Welcome Back MYP Math 9!

Reflect on last night's exercises.

	Assignment Effort Grade (Circle One)	Comments (What was interesting or challenging?)
Monday Date: 11/27 Topic: 13F Bearings & Trigonometry	0 1 2	
Tuesday Date: 11/28 Topic: Review Set A/B Trigonometry Applications	0 1 2	
Wednesday Date: 11/29 Topic: 25B: (Sine Rule) Area of a Triangle	0 1 2	
Thursday Date: ______ Topic: ____________	0 1 2	
Friday Date: ______ Topic: ____________	0 1 2	

Class Plan:

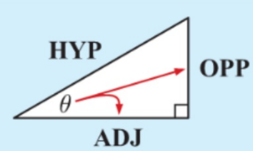
1. Warm-up
2. Law of Sines Investigation

C Chapter 25 **THE SINE RULE**

3. Example
4. Practice

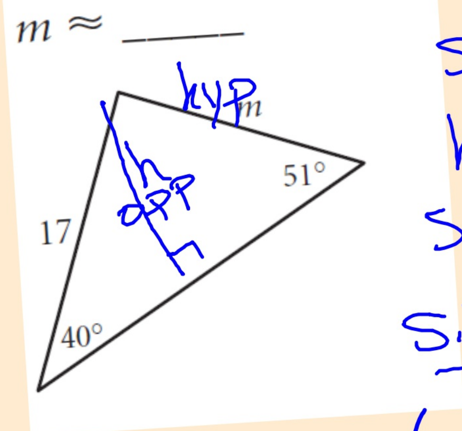
In any right angled triangle with one angle θ, we have:

$$\sin\theta = \frac{\textbf{OPP}}{\textbf{HYP}}, \quad \cos\theta = \frac{\textbf{ADJ}}{\textbf{HYP}}, \quad \tan\theta = \frac{\textbf{OPP}}{\textbf{ADJ}}$$



Warm-up: Solve for m.

Hint: Divide into two right triangles.



$$\sin 40 = \frac{h}{17}$$

$$h = 17(\sin 40)$$

$$\sin 51 = \frac{h}{m}$$

$$\frac{\sin 51}{1} = \frac{17(\sin 40)}{m}$$

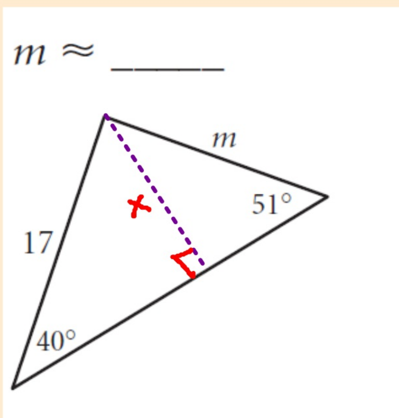
$$\frac{m(\sin 51)}{\sin 51} = \frac{17(\sin 40)}{(\sin 51)}$$

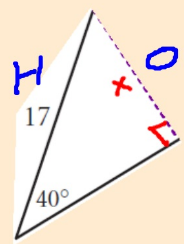
$$m \approx 14.1$$

Warm-up: The Sine Rule a.k.a Law of Sines

Solve for m.

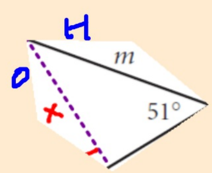
Hint: Divide triangle into two right triangles and use Right Triangle Trigonometric Ratios.





$\sin(40°) = \frac{x}{17}$

$17\sin(40°) = x$



$\sin(51°) = \frac{x}{m}$

$\sin(51°) = \frac{17\sin(40°)}{m}$

$m = \frac{17\sin(40°)}{\sin(51°)}$

$\boxed{m \approx 14.06}$

C THE SINE RULE

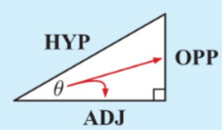
Driving Question:

Given a non-right triangle,
How can we use trig to solve for sides lengths and angle measures?

Recall...

In any right angled triangle with one angle θ, we have:

$$\sin\theta = \frac{\textbf{OPP}}{\textbf{HYP}}, \quad \cos\theta = \frac{\textbf{ADJ}}{\textbf{HYP}}, \quad \tan\theta = \frac{\textbf{OPP}}{\textbf{ADJ}}$$



...only works on ***<u>right triangles</u>***

C THE SINE RULE

The **sine rule** is a set of equations which connects the lengths of the sides of any triangle with the sines of the opposite angles.

INVESTIGATION 1 THE SINE RULE

Derive The Sine Rule (Law of Sines)

1. Guided derivation (handout)

If stuck..The Sine Rule Verification (handout)

(plugging in lengths/angles to see it works)

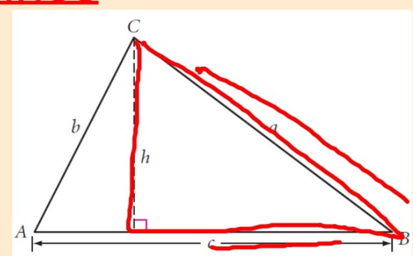
When done: Show teacher your investigation!

INVESTIGATION: Law of Sines

Consider $\triangle ABC$ with height h.

Step 1 | Find h in terms of a and the sine of an angle. B

$$h = a \cdot \sin B$$



Step 2 | Find h in terms of b and the sine of an angle.

$$h = b \cdot \sin A$$

$$\frac{a \cdot \sin B}{a \cdot b} = \frac{b \cdot \sin A}{a \cdot b}$$

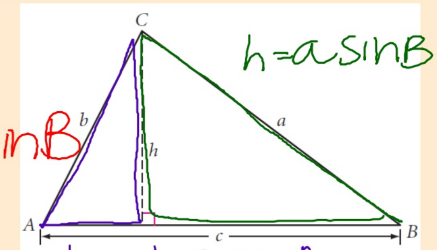
Step 3 | Use algebra to show

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

INVESTIGATION: Law of Sines

Consider $\triangle ABC$ with height h.



Step 1 | Find h in terms of a and the sine of an angle.

$\frac{\sin B}{1} = \frac{h}{a}$ $h = a\sin B$

Step 2 | Find h in terms of b and the sine of an angle.

$\frac{\sin A}{1} = \frac{h}{b}$ $h = b \cdot \sin A$

Step 3 | Use algebra to show

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

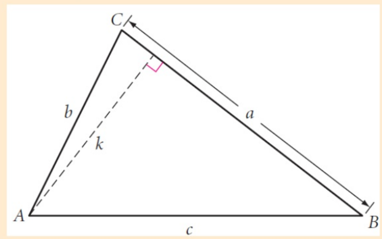
$$\frac{a\sin B}{b} = \frac{b\sin A}{b}$$

$$\frac{a\sin B}{ab} = \frac{\sin A}{a}$$

$$\boxed{\frac{\sin B}{b} = \frac{\sin A}{a}}$$

INVESTIGATION: Law of Sines

Now consider the same $\triangle ABC$ using a different height, k.



Step 4 | Find k in terms of c and the sine of an angle.

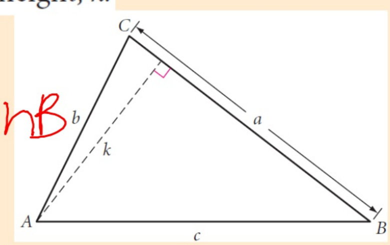
Step 5 | Find k in terms of b and the sine of an angle.

Step 6 | Use algebra to show

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

INVESTIGATION: Law of Sines

Now consider the same $\triangle ABC$ using a different height, k.



Step 4 | Find k in terms of c and the sine of an angle.

$\sin B = \frac{k}{c}$ $\quad k = c\sin B$

Step 5 | Find k in terms of b and the sine of an angle.

$\sin C = \frac{k}{b}$ $\quad k = b\sin C$

Step 6 | Use algebra to show

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{c\sin B}{cb} = \frac{b\sin C}{cb}$$

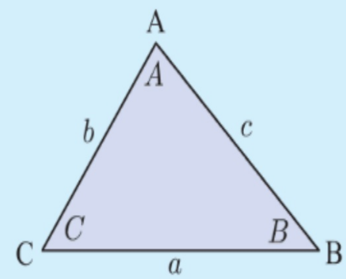
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

RULE (Any Δ)

angle ABC with sides a, b, and c units,
site angles A, B, and C respectively,

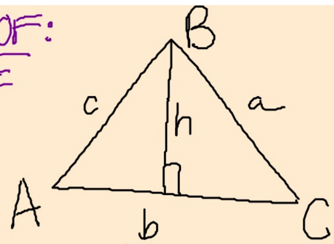
$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

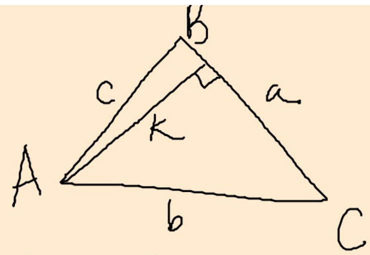


Proof:

https://www.youtube.com/watch?v=APNkWrD-U1k

PROOF:





$$\sin A = \frac{h}{c} \rightarrow h = c \cdot \sin A \qquad \sin B = \frac{k}{c} \rightarrow k = c \cdot \sin B$$

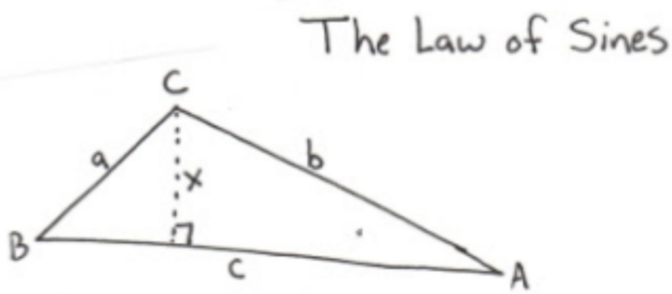
$$\sin C = \frac{h}{a} \rightarrow h = a \cdot \sin C \qquad \sin C = \frac{k}{b} \rightarrow b \cdot \sin C$$

$$\frac{c \cdot \sin A}{ca} = \frac{a \cdot \sin C}{ca} \qquad \frac{c \cdot \sin B}{cb} = \frac{b \cdot \sin C}{cb}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Proof - condensed



$\sin A = \frac{x}{b} \rightarrow x = b \sin A$

$\sin B = \frac{x}{a} \rightarrow x = a \sin B$

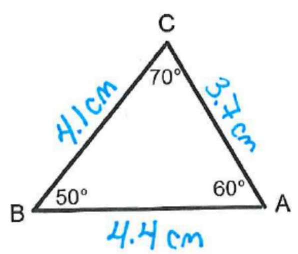
$x = x \rightarrow b \sin A = a \sin B \rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$

Textbook Investigation...

INVESTIGATION 1 **THE SINE RULE**

1. Explore measurements

<u>Step 1:</u> Use your ruler to measure the three sides of triangle *ABC*, in centimeters.



$AB =$ 4.4 cm
$BC =$ 4.1 cm
$CA =$ 3.7 cm

<u>Step 2:</u> Identify the side opposite of each angle.

a) $\angle A$ is opposite side $\overline{BC}$ b) $\angle B$ is opposite side $\overline{CA}$ c) $\angle C$ is opposite side $\overline{AB}$

Textbook Investigation...

2. Explore ratios

<u>Step 3:</u> Substitue the appropriate measures of triangle *ABC* into the ratios below and use your calculator to find the answer. (*The first one is partially started for you...*) **Round to 3 decimals.**

a) $\frac{\sin A}{BC} = \frac{\sin 60^\circ}{4.1} \approx .21$

b) $\frac{\sin B}{CA} = \frac{\sin 50}{3.7} \approx .21$

c) $\frac{\sin C}{AB} = \frac{\sin 70}{4.4} \approx .21$

3. Conclude

<u>Conclusions</u>

a) Examine step 3 and the final measures for parts ***a, b, c.*** What do you notice?

The ratios are equal

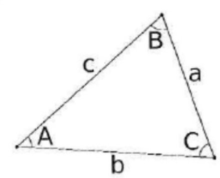
Textbook Investigation...

3. Conclude

Generalize

b) Using the triangle at the right, complete the relationship you noticed in the **conclusion.**

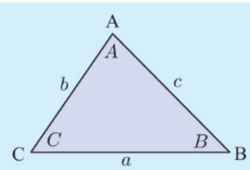
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



THE SINE RULE

In any triangle ABC with sides a, b, and c units, and opposite angles A, B, and C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



Law of Sines to Find Sides

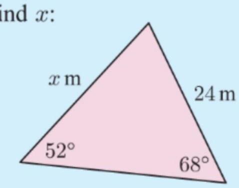
Example #1

If we are given two angles and one side of a triangle, we can use the sine rule to find another side length.

When finding sides, we use the form $\frac{a}{\sin A} = \frac{b}{\sin B}$ so that the unknown is in the numerator.

Example 4

Find x:



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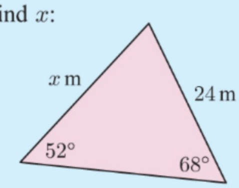


$$\frac{\sin(68°)}{x} = \frac{\sin(52°)}{24}$$

$$\frac{24 \cdot \sin(68°)}{\sin(52°)} = \frac{x \cdot \sin(52°)}{\sin(52°)}$$

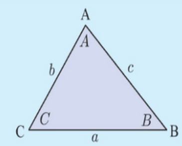
$$\frac{24 \cdot \sin(68°)}{\sin(52°)} = x$$

$$28.24 \text{ m} \approx x$$

Example #2

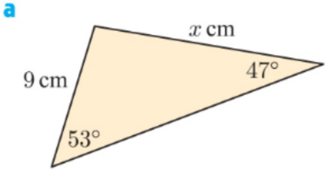
In any triangle ABC with sides a, b, and c units, and opposite angles A, B, and C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



1 Find the value of x:

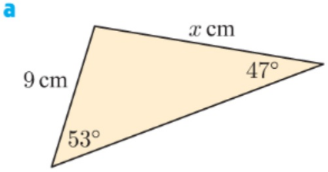
a



Example #2

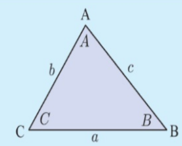
1 Find the value of x:

a



In any triangle ABC with sides a, b, and c units, and opposite angles A, B, and C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



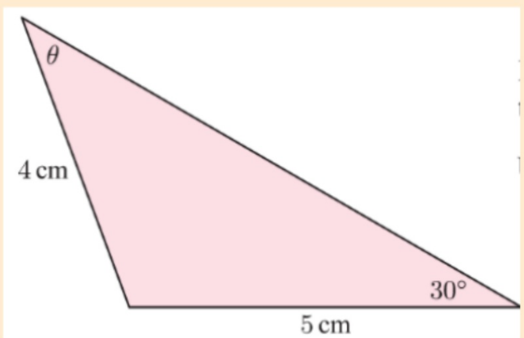
$$\frac{\sin 43}{9} = \frac{\sin 53}{X}$$

$$X \cdot \sin 43 = 9 \cdot \sin 53$$

$$X = \frac{9 \cdot \sin 53}{\sin 43} \approx \boxed{10.5\text{cm}}$$

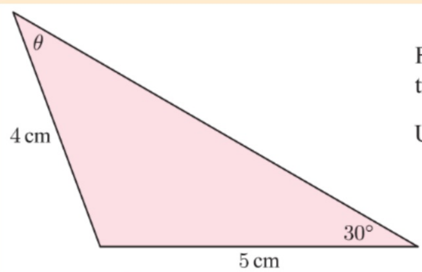
Law of Sines to Find Angles

Example #3 *Solve for theta.*



Law of Sines to Find Angles

Example #3 *Solve for theta.*



For example, suppose we want to find the angle θ in the triangle alongside.

Using the sine rule, $\frac{\sin\theta}{5} = \frac{\sin 30^\circ}{4}$

$\therefore \ \sin\theta = \frac{5 \times \sin 30^\circ}{4}$

$\therefore \ \sin\theta = \frac{5}{8}$

$$5 \cdot \frac{\sin(\theta)}{5} = \frac{\sin(30^\circ)}{4} \cdot 5$$

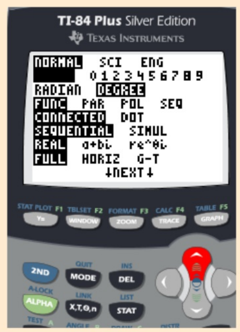
$$\sin(\theta) = \frac{5\sin(30^\circ)}{4}$$

$$\theta = \sin^{-1}\left(\frac{5\sin(30^\circ)}{4}\right)$$

$$\boxed{\theta \approx 38.68^\circ}$$

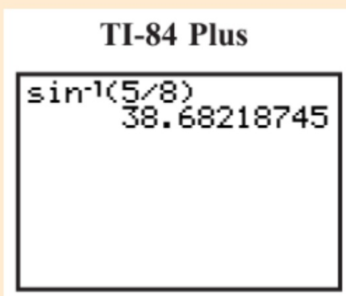
Calculator

1. **MODE**: DEGREE

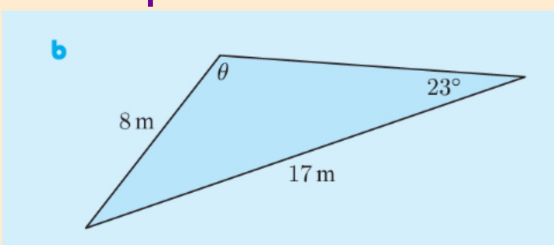


Example:

2. **2ND - SIN**



Example #4



Note: Theta appears obtuse.

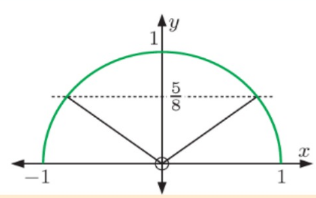
Note: Acute vs. Obtuse

If you are solving for acute angle...
simply take inverse of ratio (like example)

If you are solving for an obtuse angle...
(1) take inverse of ratio
(2) subtract from 180 degrees

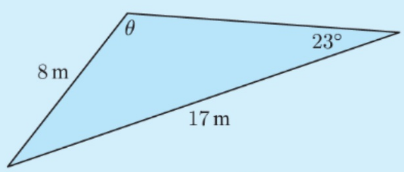
textbook explanation...

However, we know from **Section A** that there is also an obtuse angle with sine $\frac{5}{8}$. So, the two solutions to $\sin\theta = \frac{5}{8}$ are $\theta \approx 38.7°$ and $180° - 38.7° \approx 141.3°$.



Textbook Solution

b



b Using the sine rule,

$$\frac{\sin\theta}{17} = \frac{\sin 23^\circ}{8}$$

$$\therefore \quad \sin\theta = \frac{17 \times \sin 23^\circ}{8}$$

$$\therefore \quad \theta = 180^\circ - \sin^{-1}\left(\frac{17 \times \sin 23^\circ}{8}\right)$$

{as θ is clearly obtuse}

$$\therefore \quad \theta \approx 123.9^\circ$$

Exercises for tonight... p.494-6

"I am still trying to sort this out"

25C.1 (#2, #3 a) 25C.2 (#1, pick 2)

"I am feeling fairly good with the examples. I want to get more practice with problems like that!"

25C.1 (#2, #3) 25C.2 (#1, pick 2, #2 a&b)

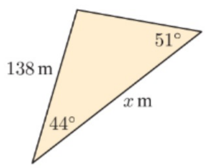
"I am feeling really comfortable & am ready for a challenge"

25C.1 (#3, #4) 25C.2 (#2)

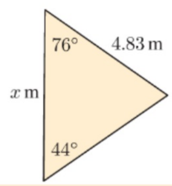
EXERCISE 25C.1

2 Find the value of x:

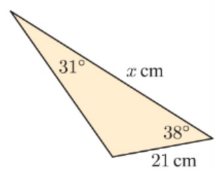
a



b



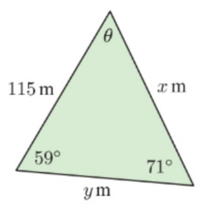
c



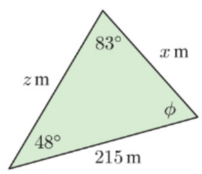
EXERCISE 25C.1

3 Find *all* unknown angles and sides of:

a



b



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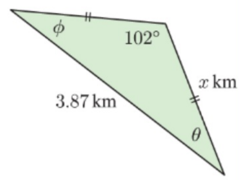

EXERCISE 25C.1

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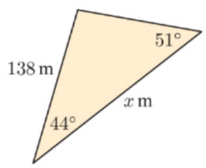
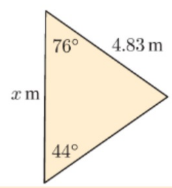
a



b



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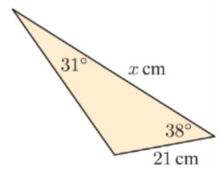
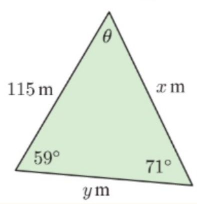
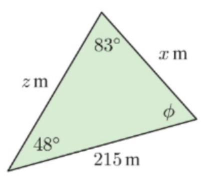


3 Find *all* unknown angles and sides of:

a



b



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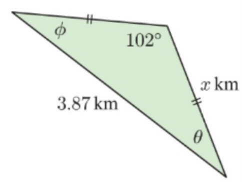
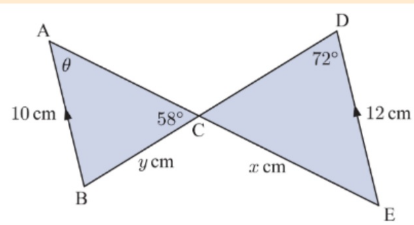


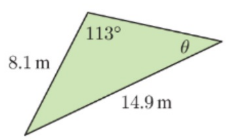
4 Find the unknown side lengths and angles:



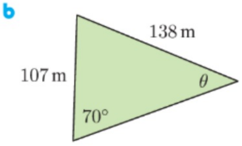
EXERCISE 25C.2

1 The diagrams below are drawn approximately to scale. Use the sine rule to find θ, rounding your answer to one decimal place.

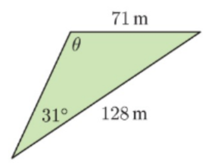
a



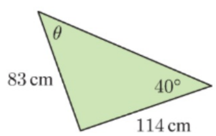
b



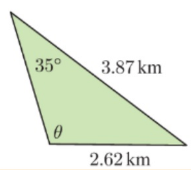
c



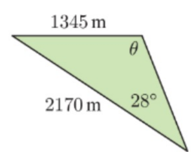
d



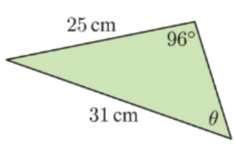
e



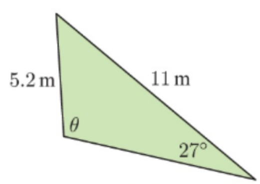
f



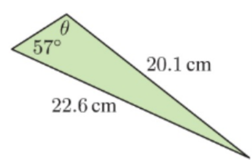
g



h

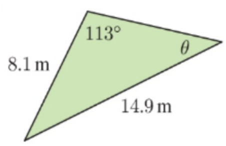


i

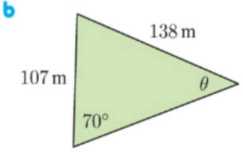


The diagrams below are drawn approximately to scale. Use the sine rule to find θ, rounding your answer to one decimal place.

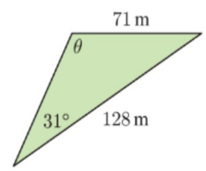
a



b



c



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 31}{71} = \frac{\sin\theta}{128}$$

$$\frac{128(\sin 31)}{71} = \frac{71(\sin\theta)}{71}$$

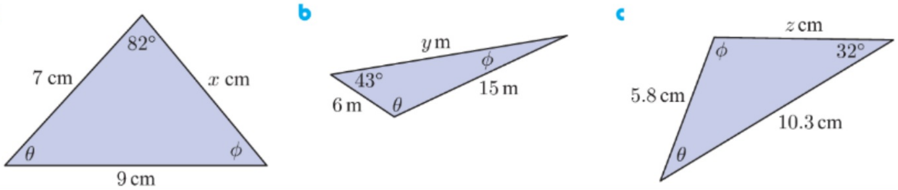
$$\sin^{-1}\left(\frac{128(\sin 31)}{71}\right) = \sin^{-1}(\sin\theta)$$

$\theta = \sin$

$$180 - 68 = 112°$$

2 The diagrams below are drawn approximately to scale. Find all unknown angles and sides.

a **b** **c**



Solutions

EXERCISE 25C.1

1 a $x \approx 9.83$ b $x \approx 11.9$ c $x \approx 8.31$

2 a $x \approx 177$ b $x \approx 6.02$ c $x \approx 38.1$

3 a $\theta = 50°$, $x \approx 104$, $y \approx 93.2$

b $\phi = 49°$, $x \approx 161$, $z \approx 163$

c $\phi = \theta = 39°$, $x \approx 2.49$

4 $\theta = 50°$, $x \approx 13.5$, $y \approx 9.03$

EXERCISE 25C.2

1 a $\theta \approx 30.0°$ b $\theta \approx 46.8°$ c $\theta \approx 112°$

d $\theta \approx 62.0°$ e $\theta \approx 122°$ f $\theta \approx 131°$

g $\theta \approx 53.3°$ h $\theta \approx 106°$ i $\theta \approx 109°$

2 a $\phi \approx 50.4°$, $\theta \approx 47.6°$, unknown side ≈ 6.71 cm

b $\phi \approx 15.8°$, $\theta \approx 121°$, unknown side ≈ 18.8 m

c $\phi \approx 110°$, $\theta \approx 38.2°$, unknown side ≈ 6.77 cm