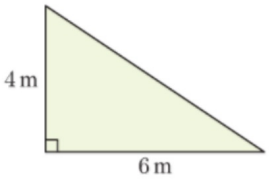
$A = \frac{1}{2}ab\sin C$

Exercises...

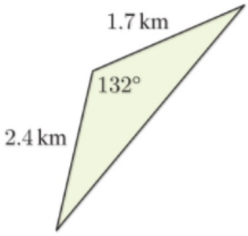
d



e



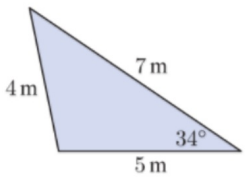
f



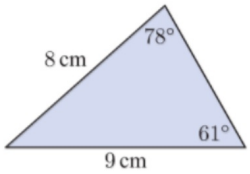
Exercises...

2 Find the area of:

a



b



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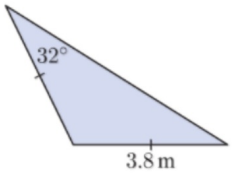
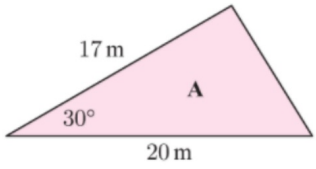
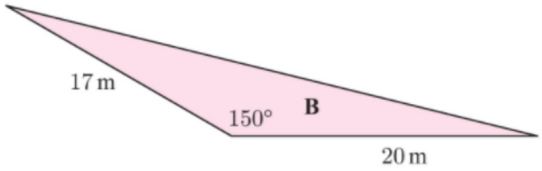


Exercises...

3





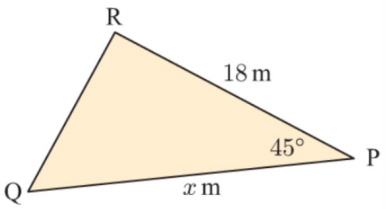
a Find the areas of triangles **A** and **B**.

b Explain why the triangles have the same area.

DE

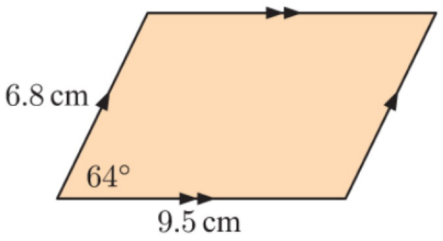
Exercises...

4 The area of this triangle is $127.3\ \text{m}^2$. Find x.



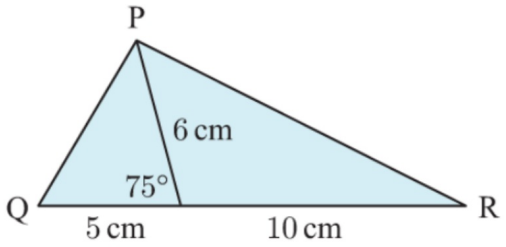
Exercises...

5 Find the area of this parallelogram:

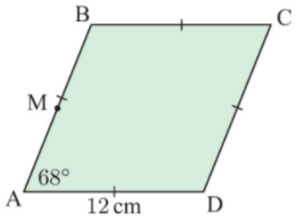


Exercises...

6 Find the area of triangle PQR.



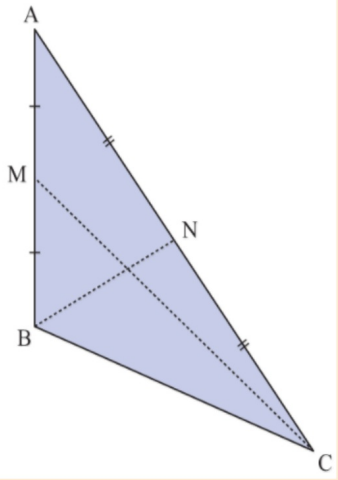
Exercises...

7 

In the rhombus ABCD, M is the midpoint of [AB]. Find the area of $\triangle BMC$.

Exercises...

8 In triangle ABC, M is the midpoint of [AB], and N is the midpoint of [AC]. Show that triangles AMC and ANB have the same area.



Exercises Solutions...

EXERCISE 25B

1 **a** $\approx 25.7\ \text{m}^2$ **b** $\approx 21.0\ \text{cm}^2$ **c** $\approx 30.9\ \text{cm}^2$

d $\approx 33.0\ \text{m}^2$ **e** $12\ \text{m}^2$ **f** $\approx 1.52\ \text{km}^2$

2 **a** $\approx 9.79\ \text{m}^2$ **b** $\approx 23.6\ \text{cm}^2$ **c** $\approx 6.49\ \text{m}^2$

3 **a** Area **A**: $85\ \text{m}^2$, Area **B**: $85\ \text{m}^2$

b Triangles have the same area because the sides used are the same length and $\sin 30° = \sin 150°$.

4 $x \approx 20.0\ \text{m}$ **5** $\approx 58.1\ \text{cm}^2$ **6** $\approx 43.5\ \text{cm}^2$

7 $\approx 33.4\ \text{cm}^2$

8 **Hint:** Let $\text{AM} = a$, $\text{AN} = b$, and $\widehat{\text{BAC}} = \theta$.
Show that both triangles have area $ab \sin\theta$.