

Trigonometric Relationship & tool to solve for sides and angles!

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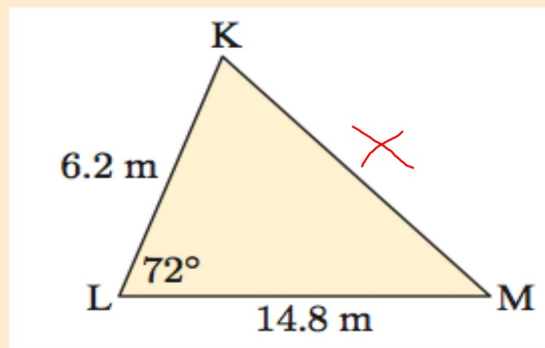
THE COSINE RULE

The **cosine rule** relates the three sides of a triangle and one of the angles.

INVESTIGATION 2

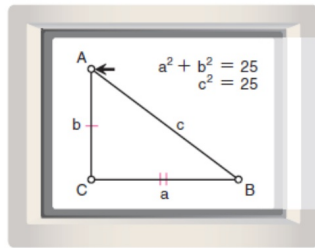
THE COSINE RULE

Question... Why can't we use the law of sines to find the length of KM ?

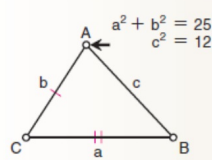


If we are given two sides of a triangle and the included angle, we can use the cosine rule to find the third side.

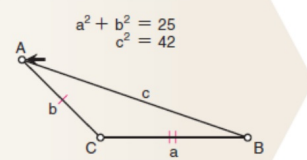
Law of Cosines



In this right triangle
 $c^2 = a^2 + b^2$



In this acute triangle
 $c^2 < a^2 + b^2$



In this obtuse triangle
 $c^2 > a^2 + b^2$

$c^2 = a^2 + b^2 - \text{something}$
 $c^2 = a^2 + b^2 + \text{something}$

INVESTIGATION 2

THE COSINE RULE

In this Investigation you will discover the cosine rule for a triangle which is acute angled.

What to do:

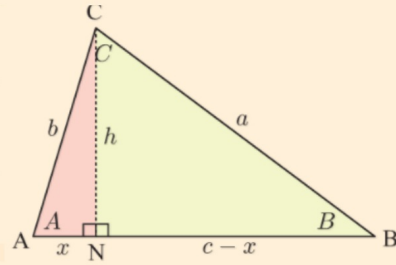
- 1 For the figure alongside, copy and complete:

$$\text{In } \triangle ANC, \quad b^2 = \dots + \dots \quad \{\text{Pythagoras}\}$$

$$\therefore h^2 = \dots - \dots \quad (1)$$

$$\text{In } \triangle BNC, \quad a^2 = \dots + \dots \quad \{\text{Pythagoras}\}$$

$$\therefore h^2 = \dots - \dots \quad (2)$$



By equating (1) and (2), $a^2 - \dots = b^2 - \dots$

Expanding and collecting terms gives $a^2 = \dots$

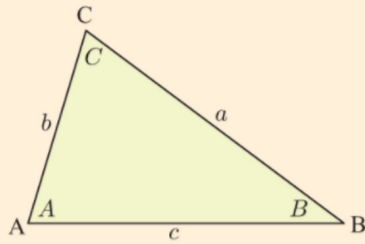
$$\text{But in } \triangle ACN, \quad \cos A = \frac{\dots}{\dots}$$

$$\therefore x = \dots$$

$$\therefore a^2 = \dots$$

$$\therefore a^2 = b^2 + c^2 - 2bc \cos A.$$

2 Copy and complete the following:



In any triangle with sides a , b , and c units and opposite angles A , B , and C respectively,

$$a^2 = b^2 + c^2 - \dots$$

$$b^2 = a^2 + c^2 - \dots$$

$$c^2 = a^2 + b^2 - \dots$$

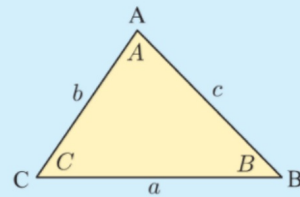
THE COSINE RULE

In any triangle ABC with sides a , b , and c units, and opposite angles A , B , and C respectively,

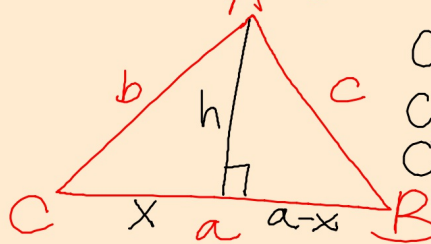
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C.$$



PROOF: GIVEN $\triangle ABC$, SHOW
 $C^2 = a^2 + b^2 - 2ab \cos C$



$$C^2 = h^2 + (a-x)^2$$

$$C^2 = h^2 + (a-x)(a-x)$$

$$C^2 = h^2 + a^2 - ax - ax + x^2$$

$$C^2 = a^2 + \underbrace{h^2 + x^2} - 2ax$$

$$C^2 = a^2 + b^2 - 2ax$$

$$C^2 = a^2 + b^2 - 2ab \cos C$$

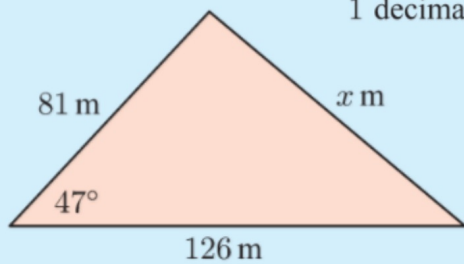
$$b^2 = h^2 + x^2$$

$$\cos C = \frac{x}{b}$$

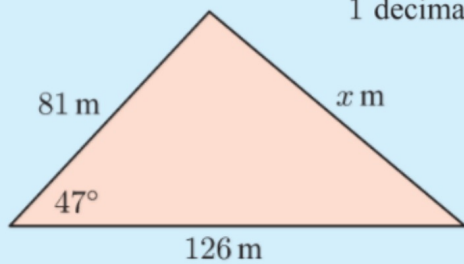
$$x = b \cos C$$



Example 1: Use the cosine rule to find x , rounding your answer to 1 decimal place.



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Using the cosine rule, $x^2 = 126^2 + 81^2 - 2 \times 126 \times 81 \times \cos 47^\circ$

$$\therefore x = \sqrt{126^2 + 81^2 - 2 \times 126 \times 81 \times \cos 47^\circ}$$

$$\therefore x \approx 92.3$$

Solve for Angle C

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Solve for Angle C

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\frac{-a^2 \quad -a^2}{-a^2 \quad -a^2}$$

$$\frac{c^2 - a^2 = b^2 - 2ab \cos C}{-b^2 \quad -b^2}$$

$$\frac{c^2 - a^2 - b^2 = -2ab \cos C}{-2ab \quad -2ab}$$

$$\frac{c^2 - a^2 - b^2}{-2ab} = \cos C$$

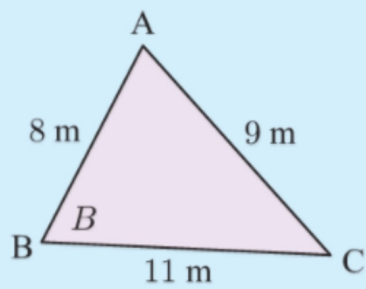
$$\cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right) = m\angle C$$

If we know all three side lengths of a triangle, we can use the cosine rule to find any of the angles. To do this, we rearrange the original cosine rule formulae:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}, \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

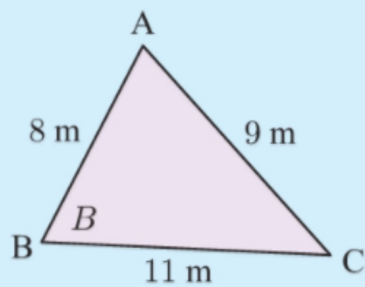
Example 2:

Find the measure of angle B in the given figure.



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Find the measure of angle B in the given figure.



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\therefore \cos B = \frac{11^2 + 8^2 - 9^2}{2 \times 11 \times 8}$$

$$\therefore \cos B = \frac{104}{176}$$

$$\therefore B = \cos^{-1} \left(\frac{104}{176} \right)$$

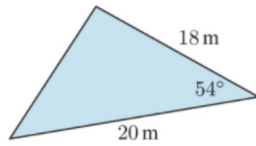
$$\therefore B \approx 53.8^\circ$$

Exercises... 25D (see 25E as well!)

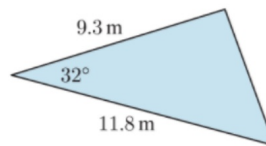
EXERCISE 25D

1 Find the length of the remaining side of each triangle:

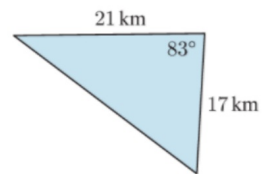
a



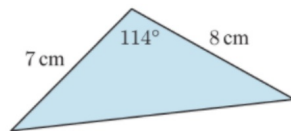
b



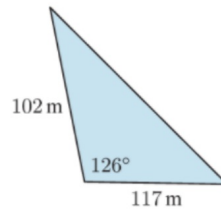
c



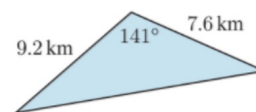
d



e



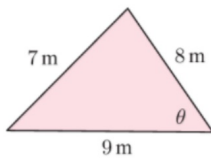
f



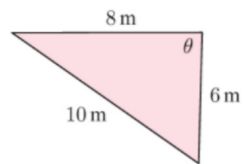
Exercises...

2 Find, to 1 decimal place, the value of the unknown:

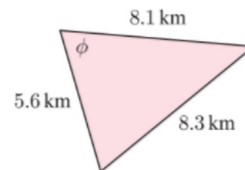
a



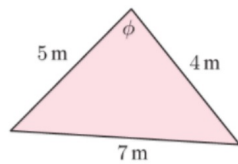
b



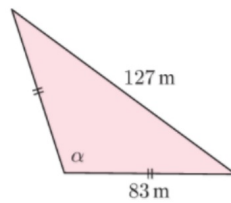
c



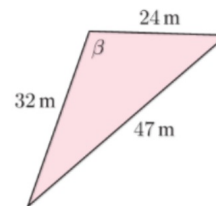
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e



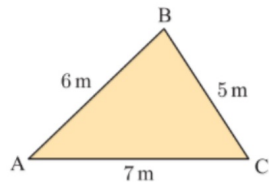
f



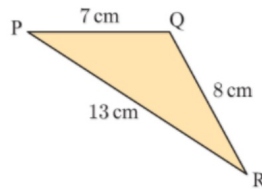
Exercises...

3 Find all of the angles of each triangle:

a

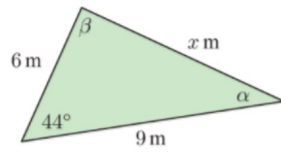


b

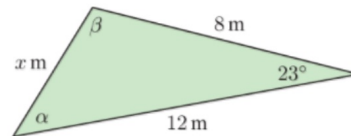


4 Find the measure of all unknown sides and angles:

a

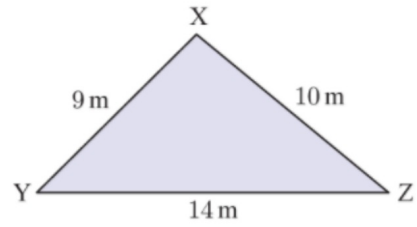


b



Exercises...

- 5 Find the area of this triangle:



Exercises...Solutions

EXERCISE 25D

- 1** **a** ≈ 17.3 m **b** ≈ 6.29 m **c** ≈ 25.4 km
 d ≈ 12.6 cm **e** ≈ 195 m **f** ≈ 15.8 km
- 2** **a** $\theta \approx 48.2^\circ$ **b** $\theta = 90^\circ$ **c** $\phi \approx 72.0^\circ$
 d $\phi \approx 101.5^\circ$ **e** $\alpha \approx 99.8^\circ$ **f** $\beta \approx 113.4^\circ$
- 3** **a** $A \approx 44.4^\circ$, $B \approx 78.5^\circ$, $C \approx 57.1^\circ$
 b $P \approx 32.2^\circ$, $Q = 120^\circ$, $R \approx 27.8^\circ$
- 4** **a** $x \approx 6.27$, $\alpha \approx 41.7^\circ$, $\beta \approx 94.3^\circ$
 b $x \approx 5.59$, $\alpha \approx 34.0^\circ$, $\beta \approx 123.0^\circ$
- 5** ≈ 44.8 m²